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Abstract

Labor market frictions are crucial for the equity premium in production economies. A dynamic stochastic general equilibrium model with recursive utility, search frictions, and capital accumulation yields a high equity premium of 4.26% per annum, a stock market volatility of 11.8%, and a low average interest rate of 1.59%, while simultaneously retaining plausible business cycle dynamics. The equity premium and stock market volatility are strongly countercyclical, while the interest rate and consumption growth are largely unpredictable. Because of wage inertia, dividends are procyclical despite consumption smoothing via capital investment. The welfare cost of business cycles is huge, 29%.

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1 Introduction

Mehra and Prescott (1985) show that the equity premium (the average difference between the stock market return and risk-free interest rate) in the Arrow-Debreu economy is negligible relative to its historical average. Subsequent studies have largely succeeded in specifying preferences and cash flow dynamics to explain the equity premium in endowment economies (Campbell and Cochrane 1999; Bansal and Yaron 2004; Barro 2006). Unfortunately, explaining the equity premium in general equilibrium production economies, in which cash flows are endogenously determined, has proven more challenging.[1] To date, no consensus general equilibrium framework has emerged. Consequently, finance and macroeconomics have largely developed in a dichotomic fashion. Finance specifies "exotic" preferences and exogenous cash flow dynamics to match asset prices but ignore firms, whereas macroeconomics analyzes full-fledged general equilibrium production economies but ignore asset prices with simple preferences (Christiano, Eichenbaum, and Evans 2005; Smets and Wouters 2007).

This macro-finance dichotomy has left many important questions unanswered. What are the microeconomic foundations underlying the exogenously specified, often complicated cash flow dynamics in finance models (Bansal, Kiku, and Yaron 2012; Nakamura, Steinsson, Barro, and Ursua 2013)? What are the essential ingredients in the production side that can endogenize the key elements of cash flow dynamics necessary to explain the equity premium? To what extent do time-varying risk premiums matter quantitatively for macroeconomic dynamics? How large is the welfare cost of business cycles in an equilibrium production economy that replicates the equity premium?

Our long-term objective is to formulate a unified equilibrium theory that explains the equity

[1] Rouwenhorst (1995) shows that the standard real business cycle model cannot explain the equity premium because optimal investment of firms provides a powerful mechanism for the representative household to smooth consumption, yielding little consumption risks. With internal habit preferences, Jermann (1998) and Boldrin, Christiano, and Fisher (2001) adopt capital adjustment costs and cross-sector immobility, respectively, to restrict consumption smoothing to match the equity premium. However, both models struggle with excessively high interest rate volatilities because of low elasticities of intertemporal substitution. Using recursive utility, Tallarini (2000) shows that increasing risk aversion in a real business cycle model improves its fit with the market Sharpe ratio but does not materially affect macro quantities. However, the model fails to match the equity premium and its volatility. Kaltenbrunner and Lochstoer (2010) show that long-run consumption risks arise endogenously from consumption smoothing in a real business cycle model, but the model falls short in explaining the equity premium and stock market volatility.

premium puzzle, while simultaneously retaining plausible business cycle dynamics. We embed the standard Diamond-Mortensen-Pissarides search model of equilibrium unemployment into a dynamic stochastic general equilibrium framework with recursive utility and capital accumulation. When calibrated to the consumption growth volatility in the Jordà-Schularick-Taylor macrohistory database, the model succeeds in yielding an equity premium (adjusted for financial leverage) of 4.26% per annum, which is close to 4.36% in the historical data. The average interest rate is 1.59%, which is not far from 0.82% in the data (the difference is insignificant). However, the stock market volatility is 11.8% in the model, which, although sizeable, is still significantly lower than 16% in the data. Also, the model implies strong time series predictability for stock market excess returns and volatilities, some predictability for consumption volatility, and weak to no predictability for consumption growth and the real interest rate. Quantitatively, the model explains stock market predictability but somewhat overstates consumption growth predictability in the historical data.

Wage inertia plays a key role in our model. To keep the model parsimonious, we work with the Nash wage that features a low bargaining weight of workers and a high flow value of unemployment. This calibration implies a wage elasticity to labor productivity of 0.256 in the model. Hagedorn and Manovskii (2008) estimate this elasticity to be 0.449 in the U.S. postwar 1951–2004 sample. Drawing from historical sources (Kendrick 1961; Officer 2009), we extend the Hagedorn-Manovskii evidence and estimate the wage elasticity to be 0.267 in the historical 1890–2015 sample.

Unlike endowment economies, in which cash flows can be exogenously specified to fit the equity premium, the main challenge facing general equilibrium production economies is that cash flows are often endogenously countercyclical. With frictionless labor market, wages equal the marginal product of labor, which is almost as procyclical as output and profits (output minus wages). Alas, investment is more procyclical than output because of consumption smoothing, making dividends (profits minus investment) countercyclical (Kaltenbrunner and Lochstoer 2010). With wage inertia, profits are more procyclical than output. The magnified procyclicality of profits is sufficient to overcome the procyclicality of investment (and vacancy costs) to render dividends procyclical. In addition, wage inertia is stronger in bad times, with smaller profits. This time-varying wage inertia amplifies risks and risk premiums in bad times, giving rise to time series predictability of the equity premium and stock market volatility. Finally, despite adjustment costs, investment still absorbs a large amount of shocks, making consumption growth and the interest rate largely unpredictable.

Risk aversion strongly affects quantity dynamics, in contrast to Tallarini (2000). In comparative statics, reducing risk aversion from 10 to 5 lowers the equity premium to 0.54% per annum. More important, consumption volatility falls from 5.13% to 3.93%, and consumption disaster probability from 5.83% to 3.82%. A lower discount rate raises the marginal benefit of hiring and reduces the unemployment rate from 8.63% to 4.63%. Echoing Hall's (2017) partial equilibrium analysis, our general equilibrium results indicate that it is imperative to study quantity and price dynamics jointly.

Our model predicts downward-sloping term structures of the equity premium and equity volatility, consistent with Binsbergen, Brandt, and Koijen (2012). Intuitively, when the search economy slides into a disaster, short-maturity dividend strips take a big hit because of inertial wages. In contrast, long-maturity strips are less impacted because disasters are followed by subsequent recoveries. Also, despite recursive utility calibrated to feature the early resolution of uncertainty, the timing premium (the fraction of the consumption stream that the investor is willing to trade for the early resolution) is only 15.3% in our model. Intuitively, the expected consumption growth and conditional consumption volatility in our search economy are much less persistent than those typically calibrated in the long-run risks model, thereby avoiding its pitfall of implausibly high timing premiums.

Finally, the average welfare cost of business cycles is huge, 29.1%, which is more than 580 times of 0.05% in Lucas (2003). More important, the welfare cost is countercyclical with a long, right tail. In simulations, its 5th percentile of 18.4% is not far below its median of 24.4%, but its 95th percentile is substantially higher, 56.3%. As such, countercyclical policies aimed to dampen disaster risks are even more important than what the average welfare cost estimate of 29.1% would suggest.

We view this work as a solid progress report toward a unified theory of asset prices and business cycles. This holy grail of macro-finance has proven elusive for decades. Petrosky-Nadeau, Zhang, and Kuehn (2018) show that the standard search model exhibits disaster dynamics. However, their asset pricing results are very limited because of no capital. Capital is particularly important for asset prices because it represents the core challenge of endogenizing procyclical dividends in production economies (Jermann 1998). We embed capital and recursive utility simultaneously to study asset prices with production, while overcoming ensuing heavy computational burden. Bai (2020) incorporates defaultable bonds to study the credit spread. We instead focus on the equity premium puzzle.

Embedding rare disasters per Rietz (1988) and Barro (2006) into a real business cycle model, Gourio (2012) shows that aggregate risks significantly affect quantity dynamics. Echoing Gourio, we show that Tallarini's (2000) separation between prices and quantities does not hold under more general settings. However, we differ from Gourio in that disasters arise endogenously from labor market frictions. We also endogenize operating leverage via wage inertia to explain the equity premium and stock market volatility. In contrast, Gourio relies on exogenous leverage to generate volatile cash flows but "does not address the volatility of the unlevered return on capital (p. 2737)." Kilic and Wachter (2018) embed the exogenous Rietz-Barro diasters into the search model of unemployment to yield a high unemployment volatility and examine its relation with a high stock market volatility. While our work differs from Kilic and Wachter's in many details, the most important distinction is, again, the endogenous nature of disasters in our setting.[2]

The rest of the paper is organized as follows. Section 2 constructs the general equilibrium model. Section 3 presents the model's key quantitative results, including the equity premium, stock market volatility, and their predictability. Section 4 examines several additional implications of the model, including the welfare cost of business cycles. Section 5 concludes. Appendix A describes

[2]Several recent studies have examined the equity premium in general equilibrium production economies but outside the disasters framework. Croce (2014) embeds exogenous long-run productivity risks into a production model. While long-run risks increase the equity premium, the return volatility is only about one quarter of that in the data. Kung and Schmid (2015) endogenize long-run productivity risks via firms' research and development in an endogenous growth model. Favilukis and Lin (2016) examine the impact of infrequent wage renegotiations in a stochastic growth model with long-run productivity risks. Finally, Chen (2017) examines a general equilibrium production model with external habit and emphasizes the role of endogenous consumption volatility risks.

our algorithm. A separate Internet Appendix details data, derivations, and supplementary results.

2 A General Equilibrium Production Economy

The economy is populated by a representative household and a representative firm. Following Merz (1995), we assume that the household has perfect consumption insurance. A continuum of mass one of members is either employed or unemployed at any point in time. The fractions of employed and unemployed workers are representative of the population at large. The household pools the income of all the members together before choosing per capita consumption.

The household maximizes recursive utility, denoted J_t, given by:

$$J_t = \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left(E_t\left[J_{t+1}^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \tag{1}$$

in which C_t is consumption, β time preference, ψ the elasticity of intertemporal substitution, and γ relative risk aversion (Epstein and Zin 1989; Weil 1990). The consumption Euler equation is:

$$1 = E_t[M_{t+1} r_{St+1}], \tag{2}$$

in which r_{St+1} is the firm's stock return, and M_{t+1} the household's stochastic discount factor:

$$M_{t+1} \equiv \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}} \left(\frac{J_{t+1}}{E_t\left[J_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}. \tag{3}$$

The riskfree rate is $r_{ft+1} = 1/E_t[M_{t+1}]$, which is known at the beginning of t.

The representative firm uses capital, K_t, and labor, N_t, to product output, Y_t, with a constant elasticity of substitution (CES) production technology (Arrow et al. 1961):

$$Y_t = X_t\left[\alpha\left(\frac{K_t}{K_0}\right)^{\omega} + (1-\alpha)N_t^{\omega}\right]^{\frac{1}{\omega}}, \tag{4}$$

in which α is the distribution parameter, and $e \equiv 1/(1-\omega)$ the elasticity of substitution between capital and labor. When ω approaches zero in the limit, equation (4) reduces to the special case of the Cobb-Douglas production function with a unitary elasticity. To facilitate the model's calibration, we work with the "normalized" CES function in equation (4), in which $K_0 > 0$ is a scaler that makes the unit of K_t/K_0 comparable to the unit of N_t (Klump and La Grandville 2000). Specifically, we calibrate K_0 to ensure that $1-\alpha$ matches the average labor share in the data (Section 3.2). Doing so eliminates the distribution parameter, α, as a free parameter.[3] Finally, the CES production function is of constant returns to scale, $Y_t = K_t \partial Y_t / \partial K_t + N_t \partial Y_t / \partial N_t$ (the Internet Appendix).

The firm takes the aggregate productivity, X_t, as given, with $x_t \equiv \log(X_t)$ governed by:

$$x_{t+1} = (1-\rho_x)\overline{x} + \rho_x x_t + \sigma_x \epsilon_{t+1}, \tag{5}$$

in which $\overline{x}$ is unconditional mean, $0 < \rho_x < 1$ persistence, $\sigma_x > 0$ conditional volatility, and ϵ_{t+1} an independently and identically distributed (i.i.d.) standard normal shock. We scale $\overline{x}$ to make the average marginal product of labor around one in simulations to ease the interpretation of parameters.

The representative firm posts a number of job vacancies, V_t, to attract unemployed workers, U_t. Vacancies are filled via the Den Haan-Ramey-Watson (2000) matching function:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^{\iota} + V_t^{\iota})^{1/\iota}}, \tag{6}$$

in which $\iota > 0$. This matching function has the desirable property that matching probabilities fall between zero and one. In particular, define $\theta_t \equiv V_t/U_t$ as the vacancy-unemployment (V/U) ratio. The probability for an unemployed worker to find a job per unit of time (the job finding rate) is $f(\theta_t) \equiv G(U_t, V_t)/U_t = \left(1 + \theta_t^{-\iota}\right)^{-1/\iota}$. The probability for a vacancy to be filled per unit of time (the vacancy filling rate) is $q(\theta_t) \equiv G(U_t, V_t)/V_t = (1 + \theta_t^{\iota})^{-1/\iota}$. It follows that $f(\theta_t) = \theta_t q(\theta_t)$ and $q'(\theta_t) < 0$. An increase in the scarcity of unemployed workers relative to vacancies makes it harder to fill a vacancy. As such, θ_t is labor market tightness, and $1/q(\theta_t)$ the average duration of vacancies.

The representative firm incurs costs in posting vacancies. The unit cost per vacancy is given by

[3] In contrast, in prior applications of the CES production function in asset pricing, the distribution parameter, α, is largely treated as a free parameter (Favilukis and Lin 2016; Kilic and Wachter 2018; Bai 2020).

$\kappa > 0$. The marginal cost of hiring, $\kappa/q(\theta_t)$, increases with the mean duration of vacancies, $1/q(\theta_t)$. In expansions, the labor market is tighter for the firm (θ_t is higher), and the vacancy filling rate, $q(\theta_t)$, is lower. As such, the marginal cost of hiring is procyclical.

Jobs are destroyed at a constant rate of s per period. Employment, N_t, evolves as:

$$N_{t+1} = (1 - s)N_t + q(\theta_t)V_t, \tag{7}$$

in which $q(\theta_t)V_t$ is the number of new hires. Population is normalized to be one, $U_t + N_t = 1$, meaning that N_t and U_t are also the rates of employment and unemployment, respectively.

The firm incurs adjustment costs when investing. Capital accumulates as:

$$K_{t+1} = (1 - \delta)K_t + \Phi(I_t, K_t), \tag{8}$$

in which δ is the capital depreciation rate, I_t is investment, and

$$\Phi_t \equiv \Phi(I_t, K_t) = \left[a_1 + \frac{a_2}{1 - 1/\nu}\left(\frac{I_t}{K_t}\right)^{1-1/\nu}\right] K_t, \tag{9}$$

is the installation function with the supply elasticity of capital $\nu > 0$. We set $a_1 = \delta/(1 - \nu)$ and $a_2 = \delta^{1/\nu}$ to ensure no adjustment costs in the deterministic steady state (Jermann 1998). This parsimonious parametrization involves only one free parameter, ν.

The dividends to the firm's shareholders are given by:

$$D_t = Y_t - W_t N_t - \kappa V_t - I_t, \tag{10}$$

in which W_t is the equilibrium wage rate. Taking W_t, the household's stochastic discount factor, M_{t+1}, and the vacancy filling rate, $q(\theta_t)$, as given, the firm chooses optimal investment and the optimal number of vacancies to maximize the cum-dividend market value of equity, S_t:

$$S_t \equiv \max_{\{V_{t+\tau}, N_{t+\tau+1}, I_{t+\tau}, K_{t+\tau+1}\}_{\tau=0}^{\infty}} E_t\left[\sum_{\tau=0}^{\infty} M_{t+\tau} D_{t+\tau}\right], \tag{11}$$

subject to equations (7) and (8) as well as a nonnegativity constraint on vacancies, $V_t \geq 0$. Because $q(\theta_t) > 0$, $V_t \geq 0$ is equivalent to $q(\theta_t)V_t \geq 0$. In contrast, equation (9) implies that $\partial\Phi_t/\partial I_t = a_2(I_t/K_t)^{-1/\nu}$, which goes to infinity as investment, I_t, goes to zero. As such, I_t is always positive.

From the first-order conditions for I_t and K_{t+1}, we obtain the investment Euler equation:

$$\frac{1}{a_2}\left(\frac{I_t}{K_t}\right)^{1/\nu} = E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + \frac{1}{a_2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^{1/\nu}(1-\delta+a_1) + \frac{1}{\nu-1}\frac{I_{t+1}}{K_{t+1}}\right]\right]. \tag{12}$$

Equivalently, $E_t[M_{t+1}r_{Kt+1}] = 1$, in which r_{Kt+1} is the investment return:

$$r_{Kt+1} \equiv \frac{\partial Y_{t+1}/\partial K_{t+1} + (1/a_2)(1-\delta+a_1)\left(I_{t+1}/K_{t+1}\right)^{1/\nu} + (1/(\nu-1))(I_{t+1}/K_{t+1})}{(1/a_2)\left(I_t/K_t\right)^{1/\nu}}. \tag{13}$$

Let λ_t be the multiplier on $q(\theta_t)V_t \geq 0$. From the first-order conditions with respect to V_t and N_{t+1}, we obtain the intertemporal job creation condition:

$$\frac{\kappa}{q(\theta_t)} - \lambda_t = E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial N_{t+1}} - W_{t+1} + (1-s)\left(\frac{\kappa}{q(\theta_{t+1})} - \lambda_{t+1}\right)\right]\right]. \tag{14}$$

Equation (14) implies that $E_t[M_{t+1}r_{Nt+1}] = 1$, in which r_{Nt+1} is the hiring return:

$$r_{Nt+1} \equiv \frac{\partial Y_{t+1}/\partial N_{t+1} - W_{t+1} + (1-s)\left(\kappa/q(\theta_{t+1}) - \lambda_{t+1}\right)}{\kappa/q(\theta_t) - \lambda_t}. \tag{15}$$

Finally, the optimal vacancy policy also satisfies the Kuhn-Tucker conditions:

$$q(\theta_t)V_t \geq 0, \quad \lambda_t \geq 0, \quad \text{and} \quad \lambda_t q(\theta_t)V_t = 0. \tag{16}$$

Under constant returns to scale, the stock return of the representative firm, r_{St+1}, is a weighted average of the investment return and the hiring return (the Internet Appendix):

$$r_{St+1} = \frac{\mu_{Kt}K_{t+1}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}}r_{Kt+1} + \frac{\mu_{Nt}N_{t+1}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}}r_{Nt+1}, \tag{17}$$

in which the shadow value of capital, μ_{Kt}, equals the marginal cost of investment, $(1/a_2)(I_t/K_t)^{(1/\nu)}$, and the shadow value of labor, μ_{Nt}, equals the marginal cost of hiring, $\kappa/q(\theta_t) - \lambda_t$.

The equilibrium wage rate is determined endogenously by applying the sharing rule per the outcome of a generalized Nash bargaining process between employed workers and the firm (Pissarides 2000). Let $\eta \in (0,1)$ be the workers' relative bargaining weight and b the workers' flow value of unemployment. The equilibrium wage rate is given by (the Internet Appendix):

$$W_t = \eta\left(\frac{\partial Y_t}{\partial N_t} + \kappa\theta_t\right) + (1-\eta)b. \tag{18}$$

The wage rate increases with the marginal product of labor, $\partial Y_t/\partial N_t$, and the vacancy cost per unemployed worker, $\kappa\theta_t$. Intuitively, the more productive the workers are, and the more costly for the firm to fill a vacancy, the higher the wage rate is for the employed workers. In addition, the workers' bargaining weight, η, affects the wage elasticity to labor productivity. The lower η is, the more the equilibrium wage is tied with the constant b, reducing the wage elasticity to productivity.

The competitive equilibrium consists of optimal investment, I_t, vacancy posting, V_t, multiplier, λ_t, and consumption, C_t, such that (i) C_t satisfies the consumption Euler equation (2); (ii) I_t satisfies the investment Euler equation (12), and V_t and λ_t satisfy the intertemporal job creation condition (14) and the Kuhn-Tucker conditions (16), while taking the stochastic discount factor, M_{t+1}, in equation (3), and the equilibrium wage in equation (18) as given; and (iii) the goods market clears:

$$C_t + \kappa V_t + I_t = Y_t. \tag{19}$$

Solving for the competitive equilibrium is computationally challenging. We adapt Petrosky-Nadeau and Zhang's (2017) globally nonlinear projection method with parameterized expectations to our setting (Appendix A). The state space consists of employment, capital, and productivity. We parameterize the conditional expectation in the right-hand side of equation (14) and solve for the indirect utility, investment, and conditional expectation functions from equations (1), (12), and (14). We use Rouwenhorst's (1995) discrete state method to approximate the log productivity with 17 grid points. We use the finite element method with cubic splines on 50 nodes on the employment space and 50 nodes on the capital space and take their tensor product on each grid point of productivity. To solve the resulting system of 127,500 equations, we use the derivative-free fixed point iteration with a small damping parameter (Judd, Maliar, Maliar, and Valero 2014).

3 Quantitative Results

We describe our data in Section 3.1 and calibrate the model in Section 3.2. We examine the model's unconditional moments in Section 3.3, sources of the equity premium in Section 3.4, and time-varying risks and risk premiums in Section 3.5. Finally, we report comparative statics in Section 3.6.

3.1 Data

For business cycle moments, we use the historical cross-country panel of output, consumption, and investment from Jordà, Schularick, and Taylor (2017), who in turn build on Barro and Ursúa (2008). For asset pricing moments, we use the Jordà et al. (2019) cross-country panel. We obtain the data from the Jordà-Schularick-Taylor macrohistory database.[4] The database contains macro and return series for 17 developed countries. The only missing series are returns for Canada, which we supplement from the Dimson-Marsh-Staunton (2002) database purchased from Morningstar. Although the Dimson-Marsh-Staunton database contains asset prices and the Barro-Ursúa database provides consumption and output series for more countries, we mainly rely on the Jordà-Schularick-Taylor database because it provides quantities and asset prices for the same set of countries. More important, it also contains investment series. The sample starts as early as 1871 and ends in 2015.[5]

Table 1 shows the properties of log growth rates of real consumption, output, and investment per capita in the historical panel. From Panel A, the consumption growth is on average 1.62% per annum, with a volatility of 5.45%, and a skewness of −0.67, all averaged across 17 countries. The first-order autocorrelation is 0.12. The consumption volatility exhibits a substantial amount

[4]http://www.macrohistory.net/data.

[5]More precisely, in the Jordà-Schularick-Taylor database, the consumption, output, and investment series start in 1870, meaning that their growth rates start in 1871. The quantities series end in 2016, but asset prices end in 2015.

of cross-country variation, ranging from 2.76% in UK to 8.72% in Belgium. The first-order autocorrelations also varies widely across countries, ranging from -0.2 in Switzerland to 0.39 in France.

From Panel B, averaged across countries, the output growth has a mean of 1.78% per annum, a volatility of 5.1%, a skewness of -1.06, and a first-order autocorrelation of 0.18. The output volatility of 5.1% is lower than the consumption volatility of 5.45%.[6] Finally, Panel C shows that the investment growth volatility is high on average, 13.5% per annum, varying from 8.2% in Netherlands to 24.4% in the United States. Its first-order autocorrelation is 0.13.

Following Barro (2006), we calculate leverage-adjusted equity premium as one minus financial leverage times the unadjusted equity premium and calculate leverage-adjusted market volatility as the standard deviation of the leverage-weighted average of stock market and bill returns. We set leverage to be 0.29, which is the mean market leverage ratio in a cross-country panel reported in Fan, Titman, and Twite (2012). From Panel D, the leverage-adjusted equity premium is 4.36% per annum on average, varying from 2.71% in Portugal to 6.8% in Finland. The leverage-adjusted stock market volatility is on average 16%, ranging from 11.9% in Denmark to 23% in Finland. For the real interest rate, the mean is only 0.82% across countries. Finland has the lowest mean interest rate of -0.74%, whereas Denmark has the highest of 3.08%. Finally, the real interest rate volatility is on average 7.3%, ranging from 4.32% in Australia to 13.22% in Germany.[7]

The asset pricing literature has traditionally focused only on the postwar U.S. data. Table S2 in the Internet Appendix reports basic macro and asset pricing moments in the 1950–2015 cross-country sample. The real consumption, output, and investment growth rates are less volatile, with standard deviations of 2.4%, 2.47%, and 7.06% per annum, respectively, averaged across countries. The U.S. macro volatilities are lower still at 1.73%, 2.21%, and 4.98%, respectively. Relatedly,

[6] As explained in Barro and Ursúa (2008), government purchases rise sharply in wartime, decrease consumption relative to output, and raise the consumption volatility relative to the output volatility.

[7] When calculating the return moments, we require stock, bond, and bill returns to be nonmissing for a given year in a given country. Relaxing this restriction has little impact on the moments. In Table S1 in the Internet Appendix, we recalculate the moments with the longest sample possible for each series. The leverage-adjusted equity premium remains at 4.36% per annum, and the leverage-adjusted stock market volatility rises lightly from 16.04% to 16.08%. The mean real interest rate increases somewhat from 0.82% to 1.05%, and its volatility from 7.3% to 7.53%.

the consumption, output, and investment growth rates are more persistent in the postwar sample, with the first-order autocorrelations of 0.46, 0.39, and 0.29, respectively. However, the postwar leverage-adjusted equity premium is higher than the historical equity premium, 5.38% versus 4.36%. The leverage-adjusted stock market volatility is also higher in the postwar sample, 17.15% versus 16.04%. The evidence indicates that the postwar U.S. sample might not be representative. As such, we mostly rely on the historical cross-country panel to calibrate our model.

For labor market moments, to our knowledge, a historical cross-country panel is unavailable. As such, we work with the U.S. historical monthly series compiled by Petrosky-Nadeau and Zhang (2020).[8] Following Weir (1992), in addition to civilian unemployment rates, Petrosky-Nadeau and Zhang construct a separate series of private nonfarm unemployment rates, by subtracting farm and government employment from both civilian labor force and civilian employment. Because this unemployment series better depicts the functioning of the private economy (Lebergott 1964), we focus our calibration on this series. This series dates back to 1890, and the vacancy rate series to 1919.

From January 1890 to December 2015, the mean private nonfarm unemployment rate is 8.94%. The skewness and kurtosis of the unemployment rates are 2.13 and 9.5, respectively. In the postwar sample from January 1950 to December 2015, the mean unemployment rate is lower, 7.65%. Skewness is also smaller, 0.55, and kurtosis is close to that of the normal distribution, 2.92.

To calculate the second moments, we follow Shimer (2005) to take quarterly averages of monthly unemployment and vacancy rates to convert to quarterly series, which are detrended as Hodrick-Prescott (1997, HP) filtered proportional deviations from the mean with a smoothing parameter of 1,600. We do not take log deviations from the HP trend because the $V \geq 0$ constraint can be occasionally binding in the model. From 1890 onward, the private nonfarm unemployment volatility is 24.43% per quarter (25.9% with log deviations). From 1919 onward, the vacancy rate volatility is 18.98% (17.36% with log deviations). For labor market tightness (the ratio of the vacancy rate over the private nonfarm unemployment rate), the volatility is 61.62% (but only 38.38% with log devia-

[8]The series are available at https://ars.els-cdn.com/content/image/1-s2.0-S0304393220300064-mmc2.csv.

tions). The U-V correlations are -0.57 and -0.79 across the two detrending methods, respectively.[9]

3.2 Calibration

We calibrate the model in monthly frequency. We set the time discount factor $\beta = 0.9976$ to help match the mean real interest rate. We set the risk aversion, γ, to 10 per the long-run risks literature (Bansal and Yaron 2004). We set the elasticity of intertemporal substitution, ψ, to 2 per Barro (2009), who is in part based on Gruber's (2013) microeconomic estimates. Following Gertler and Trigari (2009), we set the persistence of the log productivity, ρ_x, to be $0.95^{1/3}$, and set its conditional volatility, σ_x, to match the consumption growth volatility in the data. Instead of the output volatility, we target the consumption volatility, which is more important for the model's asset pricing properties. This procedure yields a value of 0.015 for σ_x. This value implies a consumption volatility of 5.13% per annum, which is close to but lower than 5.45% in the data (Table 1). However, the output volatility is 6.43%, which is higher than 5.1% in the data.

For the CES production function, we set $\omega = -1.5$. This ω value implies an elasticity of capital-labor substitution of 0.4, which is the point estimate in Chirinko and Mallick (2017). When calibrating the distribution parameter, α, we target the average labor share. Gollin (2002) shows that factor shares are approximately constant across time and space. Table S3 in the Internet Appendix reports the labor shares for the 12 countries that are in both the Gollin and the Jordà-Schularick-Taylor databases. The average labor shares across the countries from Gollin's first two adjustment methods are 0.765 and 0.72, respectively, with an average of 0.743. Gollin emphasizes that these two adjustments "give estimated labor shares that are essentially flat across countries and over time (p. 471)." As such, we set $\alpha = 0.25$, which yields an average labor share of 0.746 in simulations.

The distribution parameter, α, is close to one minus the average labor share only in the "normalized" CES production function, in which the capital unit is comparable to the labor unit

[9] Labor market volatilities are lower in the postwar sample. From 1950 onward, the private nonfarm unemployment volatility is 13.81% per quarter, and the vacancy rate volatility is 13.49%. The market tightness volatility is 26.17%, and the U-V correlation -0.9. Detrending with log deviations from the HP trend yields very similar estimates.

(Klump and La Grandville 2000). We calibrate the capital scaler, K_0, at 13.75 to set the labor share at the deterministic steady state at 0.75. For comparison, the value of capital at the deterministic steady state is 16.14. Despite the model's nonlinearity, the labor share is very close across the deterministic and stochastic steady states. We calibrate the long-run mean of the productivity, $\overline{x} = 0.1887$, to target the marginal product of labor, $\partial Y_t / \partial N_t$, around one on average in simulations.[10]

The supply elasticity of capital, ν, governs the magnitude of adjustment costs. A lower ν implies higher adjustment costs, which reduce the investment volatility but raise the consumption volatility. Alas, direct estimates of ν seem scarce. We set ν to 1.25 and the depreciation rate, δ, to 1.25%. We set the separation rate, s, to 0.035, which is the average total nonfarm separation rate in the Job Openings and Labor Turnover Survey (JOLTS) at Bureau of Labor Statistics (BLS). The curvature of the matching function, ι, is 1.25, which is based on Den Haan, Ramey, and Watson (2000).

3.2.1 Wage Inertia

We are left with the bargaining weight of workers, η, the flow value of unemployment activities, b, and the unit cost of vacancy posting, κ. To match the equity premium without overshooting the mean unemployment rate, we combine inertial wages and low vacancy costs. Specifically, we set $\eta = 0.015$ and $b = 0.91$, which yield a wage elasticity to labor productivity of 0.256 in the model. We set the unit vacancy cost, κ, to 0.01, to obtain a mean unemployment rate of 8.63%, which is close to the average private nonfarm unemployment rate of 8.94% in the 1890–2015 sample.

Is the model implied wage elasticity to labor productivity empirically plausible? Hagedorn and Manovskii (2008), for example, estimate the wage elasticity to labor productivity to be 0.449 in the postwar 1951–2004 quarterly sample from BLS.[11] However, a voluminous literature on economic history documents severe wage inertia and quantifies its large impact during the Great Depression.[12]

[10]Setting $\partial Y_t / \partial N_t = 1$ at the deterministic steady state yields $\overline{x} = 0.1787$. However, $\partial Y_t / \partial N_t$ at the stochastic steady state is somewhat lower than one. As such, we manually adjust $\overline{x}$ to 0.1887 to yield the desired outcome.

[11]Both real wages and labor productivity are in logs and HP-filtered with a smoothing parameter of 1,600.

[12]Prominent examples include Eichengreen and Sachs (1985), Bernanke and Powell (1986), Bernanke and Carey (1996), Hanes (1996), Dighe (1997), Bordo, Erceg, and Evans (2000), Cole and Ohanian (2004), and Ohanian (2009).

As such, we extend the Hagedorn-Manovskii evidence to a historical U.S. sample.

To construct a historical series of real wages, we draw elements from Gordon (2016). From 1929 to 2015, we obtain compensation of employees from National Income and Product Accounts (NIPA) Tables 6.2A–D (line 3, private industries, minus line 5, farms) at Bureau of Economic Analysis. We obtain the number of full-time equivalent employees from NIPA Tables 6.5A–D (line 3, private industries, minus line 5, farms). Dividing the compensation of employees by the number of employees yields nominal wage rates (compensation per person). We deflate nominal wage rates with the personal consumption deflator from NIPA Table 1.1.4 (line 2) to obtain real wage rates.

From 1890 to 1929, we obtain the average (nominal) hourly compensation of production workers in manufacturing and consumer price index from measuringworth.com (Officer and Williamson 2020a, 2020b). The nominal compensation series from their Web site only has two digits after the decimal. We instead use the average hourly compensation series, with three digits after the decimal, from Officer (2009, Table 7.1). To obtain an index of hours, we divide the index of manhours by the index of persons engaged in manufacturing from Kendrick (1961, Table D-II). We multiply the average hourly compensation series with the hours index to obtain the nominal compensation per person, which we then deflate with the Officer-Williamson consumer price index to obtain the series of real wages. Finally, we splice this series in 1929 to the NIPA series from 1929 onward to yield an uninterrupted series from 1890 to 2015. Splicing means that we rescale the pre-1929 series so that its value in 1929 is identical to that for the NIPA post-1929 series.[13] Finally, for labor productivity, we use the historical 1890–2015 series from Petrosky-Nadeau and Zhang (2020).[14] We

[13]We differ from Gordon (2016) in two aspects. First, Gordon measures real wages as real compensation per manhour. We instead use real compensation per person that better fits our model with no hours. This practice seems standard in the macro labor literature (Shimer 2005). Second, Gordon measures nominal compensation as total compensation of employees from NIPA Table 1.10 (line 2), which includes government and farm employees. We instead use employee compensation for the private nonfarm sector, which matches the measurement of labor productivity.

[14]The monthly series is the ratio of a nonfarm business real output series over a private nonfarm employment series. The real output series draws from Kendrick (1961) and NIPA (from 1929 onward) as well as monthly industrial production series (as monthly indicators) from Miron and Romer (1990) and Federal Reserve Bank of St. Louis (from 1919 onward). The private nonfarm employment series draws from Weir (1992) and Current Employment Statistics at BLS as well as monthly employment indicators from NBER macrohistory files. From January 1947 onward, the monthly labor productivity series is benchmarked to the quarterly nonfarm business real output per job series from BLS.

time-aggregate their monthly series into annual by taking the monthly average within a given year.

We detrend the annual real wages and labor productivity series as log deviations from their HP-trends with a smoothing parameter of 6.25, which is equivalent to a quarterly smoothing parameter of 1,600.[15] In our postwar 1950–2015 annual sample, regressing the log real wages on the log labor productivity yields a wage elasticity of 0.406, with a standard error of 0.081. The elasticity estimate is not far from the Hagedorn-Manovskii estimate of 0.449 in their 1951–2004 quarterly sample.

More important, in our 1890–2015 historical sample, the wage elasticity to labor productivity is estimated to be 0.267, with a standard error of 0.066. Deflating the pre-1929 nominal compensation series with the Johnston-Williamson (2020) implicit GDP deflator, as opposed to the Officer-Williamson (2020b) consumer price index, yields a similar wage elasticity of 0.263, with a standard error of 0.062. Our evidence that real wages are more inertial in the historical sample accords well with the economic history literature (footnote 12). In particular, the low wage elasticity to labor productivity, 0.256, in our model is empirically plausible.

Our value of $b = 0.91$ might seem high, as the marginal product of labor is around one in the model's simulations. However, the value of b includes unemployment benefits, the value of home production, self-employment, leisure, and disutility of work. Hagedorn and Manovskii (2008) argue that b should equal the marginal product of capital in a perfectly competitive labor market. Ljungqvist and Sargent (2017) show that to explain the unemployment volatility, a search model must diminish the fundamental surplus, which is the fraction of output allocated to the firm by the labor market. We view our high-b calibration as perhaps the simplest way to achieve this goal. More important, we view our high-b-low-η calibration as a parsimonious metaphor for real wage inertia. More explicit structures of wage inertia, such as alternating offer bargaining in Hall and Milgrom (2008) or staggered multiperiod Nash bargainng in Gertler and Trigari (2009), are likely to deliver similar quantitative results but would complicate our model greatly.[16]

[15] Ravn and Uhlig (2002) show that the smoothing parameter should be adjusted by the fourth power of the observation frequency ratio, which equals four going from the quarterly to annual frequency. In particular, $1600/4^4 = 6.25$.

[16] The high-b calibration is also of contemporary interest. Ganong, Noel, and Vavra (2020) document that under

3.3 Unconditional Moments

We report basic business cycle, labor market, and asset pricing moments from the model economy.

3.3.1 Business Cycle Moments

From the model's stationary distribution (after a burn-in period of 1,200 months), we repeatedly simulate 10,000 artificial samples, each with 1,740 months (145 years). The length of each sample matches the length of the Jordà-Schularick-Taylor database (1871–2015). On each artificial sample, we time-aggregate monthly consumption, output, and investment into annual observations. We add up 12 monthly observations within a given year and treat the sum as the year's annual observation. For each annual series, we compute its volatility, skewness, kurtosis, and autocorrelations of up to five lags of log growth rates. For each moment, we report the mean as well as the 5th, 50th, and 95th percentiles across the 10,000 simulations. We also report the p-value that is the fraction with which a given moment in the model is higher than its matching moment in the data. The fraction can be interpreted as the p-value for a one-sided test of our model using the moment in question.

Panel A of Table 2 shows that the model does a good job in matching consumption moments. None of the p-values for one-sided tests are significant at the 5% level. The consumption growth volatility in the model is 5.13% per annum, which is close to 5.45% in the data ($p = 0.41$). Kurtosis is 8.09 in the model, which is close to 10.34 in the data ($p = 0.18$). The first-order autocorrelation is 0.21 in the model, which is higher than 0.12 in the data, but the difference is insignificant ($p = 0.78$). The autocorrelations at higher orders are close to zero in the model as in the data.

From Panel B, the output volatility in the model is 6.43% per annum, which is higher than 5.1% in the data, but the difference is insignificant ($p = 0.86$). The model falls short in explaining the skewness, 0.09 versus -1.06, and kurtosis, 5.45 versus 14.09, of the output growth. Both differences are significant. The model comes close to match the first-order autocorrelation, 0.2 versus 0.18.

the 2020 Coronavirus Aid, Relief, and Economic Security Act, the ratio of mean benefits to mean earnings in the data is roughly 100%. The median replacement ratio is even higher at 134%. Finally, 68% of eligible unemployed workers have replacement ratios higher than 100%, and 20% of the workers have replacement ratios higher than 200%.

From Panel C, the investment volatility in the model is only 8.59% per annum, which is lower than 13.53% in the data. The difference is significant, but none of the p-values for other investment moments are significant. The kurtosis in the model is 7.12, relative to 10.75 in the data ($p = 0.08$). The first-order autocorrelation is 0.15 in the model, which is close to 0.13 in the data.

3.3.2 Labor Market Moments

Panel D of Table 2 shows that the model does a good job matching the first four moments of the unemployment rate. The mean unemployment rate is 8.63% in the model, which is close to 8.94% in the data ($p = 0.37$). The skewness is 2.64, relative to 2.13 in the data ($p = 0.53$), and the kurtosis, 13.45 versus 9.5 ($p = 0.35$). The unemployment volatility is 32.2% per quarter, which is higher than 24.43% in the data. However, the difference is not significant ($p = 0.76$).

The vacancy rate volatility is 33.73% per quarter in the model, which is significantly higher than 18.98% in the data. The volatility of labor market tightness is 33.98%, which is significantly lower than 61.62% in the data. However, as noted, this data moment is sensitive to detrending method and is only 38.38% with log deviations from the HP-trend. The unemployment-vacancy correlation is only -0.07 in the model, which is lower in magnitude than -0.57 in the data. However, this moment is also sensitive to detrending method. Using the monthly data simulated from the model with no detrending yields a U-V correlation of -0.475, which is close to the matching data moment of -0.517, and the difference is insignificant ($p = 0.66$). Finally, the wage elasticity to labor productivity is 0.256, and the data moment of 0.267 yields an insignificant p-value of 0.23.

3.3.3 Asset Pricing Moments

Most important, Panel E shows that our general equilibrium production economy succeeds in yielding an equity premium of 4.26% per annum, which is close to 4.36% in the data. The data moment lies comfortably within the model's 90% confidence interval, with a p-value of 0.34. The mean interest rate is 1.59% in the model, which is not far from 0.82% in the data. The data moment is again lies within the model's 90% confidence interval ($p = 0.87$).

The model implies a stock market volatility of 11.77% per annum, which is significantly lower than the data moment of 16.04%, although the U.S. volatility of 13.66% (Table 1) falls within the model's 90% confidence interval. The model's performance in matching stock market volatility improves over prior attempts in general equilibrium production economies (Gourio 2012).

The interest rate volatility in the model is 3.13% per annum, which is significantly lower than 7.3% in the data. The most likely reason is that we do not model sovereign default and hyperinflation that are the driving forces behind the historically high interest rate volatilities in Germany, Italy, and Japan. These destructive forces play only a limited role in the U.S., which has an interest rate volatility of only 4.65% (Table 1). It is well within the model's 90% confidence interval.

3.4 Sources of the Equity Premium

In this subsection we examine the driving forces behind the model's equity premium.

3.4.1 Dividend Dynamics

Rouwenhorst (1995) points out the difficulty in explaining the equity premium in production economies. Unlike endowment economies, in which dividends are exogenously specified to fit the data, dividends are often endogenously countercyclical in production economies. Dividends equal profits (output minus wages) minus investment. Intuitively, with frictionless labor market, wages equal the marginal product of labor, which is almost as procyclical as output. With the Cobb-Douglous production function, the marginal product of labor is exactly proportional to output. As such, profits are no more procyclical than output. However, due to consumption smoothing, investment is more procyclical than output and profits, rendering dividends countercyclical. Kaltenbrunner and Lochstoer (2010) demonstrate this insight in a stochastic growth model.

In contrast, dividends are endogenously procyclical in our search economy. Under the benchmark calibration, wages are more inertial than the marginal product of labor, making profits more procyclical than output. The magnified procyclical dynamics of profits then overpower the procyclical dynamics of vacancy costs and capital investment to make dividends procyclical.[17]

To what extent are the model's implied dividend dynamics empirically plausible? For each country, the Jordà-Schularick-Taylor macrohistory database provides separate capital gain, dividend-to-price, and consumer price index series, from which we construct the real dividend series (the Internet Appendix). Table S4 shows that dividends are procyclical in the historical cross-country panel. The correlation between the cyclical components of annual dividends and output is on average 0.11 across the countries, ranging from -0.02 from Portugal to 0.47 in the U.S. Only 3 out of 17 countries have negative correlations, all of which are small in magnitude. The relative volatility of dividends (the ratio of the dividend volatility over the output volatility) is 8.61 across the countries, varying from 3.06 from Portugal to 16.81 in Netherlands (3.18 in the U.S.).[18] Time-aggregating annual observations into 3- and 5-year observations raises the dividend-output correlation to 0.31 and 0.35 and lowers the relative volatility of dividends to 6.54 and 5.69, respectively.

The model explains procyclical dividends but overshoots the dividend-output correlation, 0.947. The model also underestimates the relative volatility of dividends at 2.89. Both differ significantly from their data moments. Time-aggregating does not materially affect the model's estimates. The dividend-output correlations are 0.954 and 0.952, and the relative volatility of dividends 2.83 and 2.74 at the 3- and 5-year frequencies, respectively. In the historical data, there are likely measurement errors in real dividends, which tend to average out over time, yielding higher dividend-output correlations at longer horizons. In contrast, no such measurement errors exist within the model.

A possible reason why the model overshoots the dividend-output correlation is that dividends in the data refer only to cash dividends, but dividends in the model match more closely to net payouts. Net payouts in the data include not only cash dividends but also share repurchases net of

[17] Petrosky-Nadeau, Zhang, and Kuehn (2018) examine this mechanism in a baseline search model without capital. However, with capital, consumption smoothing via investment strengthens the countercyclicality of dividends. We overcome this core challenge via wage inertia, for which we also provide new, supportive evidence (Section 3.2.1).

[18] Due to a few zero-dividend observations (7 out of 2,034), we detrend dividend and output series with HP-filtered proportional deviations from the mean. Using HP-filtered log deviations after discarding the 7 observations yields a higher dividend-output correlation of 0.24 and a relative dividend volatility of 7.92 averaged across the countries.

equity issuances (Boudoukh et al. 2007). Alas, to our knowledge, a historical sample of net payouts is not available. Perhaps more important, our model has only one shock, which drives the high dividend-output correlation, but there exist most likely multiple shocks in the data.

3.4.2 Disaster Dynamics

As shown in Petrosky-Nadeau, Zhang, and Kuehn (2018), the search model of equilibrium unemployment gives rise endogenously to rare disasters. To explain the equity premium, we formulate a more general model by incorporating both recursive utility and capital accumulation. Disaster risks in consumption play a key role in explaining the equity premium in our framework.

To characterize disasters in the data, we apply the Barro-Ursúa (2008) peak-to-trough method on the Jordà-Schularick-Taylor cross-country panel of consumption and output. Disasters are identified as episodes, in which the cumulative fractional decline in consumption or output exceeds a predetermined hurdle rate. We adopt two such hurdle rates, 10% and 15%.[19] We adjust for trend growth in the data because our model abstracts from growth. We subtract the mean log annual consumption growth of 1.62% from each consumption growth observation and subtract the mean log annual output growth of 1.78% from each output growth in the historical data (Table 1).

Table 3 shows that with a disaster hurdle rate of 10%, the consumption disaster probability is 6.4%, and the output disaster probability 5.78% in the cross-country panel. With a higher hurdle rate of 15%, the probabilities drop to 3.51% and 2.62%, respectively. The disaster size is 23.2% and 22.3% for consumption and output with a hurdle rate of 10%, but higher, 30.4 and 32.9, respectively, with a higher hurdle rate of 15%. The duration for consumption and output disasters lasts 4.2 and 4.1 years with a hurdle rate of 10%, but 4.5 and 5 years with a hurdle rate of 15%.

The model implied consumption disaster dynamics, which are crucial for the equity premium,

[19] Suppose there are two states, normalcy and disaster, in a given period. The number of disaster years is the number of years in the interval between peak and trough for each disaster event. The number of normalcy years is the total number of years in the sample minus the number of disaster years. The disaster probability is the likelihood with which the economy switches from normalcy to disaster in a given year. We calculate this probability as the ratio of the number of disasters over the number of normalcy years. For each disaster event, the disaster size is the cumulative fractional decline in consumption or output from peak to trough. Duration is the number of years from peak to trough.

are empirically plausible. We simulate 10,000 artificial samples from the model's stationary distribution, each with 1,740 months, matching the 1871–2015 sample length. On each sample, we time-aggregate monthly into annual consumption and apply the exact peak-to-trough method as in the data. From Panel A of Table 3, the disaster probabilities are 5.83% and 3.64%, which are relatively close to 6.4% and 3.51% in the data, with the hurdle rates of 10% and 15%, respectively. The size and duration of consumption disasters in the model are also close to those in the data, 23.4% versus 23.2% for size, and 4.1 versus 4.2 years for duration, with a hurdle rate of 10%, for example. The p-values all indicate that the differences between the model and data moments are insignificant.

As noted, consumption is more volatile than output in the cross-country panel, likely due to government purchases during wartime (Barro and Ursúa 2008). In contrast, consumption is naturally less volatile than output in production economies because of consumption smoothing. We focus on matching consumption dynamics because of their paramount importance for the equity premium. Consequently, the model overshoots output disasters. From Panel B, the output disaster probability is 10.9%, which is higher than 5.78% in the data ($p = 0.97$), with a hurdle rate of 10%. With a higher hurdle of 15%, the disaster probability is 6.1% in the model, which is still higher than 2.62% in the data ($p = 0.94$). However, disaster size and duration are relatively close to their data moments.

3.4.3 Consumption Dynamics

We dig deeper by comparing consumption dynamics in the search economy with those specified in the long-run risks literature (Bansal and Yaron 2004). Kaltenbrunner and Lochstoer (2010) show that long-run risks (high persistence in expected consumption growth) arise endogenously in production economies with frictionless labor market via consumption smoothing. Because of persistent aggregate productivity and consumption smoothing, long-run risks might also be present in our model. What is the relative role of long-run risks compared with disaster risks in our model? This economic question is important because different specifications of consumption dynamics can largely accord with observed moments of consumption growth, such as volatilities and autocorrelations, in the data. However, different specifications imply vastly different economic mechanisms.

We calculate the expected consumption growth and conditional consumption growth volatility in the model's state space. We use these solutions to simulate one million monthly periods from the model's stationary distribution. Fitting the consumption growth process specified by Bansal and Yaron (2004) on the simulated data yields:

$$g_{Ct+1} = E_t[g_{Ct+1}] + \sigma_{Ct}\,\epsilon^g_{t+1} \tag{20}$$

$$E_{t+1}[g_{Ct+2}] = 0.288\,E_t[g_{Ct+1}] + 0.705\,\sigma_{Ct}\,\epsilon^e_{t+1} \tag{21}$$

$$\sigma^2_{Ct+1} = 0.008^2 + 0.964(\sigma^2_{Ct} - 0.008^2) + 0.421 \times 10^{-5}\epsilon^V_{t+1}, \tag{22}$$

in which g_{Ct+1} is realized consumption growth, $E_t[g_{Ct+1}]$ expected consumption growth, σ_{Ct} conditional volatility of g_{Ct+1}, and ϵ^g_{t+1}, ϵ^e_{t+1}, and ϵ^V_{t+1} are i.i.d. standard normal shocks. In addition, the unconditional correlation between ϵ^g_{t+1} and ϵ^e_{t+1} is 0.048, the unconditional correlation between ϵ^e_{t+1} and ϵ^V_{t+1} is 0.024, and the unconditional correlation between ϵ^g_{t+1} and ϵ^V_{t+1} is 0.079 in simulations.

Equation (21) shows that the persistence in expected consumption growth is only 0.288 in our model, which is substantially lower than 0.979 in Bansal and Yaron (2004).[20] However, our expected consumption growth is more volatile, with its conditional volatility about 70.5% of the conditional volatility of realized consumption growth. This fraction is much higher than 4.4% in Bansal and Yaron. Similarly, our persistence of expected consumption growth, 0.288, is also much lower than that implied by baseline production economies in Kaltenbrunner and Lochstoer (2010).[21] As such, despite recursive utility and autoregressive productivity shocks, long-run risks (in the sense of highly persistent expected consumption growth) do not play an important role in our economy.

Equation (22) shows that the search economy gives rise endogenously to time-varying volatil-

[20] Bansal and Yaron (2004) specify the monthly consumption growth process to be $E_{t+1}[g_{Ct+2}] = 0.979\,E_t[g_{Ct+1}] + 0.044\,\sigma_{Ct}\,\epsilon^e_{t+1}$, $g_{Ct+1} = 0.0015 + E_t[g_{Ct+1}] + \sigma_{Ct}\,\epsilon^g_{t+1}$, and $\sigma^2_{Ct+1} = 0.0078^2 + 0.987(\sigma^2_{Ct} - 0.0078^2) + 0.23 \times 10^{-5}\epsilon^V_{t+1}$, in which $\epsilon^e_{t+1}, \epsilon^g_{t+1}$, and ϵ^V_{t+1}, are i.i.d. and mutually uncorrelated standard normal shocks.

[21] Kaltenbrunner and Lochstoer (2010, Table 6) show that the consumption growth follows $E_{t+1}[g_{Ct+2}] = 0.986\,E_t[g_{Ct+1}] + 0.093\,\sigma_{Ct}\,\epsilon^e_{t+1}$ and $g_{Ct+1} = 0.0013 + E_t[g_{Ct+1}] + \sigma_{Ct}\,\epsilon^g_{t+1}$, with transitory productivity shocks. With permanent shocks, $E_{t+1}[g_{Ct+2}] = 0.99\,E_t[g_{Ct+1}] + 0.247\,\sigma_{Ct}\epsilon^e_{t+1}$. However, σ_{Ct} is largely constant in both models.

ities (Bloom 2009). The consumption conditional variance appears "stochastic" in our model. Its persistence is 0.964, which is lower than 0.987 calibrated in Bansal and Yaron (2004) and 0.999 in Bansal, Kiku, and Yaron (2012). However, the volatility of our stochastic variance is 0.42×10^{-5}, which is higher than 0.23×10^{-5} in Bansal and Yaron and 0.28×10^{-5} in Bansal, Kiku, and Yaron. The time-variation of volatilities is another important dimension along which our search economy differs from stochastic growth models. These models with frictionless labor market yield largely constant volatilities (Kaltenbrunner and Lochstoer 2010). Perhaps more important, our quantitative results in equation (22) suggest that long-run risks in consumption volatility can be observationally equivalent to consumption disaster risks, potentially lending support to disaster models.

3.5 Time-varying Risks and Risk Premiums

We quantify the model's implications on time-varying equity premium and stock market volatility.

3.5.1 Equilibrium Properties

We first evaluate qualitative implications of the model's competitive equilibrium. From the model's stationary distribution (after a burn-in period of 1,200 months), we simulate a long sample of one million months. Figure 1 shows the scatterplots of key conditional moments against productivity. From Panel A, the price-to-consumption ratio, P_t/C_t, increases with productivity. In the 1-million-month sample, the correlations of P_t/C_t with productivity, output, unemployment, vacancy, and the investment rate are 0.97, 0.78, -0.48, 0.9, and 0.6, respectively. Clearly, P_t/C_t is procyclical.

In contrast, Panel B shows that the expected equity premium, $E_t[r_{St+1}]-r_{ft+1}$, is countercyclical. Its correlations with productivity, output, unemployment, vacancy, and the investment rate are -0.84, -0.86, 0.66, -0.87, and -0.36, respectively. In addition, the correlation between the expected equity premium and price-to-consumption is -0.88. Stock market volatility, σ_{St}, is also countercyclical (Panel C). Its correlations with productivity, output, unemployment, vacancy, and the investment rate are -0.91, -0.83, 0.57, -0.92, and -0.42, respectively. In addition, its correlations with the expected equity premium and price-to-consumption are 0.98 and -0.95, respectively.

Panel D shows that the riskfree rate, r_{ft+1}, is weakly procyclical in the model. Its correlations with productivity, output, unemployment, vacancy, and the investment rate are 0.23, 0.22, -0.2, 0.1, and 0.27, respectively. In addition, its correlations with the expected equity premium, stock market volatility, and price-to-consumption are $-0.15, -0.13$, and 0.28, respectively. Panel E shows that expected consumption growth, $E_t[g_{Ct+1}]$, behaves similarly as the risk-free rate. The correlation between $E_t[g_{Ct+1}]$ and r_{ft+1} is 0.998. Panel F shows that consumption volatility, σ_{Ct}, is weakly countercyclical. Although its correlations with output and unemployment are high, -0.48 and 0.74, its correlations with productivity and investment rate are low, -0.05 and 0.1, respectively.

In all, the model implies strong predictability for stock market excess return and volatility, some predictability for consumption volatility, and weak to no predictability for consumption growth and the interest rate. Intuitively, wage inertia yields operating leverage. In bad times, output falls, but wage inertia causes profits to drop disproportionately more than output, thereby magnifying the procyclical covariation of profits and dividends, causing the expected equity premium to rise.

More important, the impact of wage inertia is stronger in bad times, when the profits are even smaller because of low productivity. This time-varying wage inertia amplifies the risks and risk premiums, making the expected equity premium and stock market volatility countercyclical.[22] In contrast, consumption growth and consumption volatility are less predictable because of consumption smoothing via capital investment. Despite adjustment costs, investment absorbs a large amount of shocks to render the first two moments of consumption growth less predictable.

3.5.2 Data

Before quantifying the model's implications on time-varying risks and risk premiums, Table 4 shows long-horizon regressions of stock market excess returns and log consumption growth on log price-to-consumption in the historical data. We follow Beeler and Campbell (2012) but implement the

[22]Relatedly, Favilukis and Lin (2016) study this time-varying mechanism in a general equilibrium production economy with (exogenously specified) infrequent wage renegotiation, long-run risks, and labor adjustment costs. In contrast, wage inertia arises endogenously in our economy, and the equity premium arises from endogenous disaster risks.

tests on the Jordá-Schularick-Taylor historical cross-country panel. We perform the regressions on log price-to-consumption, as opposed to log price-to-dividend, because dividends (net payouts) can be negative in the model. To align the data moments with the model moments, we adjust excess returns in the data for financial leverage (by multiplying unadjusted excess returns with 0.71).

Panel A shows long-horizon predictive regressions of market excess returns:

$$\sum_{h=1}^{H} [\log(r_{St+h}) - \log(r_{ft+h})] = a + b \log(P_t/C_t) + u_{t+H}, \tag{23}$$

in which H is the forecast horizon, P_t real market index, C_t real consumption at the beginning of period t, and u_{t+H} the residual. Panel B shows long-horizon regressions of log consumption growth:

$$\sum_{h=1}^{H} \log(C_{t+h}/C_t) = a + b \log(P_t/C_t) + v_{t+H}, \tag{24}$$

in which v_{t+H} is the residual. In both long-horizon regressions, $\log(P_t/C_t)$ is standardized to have a mean of zero and a volatility of one. H ranges from one to five years. Finally, the t-values are adjusted for heteroscedasticity and autocorrelations of $2(H-1)$ lags.

Panel A shows some evidence of predictability of market excess returns. The slopes are largely negative across the countries and forecast horizons from one to five years, and their t-values are often significant, especially at the longer horizons. The R-squares averaged across the countries vary from 1.87% to 9% as the forecast horizon goes from one to five years. The prior asset pricing literature has mostly focused on the U.S. sample, which is an outlier in Panel A. In particular, the U.S. features the strongest evidence of predictability in terms of the t-values of slopes and R-squares. For example, in the 5-year horizon, the R^2 is 33.6% in the U.S. and 28% in the U.K., in contrast to 0% in Germany, 1% in Italy and Portugal, and 2% in France.

In the Internet Appendix (Table S5, Panel A), we document stronger stock market return predictability in the post-1950 sample. The slopes are all negative and mostly significant across the countries and forecast horizons. On average, the slopes are significant for all horizons except year one. The R-squares range from 4.9% in year one to 17.8% in year five.

Panel B of Table 4 shows that consumption growth is largely unpredictable. In the historical sample, the slopes averaged across the countries are all negative but insignificant. Even at the 5-year horizon, the R^2 is only 5.77% on average. In the post-1950 sample, the average slopes all flip to positive but remain insignificant, although the average R-squares increase somewhat, for example, to 9.1% in year five (Table S5, Panel B, the Internet Appendix).

Table 5 shows long-horizon regressions of excess return and consumption growth volatilities on log price-to-consumption. For a given forecast horizon, H, we measure excess return volatility as $\sigma_{St,t+H-1} = \sum_{h=0}^{H-1} |\epsilon_{St+h}|$, in which ϵ_{St+h} is the h-period-ahead residual from the first-order autoregression of log excess returns, $\log(r_{St+1}) - \log(r_{ft+1})$ (again adjusted for financial leverage). Panel A performs long-horizon predictive regressions of excess return volatilities:

$$\log \sigma_{St+1,t+H} = a + b \log(P_t/C_t) + u^{\sigma}_{t+H}. \tag{25}$$

In Panel B, the consumption volatility is $\sigma_{Ct,t+H-1} = \sum_{h=0}^{H-1} |\epsilon_{Ct+h}|$, in which ϵ_{Ct+h} is the h-period-ahead residual from the first-order autoregression of log consumption growth, $\log(C_{t+1}/C_t)$. We then perform long-horizon predictive regressions of consumption volatilities:

$$\log \sigma_{Ct+1,t+H} = a + b \log(P_t/C_t) + v^{\sigma}_{t+H}. \tag{26}$$

Panel A of Table 5 shows weak predictability for excess return volatilities. The average slopes are all negative and marginally significant in the first two years. The average R-squares range from 6.3% in year one to 19% in year five. However, the evidence is sensitive to sample period. In the post-1950 sample, the average slopes are all insignificant, with mixed signs (Table S6, Panel A, the Internet Appendix). Consumption volatilities are essentially unpredictable with log price-to-consumption. In the historical sample, the average slopes are all positive and, in long horizons, marginally significant. However, in the post-1950 sample, the slopes all flip to negative and insignificant.

3.5.3 The Model's Performance

We simulate 10,000 samples from the model's stationary distribution, each with 1,740 months. On each sample, we time-aggregate monthly returns and consumption into annual observations and implement the same procedures as in the data. Overall, the model succeeds in explaining stock market predictability but somewhat overstates consumption growth predictability, especially its volatility.

Table 6 shows the details. From Panel A, market excess returns are predictable in the model. The slopes are all significantly negative, and the R-squares range from 3.9% in year one to 13.5% in year five. None of the p-values for the slopes, their t-values, and R-squares are significant at the 5% level. From Panel B, the model overstates somewhat the consumption growth predictability. The slopes are all significantly negative. However, except for year one, the p-values for the slopes and their t-values indicate only insignificant differences between the model and data moments.

Panel C shows that stock market volatility is weakly predictable with log price-to-consumption in the model. As in the data, the slopes are all negative but insignificant. None of the p-values for slopes and their t-values suggest that the model moments deviate significantly from their data counterparts. However, the R-squares in the model are significantly lower than those in the data. More important, from Panel D, the model overstates the predictability of consumption growth volatility. While the slopes are mostly insignificant and positive in the data, the slopes in the model are significantly negative, and the p-values for the slopes and their t-values are significant.

3.6 Comparative Statics

In this subsection, we conduct comparative statics to shed light on the inner workings of our model. In each experiment, we vary one parameter only, while keeping all the other parameters identical to those in the benchmark calibration. (For log utility, we set both the risk aversion and intertemporal elasticity of substitution to one.) In all experiments, we recalibrate the capital scalar, K_0, to ensure the average labor share is unchanged from the benchmark calibration. Otherwise, the impact from changing a given parameter would be confounded with the impact of changing the labor share. The only exception is the $\alpha = 0.3$ experiment, in which we recalibrate K_0 to match the average labor share of 0.7. The simulations follow the same design as in the benchmark model.

3.6.1 Preference Parameters

Table 7 details the results. Not surprisingly, the risk aversion, γ, has a quantitatively important impact on the equity premium. Reducing γ from 10 to 7.5 and further to 5 lowers the equity premium from 4.26% per annum in the benchmark calibration to 1.55% and further to 0.54%. Stock market volatility also falls from 11.8% to 9.5% and further to 8%.

Most important, risk aversion also affects quantities. Reducing γ from 10 to 7.5 and further to 5 lowers consumption volatility from 5.13% to 4.24% and further to 3.93%. The probability of consumption disasters falls from 5.83% to 4.28% and further to 3.82%, and the disaster size also drops somewhat. A lower discount rate (the equity premium plus the interest rate) raises the marginal benefit of hiring, stimulating employment. Consequently, the mean unemployment rate falls from 8.63% to 5.71% and further to 4.63%. Although the unemployment volatility remains stable, the vacancy and labor market tightness volatilities both fall by about one-third. As such, echoing Gourio (2012) and Hall (2017) but differing from Tallarini (2000), our results indicate the necessity to jointly study macro quantities and asset prices, which do not seem to be determined separately.

The intertemporal elasticity of substitution, ψ, governs the willingness of the representative investor to substitute consumption over time. A lower elasticity indicates stronger incentives for consumption smoothing. Consequently, reducing ψ from 2 to 1.5 and further to 1 lowers the consumption volatility from 5.13% per annum to 4.89% and further to 4.51%. The consumption disaster probability falls from 5.83% to 5.4% and further to 4.77%. The disaster size also drops somewhat. The lower consumption risks reduce the equity premium from 4.26% to 3.82% and further to 3.17%. The lower discount rate again raises the marginal benefit of hiring to reduce the unemployment rate to 7.9% and further to 6.87%. However, labor market volatilities remain largely unchanged.

Finally, the log utility ($\gamma = \psi = 1$) implies lower consumption, output, and investment volatilities, 3.83%, 5.21%, and 5.32% per annum, than the benchmark calibration with recursive utility, 5.13%, 6.43%, and 8.59%, respectively. Although the unemployment volatility is largely unaffected, the vacancy and labor market tightness volatilities both fall by about one-third. The equity premium drops from 4.26% to only 0.53%, and stock market volatility from 11.77% to 8.68%.

3.6.2 Labor Market Parameters

The flow value of unemployment, b, plays an important role in driving our results. Lowering its value from 0.91 to 0.85 is sufficient to reduce the unemployment rate from 8.63% to 3.45% and the unemployment volatility from 0.32 to 0.07. Intuitively, a lower b reduces wages and raises profits, stimulating hiring incentives. A lower b also enlarges the fundamental surplus allocated to the firm, dampening the unemployment volatility (Hagedorn and Manovskii 2008; Ljungqvist and Sargent 2017). This mechanism also reduces the consumption volatility from 5.13% per annum to 2.62% and the consumption disaster probability from 5.83% to 2.36%. The smaller consumption risks then reduce the equity premium to only 0.45% and stock market volatility to 7.33%.

The bargaining weight of workers, η, also plays an important role in driving our results. Raising η from 0.01 to 0.025 makes wages more sensitive to shocks. The wage elasticity to labor productivity rises from 0.26 to 0.37. Because wages become more cyclical, profits and dividends become less cyclical, and the equity premium falls to 3.98% per annum. In addition, because workers gain a larger fraction of bargaining surplus, the unemployment rate rises somewhat from 8.63% to 8.81%. However, business cycle and labor market volatilities are largely unchanged.

The results are relatively insensitive to the separation rate, s. Reducing s from 3.5% to 3.25% lowers the unemployment rate slightly from 8.63% to 8.51%. The impact on business cycle and labor market volatilities is also small. The equity premium rises slightly from 4.26% per annum to 4.41%, and stock market volatility from 11.77% to 11.91%. The results are also relatively insensitive to the curvature parameter in the matching function, ι. Raising ι from 1.25 to 1.35 makes the matching process less frictional. The unemployment rate falls slightly from 8.63% to 8.5%. The impact on business cycle and labor market volatilities is also small. The equity premium rises slightly from 4.26% per annum to 4.3%, but stock market volatility falls slightly from 11.77% to 11.72%.

Raising the unit cost of vacancy posting, κ, from 0.01 to 0.025 increases the marginal cost of hiring, causing the unemployment rate to rise from 8.63% to 8.9%. The consumption, output, and investment volatilities all go up, but labor market volatilities remain largely unchanged. The equity premium falls somewhat from 4.26% per annum to 4.02%, but stock market volatility remains stable. From equation (18), a higher κ makes wages more sensitive to procyclical labor market tightness, θ_t. Consequently, profits and dividends become less procyclical, dampening the equity premium.

3.6.3 Technology Parameters

The supply elasticity of capital, ν, governs the magnitude of capital adjustment costs. A rising ν from 1.25 to 1.5 means falling adjustment costs, which in turn imply a stronger mechanism of consumption smoothing via investment. Consequently, the consumption volatility falls from 5.13% per annum to 4.98%, but the investment volatility rises from 8.59% to 9.41%, even though the output volatility remains largely unchanged at 6.45% (6.43% in the benchmark calibration). The lower consumption risks give rise to a lower equity premium, 4.03%, echoing Jermann (1998). A lower discount rate then raises the marginal benefit of hiring, reducing the unemployment rate to 8.54%. However, similar to the output volatility, labor market volatilities are largely unchanged.

Lowering the rate of capital depreciation, δ, from 1.25% to 1% per month reduces the consumption volatility from 5.13% to 4.71% per annum and the consumption disaster probability from 5.83% to 5.26%. The output volatility also falls to 5.98%, and the investment volatility to 7.3%. The lower amount of consumption risk reduces the equity premium from 4.26% to 2.56%. The lower discount rate provides stronger hiring incentives and reduces the unemployment rate to 6.86%. Intuitively, a lower δ gives rise to a larger stochastic steady state capital than the benchmark calibration, 18.2 versus 14.7. The larger capital stock helps stabilize the economy in the presence of shocks.[23]

[23]This effect of δ on the capital stock is distinct from the impact of the capital share. As note, we recalibrate the capital scalar, K_0, to keep the average labor share unchanged. Scaling by their respective K_0 values still yields a

Raising the elasticity of capital-labor substitution, $e = 1/(1-\omega)$, from 0.4 to 0.5 increases the business cycle and labor market volatilities. The consumption volatility rises from 5.13% per annum to 5.78%, and the consumption disaster probability from 5.83% to 6.31%. From the CES production function in equation (4), $\partial Y_t/\partial X_t$ increases with ω (and e). The higher amount of consumption risk implies a higher equity premium of 4.72% and a higher stock market volatility of 12.13%. Finally, a higher discount rate in turn implies a higher unemployment rate of 9.06%.

Finally, we change the distribution parameter, α, from 0.25 to 0.3. The average labor share falls to 0.7 in simulations. Although the stochastic steady state capital rises to 20.64, its value scaled by K_0 remains at 1.07, which is identical to the benchmark calibration. Because of a smaller labor share, labor market frictions play a less prominent role in this economy. Consequently, the business cycle and labor market volatilities all fall. The consumption volatility declines to 4.26% per annum, and the consumption disaster probability to 5.1%. As a result of the lower consumption risk, the equity premium falls to only 2.27%, and stock market volatility to 9.15%. The lower discount rate raises the marginal benefit of hiring, reducing the unemployment rate to 7.2%.

4 Additional Predictions

In this section, we quantify several additional implications from the model, including the term structure of the equity premium (Section 4.1), the term structure of real interest rates (Section 4.2), the timing premium (Section 4.3), and the welfare cost of business cycles (Section 4.4).

4.1 The Term Structure of the Equity Premium

Binsbergen, Brandt, and Koijen (2012) show that short-maturity dividend strips on the aggregate stock market have higher expected returns and volatilities than long-maturity dividend strips. This downward-sloping pattern seems difficult to reconcile with leading consumption-based models.[24]

somewhat higher stochastic steady state capital for the low-δ economy than the benchmark economy, 1.1 versus 1.07.

[24]Intuitively, in the Campbell-Cochrane (1999) external habit model, the impact of shocks on slow-moving surplus consumption is more pronounced for long-maturity dividend strips than for short-maturity strips, giving rise to an upward-sloping term structure of equity returns. In the Bansal-Yaron (2004) long-run risks model, small shocks on highly persistent expected consumption growth and to stochastic consumption volatility gradually build up

Our model yields a downward-sloping equity term structure. Let P^D_{nt} denote the price of an n-period dividend strip. For $n = 1$, $P^D_{1t} = E_t[M_{t+1}D_{t+1}]$. For $n > 1$, we solve for P^D_{nt} recursively from $P^D_{nt} = E_t[M_{t+1}P^D_{n-1,t+1}]$. We calculate $r^D_{n,t+1} \equiv P^D_{n-1,t+1}/P^D_{nt}$ as the return of buying the n-period dividend strip at time t and selling it at $t+1$. However, as noted, dividends in the model are net payouts, which can be negative in certain states of the world. Negative prices on these dividend strips then render their returns undefined. In practice, dividends are all positive when $n \geq 67$ months. As such, we calculate the equity term structure from year 6 to 40. In contrast, consumption in the model is always positive in all states of the world. Accordingly, we also calculate the term structure of consumption strips from year 1 to 40. The definitions of price of an n-period consumption strip, P^C_{nt}, and its return, $r^C_{n,t+1}$, are exactly analogous to those of the n-period dividend strip.

Figure 2 shows that risk premiums, volatilities, and Sharpe ratios on dividend and consumption strips are largely downward-sloping in our model. From Panel A, the dividend risk premium falls from 7.91% per annum in year 6 to 6.64% in year 10 and further to 1.26% in year 40. The volatility of the dividend strip falls from 22.54% in year 6 to 18.6% in year 10 and further to 3.86% in year 40 (Panel B). The Sharpe ratio of the dividend strip starts at 0.35 in year 6, rises slightly to 0.36 in year 10, and then falls steadily to 0.32 in year 40 (Panel C). For the consumption strip, the risk premium starts at 2.37% in year 1, rises to 2.52% in year 6, and then falls gradually to 0.59% in year 40 (Panel D). Its volatility starts at 6.82% in year 1, rises to 7.04% in year 4, and then drops to 2.49% in year 40 (Panel E). The Sharpe ratio starts at 0.348 in year 1, rises slightly to 0.358 in year 8, and falls to 0.237 in year 40 (Panel F). Finally, for the wealth portfolio that pays the consumption stream as its dividends, its risk premium is 2.23%, and its volatility 5.17%.

Intuitively, short-maturity dividend and consumption strips are riskier in our model because of their higher exposures to disaster risks. When the economy slides into a disaster, short-maturity

over longer horizons to make long-maturity dividend strips riskier than short-maturity strips, again yielding an upward-sloping equity term structure. In the Rietz-Barro baseline disaster model, dividend strips of all maturities are exposed to the same amount of disaster risks, which are specified to be i.i.d., yielding a flat equity term structure. Finally, in the Wachter (2013) model with time-varying, but highly persistent disaster probabilities, small shocks on the disaster probabilities build up over time to yield an upward-sloping equity term structure.

dividends and consumption take a big hit because of inertial wages. Long-maturity dividend and consumption strips are less impacted because disasters are followed by subsequent recoveries.[25]

4.2 The Term Structure of Real Interest Rates

We calculate the prices of real zero-coupon bonds for maturities ranging from 1 month to 10 years. Let P_{nt} denote the price of an n-period zero-coupon bond. For $n = 1$, $P_{1t} = E_t[M_{t+1}]$. For $n > 1$, we solve for P_{nt} recursively from $P_{nt} = E_t[M_{t+1} P_{n-1,t+1}]$. The log yield-to-maturity is $y_{nt} \equiv -\log(P_{nt})/n$. Let $r_{n,t+1} \equiv P_{n-1,t+1}/P_{nt}$ be the return of buying the n-period zero-coupon bond at time t and selling it at $t+1$. Excess returns are in excess of the 1-month interest rate, $r_{n,t+1} - r_{ft+1}$.

To calculate the term structure, we simulate one million months from the model's stationary distribution. The real yield curve is downward sloping in the model. The yield-to-maturity starts at 1.53% per annum for 1-month zero-coupon bond but falls to 1.29% for 1-year, 0.95% for 5-year, and further to 0.72% for 10-year zero-coupon bond. The average yield spread is −0.81% for the 10-year zero-coupon bond relative to the 1-month bond. The real term premium is also negative, −1.11%, for the 10-year zero-coupon bond. Intuitively, long-term bonds earn lower average returns because these bonds are hedges against disaster risks. Disasters stimulate precautionary savings, which in turn drive down real interest rates and push up real bond prices. Because the prices of long-term bonds tend to rise at the onset of disasters, these bonds provide hedges against disaster risks and, consequently, earn lower average returns (Nakamura et al. 2013; Wachter 2013).

Evidence on the slope of the real yield curve seems mixed. A large and liquid market for inflation-indexed bonds (index-linked gilts) has existed in the UK since 1982. Evans (1998) and Piazzesi and Schneider (2007) document that real yield curve is downward sloping in the U.K. In the U.S., Treasury inflation-protected securities (TIPS) start trading in 1997. Piazzesi and Schneider show that the TIPS yield curve appears to be upward sloping but caution that interpreting the evidence

[25] Nakamura et al. (2013) show that a model with (exogenous) multiperiod disasters and subsequent recoveries also yields a downward-sloping equity term structure. Our work differs in that disasters and recoveries are endogenous.

might be complicated by the relatively short sample and poor liquidity in the TIPS market.[26]

4.3 The Timing Premium

Epstein, Farhi, and Strzalecki (2014) show that the representative investor in the Bansal-Yaron (2004) model would give up an implausibly high fraction, 31%, of its consumption stream for the early resolution of consumption risks. In the Wachter (2013) model with time-varying disaster probabilities, this fraction is even higher at 42%. Epstein et al. argue that the fractions (dubbed the timing premium) seem too high because the household cannot use the information from the early resolution to modify its risky consumption stream. Because we follow Bansal and Yaron when calibrating preference parameters, with risk aversion higher than the inverse of the elasticity of intertemporal substitution ($10 > 1/2$), it is natural to ask what the timing premium is in our model.

The timing premium is defined as $\pi \equiv 1 - J_0/J_0^{\star}$, in which J_0 is the household's utility with risks resolved gradually, and $J_0^{\star}$ is the utility with risks resolved in the next period. Formally,

$$J_0^{\star} = \left[(1-\beta)C_0^{1-\frac{1}{\psi}} + \beta\left(E_t\left[(J_1^{\star})^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \tag{27}$$

in which the continuation utility $J_1^{\star}$ is given by

$$J_1^{\star} = \left[(1-\beta)\sum_{t=1}^{\infty}\beta^{t-1}C_t^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-1/\psi}}. \tag{28}$$

Following Epstein, Farhi, and Strzalecki (2014), we calculate $J_0^{\star}$ via Monte Carlo simulations, with the economy's stochastic steady state ($N_t = 0.9137$, $K_t = 14.6909$, and $x_t = 0.1887$) as the initial condition. Specifically, we simulate in total 100,000 sample paths, each with $T = 2,500$ months, while pasting J_0 as the continuation value at T. J_0 is available from our projection algorithm. On each path, we calculate one realization of $J_1^{\star}$ using equation (28). The expectation in equation (27), $E_t\left[(J_1^{\star})^{1-\gamma}\right]$, is calculated as the cross-simulation average.

[26] We wish to point out that the downward sloping real yield curve in our model does not necessarily contradict the upward sloping nominal yield curve in the data. Nominal bonds are subject to inflation risks, which are left outside our model. Because long-term bonds are more exposed to persistent inflation risks, a positive inflation risk premium would give rise to an upward sloping nominal yield curve. We leave such an extension of our model to future work.

The timing premium in our model is only 15.3%. We view this estimate to be empirically plausible. For comparison, Epstein, Farhi, and Strzalecki (2014) calculate the timing premium to be 9.5% with i.i.d. consumption growth, a risk aversion of 10, and an elasticity of intertemporal substitution of 1.5. In the Barro (2009) model with a constant disaster probability, a risk aversion of 4, and an elasticity of intertemporal substitution of 2, the timing premium is 18%.

Intuitively, the long-run risks model assumes extremely high persistence in expected consumption growth (Bansal and Yaron 2004) or in conditional consumption volatility (Bansal, Kiku, and Yaron 2012). Analogously, the Wachter (2013) model assumes very high persistence in time-varying disaster probabilities. Because the risks are not resolved until much later, the investor that prefers early resolution of uncertainty would pay a high timing premium for the risks to be resolved early. In contrast, in our model, the expected consumption growth and conditional consumption volatility are much less persistent, as shown in equations (21) and (22), yielding a relatively low timing premium.

4.4 The Welfare Cost of Business Cycles

Lucas (1987, 2003) argues that the welfare cost of business cycles is negligible. Assuming log utility for the representative household and log-normal distribution for consumption growth, Lucas (2003) calculates that the agent would sacrifice a mere 0.05% of their consumption in perpetuity to eliminate consumption fluctuations. However, Lucas assumes log utility that fails to explain the equity premium puzzle. Atkeson and Phelan (1994), for example, argue that welfare cost calculations should be carried out within models that at least roughly replicate how asset markets price consumption risks. Because our model replicates the equity premium, we quantify its implied welfare cost.

Following Lucas (1987, 2003), we define the welfare cost of business cycles as the permanent percentage of the consumption stream that the representative household would sacrifice to eliminate aggregate consumption fluctuations. Formally, let ${}_tC \equiv \{C_t, C_{t+1}, \ldots\}$ be the consumption stream starting at time t. For a given state of the economy, (N_t, K_t, x_t), at date t, we calculate the welfare cost, denoted $\chi_t \equiv \chi(N_t, K_t, x_t)$, implicitly from:

$$J\left({}_tC(1+\chi_t)\right) = \overline{J}, \tag{29}$$

in which $\overline{J}$ is the recursive utility derived from the constant consumption at the deterministic steady state, $\overline{C}$. We solve for $\overline{J}$ by iterating on $\overline{J} = \left[(1-\beta)\overline{C}^{1-\frac{1}{\psi}} + \beta\overline{J}^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-1/\psi}}$. Because the recursive utility J_t is linear homogeneous, $J\left({}_tC(1+\chi_t)\right) = (1+\chi_t)J\left({}_tC\right)$, solving for χ_t yields:

$$\chi_t = \frac{\overline{J}}{J_t} - 1. \tag{30}$$

We calculate the welfare cost, χ_t, on the state space, (N_t, K_t, x_t). To evaluate its magnitude, we simulate one million months of χ_t from the model's stationary distribution. The average welfare cost in simulations is 29.1%, which is more than 580 times of the Lucas estimate of 0.05%. The consumption in the stochastic steady state is 3.13% lower than the deterministic steady state consumption.

Perhaps more important, the welfare cost is time-varying and strongly countercyclical. In simulation, its median is 24.4%, and the 2.5th, 5th, and 25th percentiles are 17.3%, 18.4%, and 21.5%, whereas the 75th, 95th, and 97.5th percentiles are 31.7%, 56.3%, and 66.1%, respectively. Figure 3 shows the scatterplot of the welfare cost against the productivity in simulations. The welfare cost is clearly countercyclical. Its correlations with productivity, output, unemployment, vacancy, and the investment rate are $-0.76, -0.97$, 0.94, -0.66, and -0.46, respectively. The countercyclicality of the welfare cost imply that optimal fiscal and monetary policies aimed to dampen disaster risks are even more important than what the average welfare cost of 29.1% would suggest.

5 Conclusion

Labor market frictions are crucial for explaining the equity premium puzzle in general equilibrium. A dynamic stochastic general equilibrium economy with recursive utility, search frictions, and capital accumulation yields a high equity premium of 4.26% per annum and a low average interest rate of 1.59%, while simultaneously obtaining plausible quantity dynamics. The equity premium and stock market volatility are both countercyclical, and the real interest rate and consumption growth are largely unpredictable. The welfare cost of business cycles is huge, 29%. Wage inertia plays a key role by amplifying the procyclical dynamics of profits, which in turn overcome the procyclical dynamics of investment and vacancy costs to make dividends endogenously procyclical.

Several directions arise for future research. First, one can embed our model into a New Keynesian framework to examine the nominal yield curve and the interaction between the equity premium and fiscal and monetary policies. Second, one can extend our model to a multi-country setting to study international asset prices and business cycles. Finally, one can incorporate heterogeneous firms to study how the cross-sectional distribution impacts on aggregate quantities and asset prices.

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A Computational Algorithm

We adapt the globally nonlinear projection method with parameterized expectations in Petrosky-Nadeau and Zhang (2017) to our more general setting.

We approximate the x_t process with the discrete state space method of Rouwenhorst (1995) with 17 grid points, which are sufficient to cover the values of x_t within four unconditional standard deviations from its unconditional mean, $\overline{x}$. The Rouwenhorst grid is symmetric around $\overline{x}$. The grid is also even-spaced, with the distance between any two adjacent grid points, d_x, given by:

$$d_x \equiv 2\sigma/\sqrt{(1-\rho^2)(n_x-1)}, \tag{A.1}$$

in which ρ is the persistence, σ the conditional volatility of x_t, and $n_x = 17$. We still need to construct the transition matrix, Π, in which the (i,j) element, Π_{ij}, is the probability of $x_{t+1} = x_j$ conditional on $x_t = x_i$. To this end, we set $p = (\rho+1)/2$, and:

$$\Pi^{(3)} \equiv \left[\begin{array}{ccc} p^2 & 2p(1-p) & (1-p)^2 \\ p(1-p) & p^2+(1-p)^2 & p(1-p) \\ (1-p)^2 & 2p(1-p) & p^2 \end{array}\right], \tag{A.2}$$

which is the transition matrix for $n_x = 3$. To obtain $\Pi^{(17)}$, we use the following recursion:

$$p\left[\begin{array}{cc} \Pi^{(n_x)} & \mathbf{0} \\ \mathbf{0}' & 0 \end{array}\right] + (1-p)\left[\begin{array}{cc} \mathbf{0} & \Pi^{(n_x)} \\ 0 & \mathbf{0}' \end{array}\right] + (1-p)\left[\begin{array}{cc} \mathbf{0}' & 0 \\ \Pi^{(n_x)} & \mathbf{0} \end{array}\right] + p\left[\begin{array}{cc} 0 & \mathbf{0}' \\ \mathbf{0} & \Pi^{(n_x)} \end{array}\right], \tag{A.3}$$

in which $\mathbf{0}$ is a $n_x \times 1$ column vector of zeros. We then divide all but the top and bottom rows by two to ensure that the conditional probabilities sum up to one in the resulting transition matrix, $\Pi^{(n_x+1)}$. Rouwenhorst (p. 306–307; p. 325–329) contains more details.

The state space of our model consists of employment, capital, and productivity, (N_t, K_t, x_t). The goal is to solve for the indirect utility function, $J(N_t, K_t, x_t)$, the optimal vacancy function, $V(N_t, K_t, x_t)$, the multiplier function, $\lambda(N_t, K_t, x_t)$, and the optimal investment function, $I(N_t, K_t, x_t)$, from the following three functional equations:

$$J(N_t,K_t,x_t) = \left[(1-\beta)C(N_t,K_t,x_t)^{1-\frac{1}{\psi}} + \beta\left(E_t\left[J(N_{t+1},K_{t+1},x_{t+1})^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} \tag{A.4}$$

$$\begin{aligned} \frac{1}{a_2}\left(\frac{I(N_t,K_t,x_t)}{K_t}\right)^{1/\nu} &= E_t\left[M_{t+1}\left[\frac{Y(N_{t+1},K_{t+1},x_{t+1})}{K_{t+1}}\frac{\alpha\left(K_{t+1}/K_0\right)^{\omega}}{\alpha\left(K_{t+1}/K_0\right)^{\omega}+(1-\alpha)N_{t+1}^{\omega}}\right.\right. \\ &\left.\left. + \frac{1}{a_2}\left(\frac{I(N_{t+1},K_{t+1},x_{t+1})}{K_{t+1}}\right)^{1/\nu}(1-\delta+a_1) + \frac{1}{\nu-1}\frac{I(N_{t+1},K_{t+1},x_{t+1})}{K_{t+1}}\right]\right] \end{aligned} \tag{A.5}$$

$$\begin{aligned} \frac{\kappa}{q(\theta_t)} - \lambda(N_t,K_t,x_t) &= E_t\left[M_{t+1}\left[\frac{Y(N_{t+1},K_{t+1},x_{t+1})}{N_{t+1}}\frac{(1-\alpha)N_{t+1}^{\omega}}{\alpha\left(K_{t+1}/K_0\right)^{\omega}+(1-\alpha)N_{t+1}^{\omega}} - W_{t+1}\right.\right. \\ &\left.\left. + (1-s)\left[\frac{\kappa}{q(\theta(N_{t+1},K_{t+1},x_{t+1}))} - \lambda(N_{t+1},K_{t+1},x_{t+1})\right]\right]\right], \end{aligned} \tag{A.6}$$

in which

$$M_{t+1} = \beta \left[\frac{C(N_{t+1}, K_{t+1}, x_{t+1})}{C(N_t, K_t, x_t)} \right]^{-\frac{1}{\psi}} \left[\frac{J(N_{t+1}, K_{t+1}, x_{t+1})}{E_t[J(N_{t+1}, K_{t+1}, x_{t+1})^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right]^{\frac{1}{\psi}-\gamma}. \tag{A.7}$$

Also, $V(N_t, K_t, x_t)$ and $\lambda(N_t, K_t, x_t)$ must satisfy the Kuhn-Tucker conditions.

Following Petrosky-Nadeau and Zhang (2017), we deal with $V_t \geq 0$ by exploiting a convenient mapping from the conditional expectation function, $\mathcal{E}_t \equiv \mathcal{E}(N_t, K_t, x_t)$, defined as the right-hand side of equation (A.6), to policy and multiplier functions to eliminate the need to parameterize the multiplier separately. After obtaining $\mathcal{E}_t$, we first calculate $\tilde{q}(\theta_t) = \kappa_0 / (\mathcal{E}_t - \kappa_1)$. If $\tilde{q}(\theta_t) < 1$, the $V_t \geq 0$ constraint is not binding, we set $\lambda_t = 0$ and $q(\theta_t) = \tilde{q}(\theta_t)$. We then solve $\theta_t = q^{-1}(\tilde{q}(\theta_t))$, in which $q^{-1}(\cdot)$ is the inverse function of $q(\theta_t)$, and $V_t = \theta_t(1 - N_t)$. If $\tilde{q}(\theta_t) \geq 1$, the $V_t \geq 0$ constraint is binding, we set $V_t = 0$, $\theta_t = 0$, $q(\theta_t) = 1$, and $\lambda_t = \kappa_0 + \kappa_1 - \mathcal{E}_t$. An advantage of the installation function, Φ_t, is that when investment goes to zero, the marginal benefit of investment, $\partial\Phi(I_t, K_t)/\partial I_t = a_2(I_t/K_t)^{-1/\nu}$, goes to infinity. As such, the optimal investment is always positive, with no need to impose the $I_t \geq 0$ constraint. We approximate $I(N_t, K_t, x_t)$ directly.

We approximate $J(N_t, K_t, x_t)$, $I(N_t, K_t, x_t)$, and $\mathcal{E}(N_t, K_t, x_t)$ on each grid point of x_t. We use the finite element method with cubic splines on 50 nodes on the N_t space, $[0.245, 0.975]$, and 50 nodes on the K_t space, $[5, 20]$. We experiment to ensure that the bounds are not binding at a frequency higher than 0.01% in the model's simulations. We take the tensor product of N_t and K_t for each grid point of x_t. We use the Miranda-Fackler (2002) CompEcon toolbox for function approximation and interpolation. With three functional equations on the 17-point x_t grid, the 50-point N_t grid, and the 50-point K_t grid, we must solve a system of 127,500 nonlinear equations. we use such a large system to ensure the accuracy of our numerical solution. Following Judd et al. (2014), we use derivative-free fixed point iteration with a damping parameter of 0.00325. The convergence criterion is set to be 10^{-4} for the maximum absolute value of the errors across the nonlinear functional equations.

Table 1 : Basic Properties of the Real Consumption, Output, and Investment Growth and Asset Prices in the Historical Sample

The historical cross-country panel is from the Jordà-Schularick-Taylor macrohistory database, except for Canada's asset prices, which we obtain from the Dimson-Marsh-Staunton (2002) database purchased from Morningstar. All (annual) series end in 2015. In Panels A and B, the column "Sample" indicates the sample's starting year. In Panels C and D, besides the starting year, the "Sample" column also reports the missing years in parentheses. For example, the real investment growth series for Australia starts in 1871 but is missing from 1947 to 1949. Other than Italy, which has missing asset prices from 1872 to 1884, in Panel D, all other missing years are in the 20th century. In Panel A, $\overline{g}_C$, σ_C, S_C, K_C, and $\rho_C^{(i)}$ denote the mean (in percent), volatility (in percent), skewness, kurtosis, and ith-order autocorrelation, for $i = 1, 2, \ldots, 5$, of log real per capita consumption growth. In Panel B, $\overline{g}_Y$, σ_Y, S_Y, K_Y, and $\rho_Y^{(i)}$ denote the mean (in percent), volatility (in percent), skewness, kurtosis, and ith-order autocorrelation for log real per capita output growth. In Panel C, $\overline{g}_I$, σ_I, S_I, K_I, and $\rho_I^{(i)}$ denote the mean (in percent), volatility (in percent), skewness, kurtosis, and ith-order autocorrelation for log real per capita investment growth. Finally, in Panel D, $E[\widetilde{r}_S]$, $\widetilde{\sigma}_S$, and $E[\widetilde{r}_S - r_f]$ are the average stock market return, stock market volatility, and the equity premium, respectively, without adjusting for financial leverage. $E[r_S - r_f]$ and σ_S are the equity premium and stock market volatility, respectively, after adjusting for financial leverage. $E[r_f]$ is the mean real interest rate, and σ_f the interest rate volatility. All asset pricing moments are in annual percent. We require nonmissing stocks, bonds, and bills.

	Panel A: Real consumption growth										Panel B: Real output growth									
	Sample	$\overline{g}_C$	σ_C	S_C	K_C	$\rho_C^{(1)}$	$\rho_C^{(2)}$	$\rho_C^{(3)}$	$\rho_C^{(4)}$	$\rho_C^{(5)}$	Sample	$\overline{g}_Y$	σ_Y	S_Y	K_Y	$\rho_Y^{(1)}$	$\rho_Y^{(2)}$	$\rho_Y^{(3)}$	$\rho_Y^{(4)}$	$\rho_Y^{(5)}$
Australia	1871	1.11	5.76	−0.77	6.35	−0.04	0.22	−0.03	0.03	−0.09	1871	1.45	4.11	−0.90	5.49	0.04	0.27	−0.10	−0.03	−0.05
Belgium	1914	1.35	8.72	−1.14	13.18	0.26	0.19	0.00	−0.40	−0.22	1871	1.63	7.45	1.26	19.01	0.33	0.05	0.00	0.03	−0.29
Canada	1872	1.77	4.62	−1.04	6.27	0.00	0.16	−0.16	−0.04	−0.14	1871	1.87	4.97	−0.78	5.11	0.26	0.11	−0.07	−0.15	−0.15
Denmark	1871	1.38	5.27	−0.83	11.44	−0.01	−0.41	0.06	0.18	−0.23	1871	1.68	3.66	−1.03	8.13	0.05	−0.17	0.08	0.08	−0.08
Finland	1871	2.07	5.54	−1.13	9.01	0.16	−0.08	0.02	−0.04	−0.23	1871	2.06	4.47	−0.78	7.15	0.25	−0.11	0.10	−0.12	−0.17
France	1871	1.37	6.57	−1.06	13.69	0.39	0.19	−0.06	−0.28	−0.14	1871	1.64	6.20	−0.60	10.30	0.09	−0.09	0.10	0.19	−0.09
Germany	1871	1.67	5.51	−0.57	7.11	0.25	0.24	0.28	−0.07	0.00	1871	1.62	10.66	−7.62	78.70	0.30	−0.04	−0.11	−0.16	−0.13
Italy	1871	1.47	3.63	0.14	7.62	0.38	0.32	0.10	0.08	0.11	1871	1.80	4.71	−1.32	13.34	0.27	−0.06	−0.03	0.14	0.01
Japan	1875	2.11	6.74	−1.53	20.90	0.21	0.10	0.18	0.20	0.20	1871	2.40	6.18	−2.23	15.50	0.27	0.03	0.16	0.09	0.01
Netherlands	1871	1.41	8.18	−0.83	19.86	0.17	0.13	−0.21	−0.21	−0.19	1871	1.54	6.75	0.97	32.58	0.25	−0.12	−0.02	−0.07	−0.16
Norway	1871	1.83	3.65	−0.32	12.65	−0.06	−0.34	0.26	0.07	−0.24	1871	2.10	3.53	−0.72	7.21	0.11	−0.08	0.12	0.06	−0.15
Portugal	1911	2.36	4.36	−0.49	3.30	0.22	0.23	−0.02	0.09	−0.16	1871	1.84	4.16	−0.01	4.23	0.01	0.18	0.02	0.18	0.04
Spain	1871	1.56	7.92	−2.20	17.20	0.00	−0.02	−0.13	−0.05	0.08	1871	1.86	4.98	−1.58	10.94	0.18	0.05	0.03	0.04	0.14
Sweden	1871	1.80	4.20	0.44	7.04	−0.15	−0.17	0.05	0.07	−0.20	1871	2.02	3.39	−1.32	7.30	−0.08	−0.04	0.02	0.18	−0.17
Switzerland	1871	1.22	5.85	0.35	7.34	−0.20	−0.10	−0.11	−0.10	0.04	1871	1.41	3.84	−0.41	4.02	0.13	−0.14	−0.05	0.09	0.05
UK	1871	1.33	2.76	−0.34	8.90	0.33	0.02	−0.06	−0.01	−0.11	1871	1.40	2.86	−0.89	5.62	0.35	0.03	−0.18	−0.22	−0.09
USA	1871	1.75	3.42	−0.07	3.99	0.08	0.09	−0.11	0.00	−0.10	1871	1.91	4.77	−0.08	4.83	0.25	0.08	−0.13	−0.19	−0.19
Mean		1.62	5.45	−0.67	10.34	0.12	0.04	0.00	−0.03	−0.09		1.78	5.10	−1.06	14.09	0.18	0.00	0.00	0.01	−0.09
Median		1.56	5.51	−0.77	8.90	0.16	0.10	−0.02	−0.01	−0.14		1.80	4.71	−0.78	7.30	0.25	−0.04	0.00	0.04	−0.09

	Sample	$\overline{g}_I$	σ_I	S_I	K_I	$\rho_I^{(1)}$	$\rho_I^{(2)}$	$\rho_I^{(3)}$	$\rho_I^{(4)}$	$\rho_I^{(5)}$
	Panel C: Real investment growth									
Australia	1871 (47–49)	1.60	13.56	−0.72	5.06	0.15	0.09	−0.07	−0.16	−0.07
Belgium	1901 (14–20, 40–46)	1.68	10.74	−0.20	3.44	−0.09	−0.06	−0.02	−0.23	0.14
Canada	1872	2.17	18.12	−0.18	10.68	0.27	0.02	−0.18	−0.19	−0.16
Denmark	1871 (15–22)	1.96	10.10	−0.52	6.63	0.21	−0.11	−0.05	0.00	−0.17
Finland	1871	2.40	13.24	−1.49	11.14	0.19	0.01	0.06	−0.27	−0.28
France	1871 (19–20, 45–46)	1.98	19.23	−1.33	16.16	−0.07	−0.31	−0.04	−0.08	0.15
Germany	1871 (14–20, 40–48)	2.69	14.42	−0.56	5.40	0.06	−0.01	−0.10	−0.11	−0.23
Italy	1871	2.50	12.42	1.82	23.10	0.11	−0.14	0.12	0.03	−0.08
Japan	1886 (45–46)	4.21	14.36	−0.77	13.61	0.14	−0.04	−0.07	0.00	0.08
Netherlands	1871 (14–21, 40–48)	1.78	8.23	−0.28	3.70	0.03	0.01	−0.15	−0.04	−0.21
Norway	1871 (40–46)	2.69	13.33	2.08	21.86	−0.13	−0.16	0.02	−0.04	−0.05
Portugal	1954	2.64	9.58	−0.22	3.08	0.22	0.21	0.06	−0.13	0.08
Spain	1871	2.85	13.23	−0.41	4.01	0.23	0.02	−0.23	−0.13	−0.12
Sweden	1871	2.65	12.43	0.10	4.88	0.07	−0.27	−0.08	0.01	−0.11
Switzerland	1871 (14–48)	2.58	11.02	0.69	5.33	0.37	0.17	−0.11	−0.33	−0.22
UK	1871	1.98	11.68	2.82	26.62	0.35	−0.14	−0.12	−0.03	−0.08
USA	1871	2.04	24.37	−1.71	18.02	0.17	−0.11	−0.32	−0.13	−0.02
Mean		2.38	13.53	−0.05	10.75	0.13	−0.05	−0.07	−0.11	−0.08
Median		2.40	13.23	−0.28	6.63	0.15	−0.04	−0.07	−0.11	−0.08

	Sample	$E[\widetilde{r}_S]$	$\widetilde{\sigma}_S$	$E[r_f]$	σ_f	$E[\widetilde{r}_S - r_f]$	$E[r_S - r_f]$	σ_S
	Panel D: Asset prices							
Australia	1900 (45–47)	7.75	17.08	1.29	4.32	6.46	4.58	12.55
Belgium	1871 (14–19)	6.31	19.88	1.21	8.43	5.10	3.62	14.62
Canada	1900	7.01	17.00	1.60	4.79	5.41	3.84	12.26
Denmark	1875 (15)	7.47	16.43	3.08	5.68	4.39	3.12	11.91
Finland	1896	8.83	30.57	−0.74	10.93	9.57	6.80	22.98
France	1871 (15–21)	3.99	22.22	−0.47	7.78	4.45	3.16	16.75
Germany	1871 (23, 44–49)	8.83	27.59	−0.23	13.22	9.05	6.43	20.22
Italy	1871 (1872–84, 15–21)	6.63	27.21	0.58	10.50	6.05	4.29	20.41
Japan	1886 (46–47)	8.86	27.69	0.00	11.20	8.87	6.29	21.10
Netherlands	1900	6.96	21.44	0.78	4.91	6.19	4.39	15.32
Norway	1881	5.67	19.82	0.90	5.98	4.77	3.39	14.53
Portugal	1880	3.81	25.68	−0.01	9.43	3.82	2.71	19.29
Spain	1900 (36–40)	6.25	21.41	−0.04	6.90	6.29	4.47	15.94
Sweden	1871	8.00	19.54	1.77	5.60	6.23	4.42	14.26
Switzerland	1900 (15)	6.69	19.08	0.89	5.00	5.79	4.11	14.00
UK	1871	6.86	17.77	1.16	4.82	5.70	4.05	12.96
USA	1872	8.40	18.68	2.17	4.65	6.23	4.43	13.66
Mean		6.96	21.71	0.82	7.30	6.14	4.36	16.04
Median		6.96	19.88	0.89	5.98	6.05	4.29	14.62

Table 2 : Basic Moments in the Model Under the Benchmark Calibration

The model moments are based on 10,000 simulated samples, each with 1,740 months. On each artificial sample, we calculate the moments and report the mean as well as the 5th, 50th, and 95th percentiles across the 10,000 simulations. p-value is the fraction with which a model moment is higher than its data moment. The data moments are from Table 1. In Panel A, σ_C, S_C, K_C, and ρ_{Ci}, for $i = 1, 2, \ldots, 5$, denote the volatility (in percent), skewness, kurtosis, and ith-order autocorrelation of the log consumption growth. The symbols in Panels B and C are defined analogously. In Panel D, $E[U]$, S_U, and K_U are the mean, skewness, and kurtosis of monthly unemployment rates, σ_U, σ_V, and σ_θ are the volatilities of quarterly unemployment, vacancy, and labor market tightness, respectively. ρ_{UV} is the cross-correlation of quarterly unemployment and vacancy rates, and $e_{w,y/n}$ the wage elasticity to labor productivity. In Panel E, $E[r_S - r_f]$, $E[r_f]$, σ_S, and σ_f are the average equity premium, average real interest rate, stock market volatility, and interest rate volatility, respectively, all of which are in annual percent.

	Data	Mean	5th	50th	95th	p		Data	Mean	5th	50th	95th	p
Panel A: Real consumption growth							Panel B: Real output growth						
σ_C	5.45	5.13	2.87	5.13	7.39	0.41	σ_Y	5.10	6.43	4.46	6.40	8.48	0.86
S_C	−0.67	0.03	−1.03	0.03	1.10	0.89	S_Y	−1.06	0.09	−0.62	0.08	0.81	0.99
K_C	10.34	8.09	4.38	7.30	14.44	0.18	K_Y	14.09	5.45	3.50	5.09	8.64	0.00
ρ_{C1}	0.12	0.21	−0.01	0.22	0.40	0.78	ρ_{Y1}	0.18	0.20	0.03	0.21	0.36	0.60
ρ_{C2}	0.04	−0.05	−0.26	−0.05	0.17	0.24	ρ_{Y2}	0.00	−0.06	−0.23	−0.06	0.12	0.31
ρ_{C3}	0.00	−0.04	−0.24	−0.04	0.16	0.35	ρ_{Y3}	0.00	−0.05	−0.22	−0.05	0.12	0.31
ρ_{C4}	−0.03	−0.04	−0.23	−0.04	0.15	0.44	ρ_{Y4}	0.01	−0.05	−0.21	−0.05	0.12	0.29
ρ_{C5}	−0.09	−0.04	−0.23	−0.04	0.14	0.67	ρ_{Y5}	−0.09	−0.05	−0.21	−0.05	0.12	0.65
Panel C: Real investment growth							Panel D: Labor market moments						
σ_I	13.53	8.59	5.29	8.61	11.83	0.00	$E[U]$	8.94	8.63	3.81	7.45	17.63	0.37
S_I	−0.05	0.31	−0.57	0.28	1.26	0.76	S_U	2.13	2.64	0.76	2.20	5.85	0.53
K_I	10.75	7.12	4.12	6.47	12.17	0.08	K_U	9.50	13.45	2.11	6.77	39.06	0.35
ρ_{I1}	0.13	0.15	−0.04	0.16	0.33	0.58	σ_U	0.24	0.32	0.16	0.32	0.48	0.76
ρ_{I2}	−0.05	−0.11	−0.29	−0.11	0.08	0.30	σ_V	0.19	0.34	0.23	0.32	0.49	1.00
ρ_{I3}	−0.07	−0.09	−0.27	−0.09	0.10	0.45	σ_θ	0.62	0.34	0.23	0.32	0.50	0.01
ρ_{I4}	−0.11	−0.07	−0.25	−0.07	0.11	0.62	ρ_{UV}	−0.57	−0.07	−0.16	−0.07	0.01	1.00
ρ_{I5}	−0.08	−0.06	−0.24	−0.06	0.12	0.56	$e_{w,y/n}$	0.27	0.26	0.23	0.26	0.27	0.22
Panel E: Asset pricing moments													
$E[r_S - r_f]$	4.36	4.26	3.52	4.12	5.49	0.34							
$E[r_f]$	0.82	1.59	0.07	1.83	2.26	0.87							
σ_S	16.04	11.77	9.19	11.74	14.46	0.00							
σ_f	7.30	3.13	1.13	3.05	5.37	0.00							

Table 3 : Disaster Moments in the Data and in the Model

The data moments are obtained by applying the Barro-Ursúa (2008) peak-to-trough method on the Jordà-Schularick-Taylor cross-country panel. We adjust for trend growth in the data (no growth in our model). We subtract each log annual consumption growth observation with its mean of 1.62% and subtract each log annual output growth with the mean of 1.78% in the historical panel. For model moments, we simulate 10,000 artificial samples from the model's stationary distribution under the benchmark calibration, each with 1,740 months, matching the number of years, 145, from 1871 to 2015. On each artificial sample, we time-aggregate consumption and output into annual observations and apply the peak-to-trough method to identify disasters as cumulative fractional declines of consumption or output of at least 10% or 15%. We report the mean, 5th, 50th, and 95th percentiles across the simulations. If no disaster appears in an artificial sample, we set its disaster probability to zero and calculate the model's disaster probability moments across all the 10,000 simulations. However, we calculate disaster size and duration across samples with at least one disaster. The disaster probability and size are in percent, and duration in the number of years.

	Data	Mean	5th	50th	95th	p	Data	Mean	5th	50th	95th	p
	Disaster hurdle = 10%						Disaster hurdle = 15%					
	Panel A: Consumption disasters											
Probability	6.40	5.83	1.55	5.31	11.32	0.37	3.51	3.64	0.71	3.20	7.69	0.45
Size	23.16	23.41	14.52	22.83	33.84	0.48	30.36	29.51	18.81	28.49	43.31	0.38
Duration	4.19	4.10	2.80	4.00	5.80	0.40	4.50	4.49	3.00	4.33	6.81	0.39
	Panel B: Output disasters											
Probability	5.78	10.9	6.14	10.58	16.48	0.97	2.62	6.10	2.33	5.88	10.68	0.94
Size	22.34	22.31	15.91	21.89	30.13	0.46	32.9	28.50	20.07	27.88	38.86	0.20
Duration	4.14	3.73	2.89	3.67	4.78	0.23	5.04	4.25	3.00	4.17	5.75	0.15

Table 4 : Predicting Excess Returns and Consumption Growth with Log Price-to-consumption in the Historical Sample

The cross-country panel is from the Jordà-Schularick-Taylor macrohistory database, except for Canada. The annuals series start as early as 1870 and end in 2015 (the Internet Appendix). Panel A performs predictive regressions of stock market excess returns on log price-to-consumption, $\sum_{h=1}^{H}[\log(r_{St+h}) - \log(r_{ft+h})] = a + b\log(P_t/C_t) + u_{t+H}$, in which H is the forecast horizon, r_{St+1} the real stock market return, r_{ft+1} the real interest rate, P_t the real stock market index, and C_t real consumption. r_{St+1} and r_{ft+1} are over the course of period t, and P_t and C_t are at the beginning of period t (the end of period $t-1$). Excess returns are adjusted for a financial leverage ratio of 0.29. Panel B performs long-horizon predictive regressions of log consumption growth on $\log(P_t/C_t)$, $\sum_{h=1}^{H}\log(C_{t+h}/C_t) = c + d\log(P_t/C_t) + v_{t+H}$. In both regressions, $\log(P_t/C_t)$ is standardized to have a mean of zero and a standard deviation of one. H ranges from one year (1y) to five years (5y). The t-values of the slopes are adjusted for heteroscedasticity and autocorrelations of $2(H-1)$ lags. The slopes and R-squares are in percent.

	Slopes					t-values					R-squares				
	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
	Panel A: Predicting stock market excess returns														
Australia	−1.42	−2.49	−2.92	−3.53	−3.77	−1.97	−1.96	−1.80	−1.74	−1.62	1.80	3.14	3.56	4.20	4.29
Belgium	−1.30	−3.26	−4.79	−5.48	−5.16	−0.82	−0.98	−1.00	−0.91	−0.76	0.58	1.62	2.47	2.58	2.01
Denmark	−0.81	−1.94	−2.87	−3.74	−4.24	−0.85	−1.18	−1.43	−1.81	−2.14	0.50	1.35	2.13	3.04	3.76
Finland	−1.38	−3.79	−5.40	−6.40	−7.36	−0.77	−1.05	−1.06	−1.07	−1.22	0.55	1.78	2.55	3.05	3.78
France	−0.12	−0.34	−0.52	−0.63	−0.43	−0.11	−0.18	−0.21	−0.20	−0.11	0.01	0.03	0.05	0.05	0.02
Germany	−1.04	−2.11	−2.06	−1.54	0.13	−0.75	−0.91	−0.54	−0.28	0.02	0.19	0.34	0.21	0.08	0.00
Italy	−0.36	−0.58	−0.36	−0.07	0.38	−0.25	−0.22	−0.10	−0.01	0.07	0.04	0.05	0.01	0.00	0.01
Japan	−0.70	−1.40	−1.60	−1.73	−1.77	−0.45	−0.56	−0.45	−0.41	−0.36	0.19	0.36	0.35	0.30	0.24
Netherlands	−3.03	−6.45	−8.88	−11.06	−13.35	−1.68	−1.88	−2.11	−2.48	−2.98	4.15	9.00	12.73	16.34	20.25
Norway	−1.77	−3.59	−5.13	−6.52	−7.92	−1.55	−2.07	−2.41	−2.76	−3.24	1.75	3.61	5.60	7.68	9.84
Portugal	−0.24	−2.39	−3.87	−3.41	0.53	−0.08	−0.39	−0.50	−0.37	0.06	0.02	0.55	0.83	0.48	0.01
Spain	−1.02	−2.77	−4.90	−6.80	−8.13	−0.74	−0.92	−1.21	−1.66	−2.38	0.59	1.68	3.13	4.42	5.25
Sweden	−1.56	−3.81	−6.04	−8.31	−10.50	−1.63	−2.29	−2.91	−3.19	−3.20	1.42	3.74	6.47	9.64	13.08
Switzerland	−3.09	−6.51	−8.50	−10.67	−12.95	−1.70	−2.30	−2.85	−3.89	−4.17	4.02	8.50	11.76	15.72	20.05
UK	−2.95	−5.64	−7.62	−8.91	−10.51	−2.33	−4.92	−5.43	−5.84	−5.92	6.35	12.49	18.14	23.18	28.03
USA	−3.50	−7.45	−9.89	−12.98	−15.75	−3.83	−4.50	−4.35	−4.59	−5.16	7.71	16.13	21.01	27.48	33.59
Mean	−1.52	−3.41	−4.71	−5.74	−6.30	−1.22	−1.64	−1.77	−1.95	−2.07	1.87	4.02	5.69	7.39	9.01
Median	−1.34	−3.01	−4.84	−5.94	−6.26	−0.83	−1.11	−1.32	−1.70	−1.88	0.59	1.73	2.84	3.63	4.03
	Panel B: Predicting consumption growth														
Australia	0.75	0.98	1.14	1.50	1.85	1.40	0.88	0.65	0.67	0.71	1.69	1.52	1.21	1.49	1.75
Belgium	−1.03	−1.38	−0.94	−0.68	−0.10	−0.91	−0.73	−0.41	−0.26	−0.04	1.41	1.05	0.30	0.11	0.00
Denmark	0.23	0.32	0.28	0.24	0.20	0.71	0.73	0.52	0.40	0.29	0.18	0.18	0.13	0.08	0.05
Finland	−0.91	−2.10	−2.90	−3.62	−4.07	−1.14	−1.46	−1.54	−1.67	−1.68	2.30	5.20	6.56	7.56	7.66
France	−0.84	−1.47	−2.02	−2.55	−3.18	−2.12	−1.81	−1.81	−1.85	−1.95	1.64	1.79	1.89	2.11	2.67
Germany	−0.95	−1.87	−2.88	−3.79	−4.70	−2.15	−1.85	−1.81	−1.74	−1.74	2.97	4.64	6.17	6.84	7.79
Italy	−0.60	−1.22	−1.74	−2.28	−2.91	−2.71	−2.21	−1.96	−1.87	−1.89	2.74	4.02	4.32	4.84	5.79
Japan	−1.76	−3.59	−5.38	−7.12	−8.78	−4.04	−3.35	−2.89	−2.60	−2.40	8.22	11.84	14.23	15.81	16.95
Netherlands	0.66	1.10	1.43	1.83	2.32	2.41	1.47	1.22	1.17	1.14	7.27	6.03	5.50	6.17	7.48
Norway	−0.35	−0.77	−1.21	−1.68	−2.10	−1.36	−1.80	−2.11	−2.40	−2.54	0.91	2.40	5.58	8.09	9.68
Portugal	−1.05	−2.20	−3.26	−4.08	−4.95	−2.18	−1.72	−1.67	−1.61	−1.53	4.82	8.98	10.91	11.55	11.55
Spain	−0.10	−0.18	−0.41	−0.67	−1.10	−0.14	−0.14	−0.27	−0.38	−0.55	0.02	0.02	0.08	0.17	0.40
Sweden	0.18	0.22	0.20	0.02	−0.17	0.56	0.44	0.28	0.02	−0.17	0.18	0.15	0.10	0.00	0.05
Switzerland	0.22	0.31	0.36	0.35	0.34	1.32	0.84	0.61	0.43	0.33	2.52	1.40	1.00	0.64	0.44
UK	−0.33	−0.89	−1.53	−2.32	−3.15	−1.78	−2.22	−2.77	−3.53	−4.16	1.44	3.94	7.06	11.86	17.32
USA	0.48	−0.09	−0.64	−1.05	−1.40	1.86	−0.18	−0.85	−1.08	−1.23	1.89	0.03	0.94	1.92	2.70
Mean	−0.34	−0.80	−1.22	−1.62	−1.99	−0.64	−0.82	−0.93	−1.02	−1.09	2.51	3.32	4.12	4.95	5.77
Median	−0.34	−0.83	−1.07	−1.36	−1.75	−1.02	−1.09	−1.20	−1.35	−1.38	1.79	2.09	3.10	3.48	4.25

Table 5 : Predicting Volatilities of Stock Market Excess Returns and Consumption Growth with Log Price-to-consumption in the Historical Sample

The cross-country panel is from the Jordà-Schularick-Taylor macrohistory database, except for Canada. The annuals series start in 1870 and end in 2015. For a given forecast horizon, H, we measure excess return volatility as $\sigma_{St,t+H-1} = \sum_{h=0}^{H-1} |\epsilon_{St+h}|$, in which ϵ_{St+h} is the h-period-ahead residual from the first-order autoregression of excess returns, $\log(r_{St+1}) - \log(r_{ft+1})$. Excess returns are adjusted for a financial leverage ratio of 0.29. Panel A performs long-horizon predictive regressions of excess return volatilities, $\log \sigma_{St+1,t+H} = a + b \log(P_t/C_t) + u^{\sigma}_{t+H}$. Consumption growth volatility is $\sigma_{Ct,t+H-1} = \sum_{h=0}^{H-1} |\epsilon_{Ct+h}|$, in which ϵ_{Ct+h} is the h-period-ahead residual from the first-order autoregression of log consumption growth, $\log(C_{t+1}/C_t)$. Panel B performs long-horizon predictive regressions of consumption growth volatilities, $\log \sigma_{Ct+1,t+H} = c + d \log(P_t/C_t) + v^{\sigma}_{t+H}$. $\log(P_t/C_t)$ is standardized to have a mean of zero and a standard deviation of one. H ranges from one year (1y) to five years (5y). The t-values are adjusted for heteroscedasticity and autocorrelations of $2(H-1)$ lags. The slopes and R-squares are in percent.

	Slopes					t-values					R-squares				
	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
	Panel A: Predicting stock market volatility														
Australia	20.04	16.84	15.73	16.20	15.82	1.89	1.91	1.81	1.81	1.80	2.15	3.55	4.28	5.51	6.55
Belgium	11.85	12.70	12.20	11.69	11.61	1.28	2.11	2.05	1.99	2.21	1.42	5.12	7.26	8.84	10.71
Denmark	−35.30	−37.64	−38.17	−37.40	−36.44	−3.70	−4.02	−3.95	−3.74	−3.43	7.90	16.11	21.11	24.13	25.79
Finland	6.94	3.22	5.56	5.49	3.54	0.66	0.45	0.97	1.02	0.65	0.42	0.23	1.26	1.56	0.76
France	−58.81	−60.73	−60.03	−58.54	−57.57	−6.19	−6.50	−5.93	−5.51	−5.17	20.49	37.29	42.82	45.17	46.37
Germany	−31.59	−35.33	−35.20	−34.06	−32.90	−2.89	−3.61	−3.42	−3.02	−2.67	5.44	12.60	16.61	17.96	18.71
Italy	−17.60	−23.51	−24.80	−24.34	−24.27	−1.92	−2.94	−2.86	−2.59	−2.41	2.45	8.22	12.62	15.43	17.69
Japan	8.99	7.32	9.12	10.91	11.75	0.80	0.74	0.94	1.15	1.26	0.48	0.72	1.89	3.41	4.74
Netherlands	7.49	8.46	11.28	10.93	8.97	0.50	0.67	1.03	1.18	1.15	0.48	1.43	4.60	5.52	5.30
Norway	−51.27	−54.63	−54.25	−53.32	−52.54	−5.44	−7.48	−7.26	−7.41	−7.81	20.22	39.57	51.02	56.54	60.54
Portugal	−50.20	−45.97	−44.35	−43.46	−39.43	−4.10	−3.57	−3.50	−3.56	−3.39	14.11	23.07	27.71	28.72	25.37
Spain	−37.40	−34.97	−34.23	−33.42	−32.51	−4.00	−5.24	−4.81	−4.51	−3.96	10.86	18.97	26.06	30.91	33.48
Sweden	−23.98	−22.89	−21.83	−21.84	−21.98	−2.75	−2.62	−2.16	−1.93	−1.79	4.88	8.45	9.78	10.82	11.85
Switzerland	7.05	11.57	9.51	11.11	11.03	0.39	0.87	0.90	1.18	1.30	0.27	2.01	3.01	5.79	7.64
UK	−35.31	−34.28	−33.22	−32.10	−31.62	−4.99	−4.69	−4.10	−3.58	−3.23	9.59	18.29	21.60	22.69	23.91
USA	0.30	5.54	6.58	7.08	8.06	0.03	0.87	1.57	2.13	2.51	0.00	0.65	1.77	2.89	4.86
Mean	−17.43	−17.77	−17.26	−16.57	−16.16	−1.90	−2.07	−1.80	−1.59	−1.44	6.32	12.27	15.84	17.87	19.02
Median	−20.79	−23.20	−23.32	−23.09	−23.12	−2.34	−2.78	−2.51	−2.26	−2.10	3.67	8.34	11.20	13.12	14.77
	Panel B: Predicting consumption growth volatility														
Australia	3.23	−3.95	−3.07	−4.13	−5.52	0.28	−0.39	−0.27	−0.32	−0.40	0.06	0.18	0.14	0.28	0.55
Belgium	48.77	54.27	55.42	58.66	59.15	2.88	3.74	3.41	3.50	3.50	11.11	23.57	29.61	36.63	40.03
Denmark	−2.11	−1.62	0.21	0.64	1.13	−0.17	−0.15	0.02	0.05	0.09	0.02	0.03	0.00	0.01	0.03
Finland	32.87	35.10	38.93	40.82	41.42	2.38	2.84	3.27	3.61	3.79	6.84	16.21	25.65	30.35	33.31
France	84.04	78.92	77.39	76.70	76.69	9.11	9.59	9.25	8.72	8.35	37.34	53.32	60.94	65.91	68.76
Germany	11.37	11.62	13.29	14.96	16.28	1.14	0.98	0.95	0.96	0.95	0.77	1.42	2.15	3.01	3.75
Italy	6.73	7.80	8.51	9.88	11.70	0.78	1.01	1.06	1.15	1.30	0.36	0.89	1.60	2.71	4.48
Japan	37.88	39.76	39.78	39.88	39.93	3.66	3.72	3.54	3.26	3.07	8.50	15.96	21.30	23.11	24.82
Netherlands	7.04	7.92	9.68	9.26	8.42	0.60	0.73	0.85	0.89	0.90	0.58	1.51	3.20	3.47	3.56
Norway	3.69	5.69	4.34	3.81	3.63	0.34	0.50	0.37	0.33	0.32	0.09	0.38	0.28	0.27	0.29
Portugal	13.68	15.62	16.08	18.09	19.63	1.43	2.51	3.57	5.20	5.93	2.03	6.49	10.05	15.19	18.62
Spain	64.78	61.73	59.39	57.46	56.29	6.29	6.16	5.80	5.51	5.46	25.68	40.34	49.05	51.18	54.04
Sweden	−1.44	1.56	3.93	6.22	7.43	−0.14	0.17	0.39	0.57	0.64	0.01	0.03	0.26	0.79	1.32
Switzerland	−13.49	−13.67	−13.37	−9.64	−6.97	−1.01	−1.06	−1.11	−0.84	−0.69	1.40	2.71	3.78	2.63	1.61
UK	0.76	0.75	1.50	2.03	2.31	0.07	0.11	0.22	0.27	0.30	0.00	0.01	0.06	0.14	0.23
USA	−18.05	−19.66	−18.25	−17.46	−15.82	−1.89	−2.05	−1.87	−1.71	−1.45	2.44	5.14	6.30	6.70	6.07
Mean	17.49	17.62	18.36	19.20	19.73	1.61	1.78	1.84	1.95	2.00	6.08	10.51	13.40	15.15	16.34
Median	6.88	7.86	9.10	9.57	10.06	0.69	0.85	0.90	0.93	0.92	1.09	2.11	3.49	3.24	4.12

Table 6 : Predicting Excess Returns, Consumption Growth, and Their Volatilities with Log Price-to-consumption in the Model

The data moments are the mean estimates in Tables 4 and 5 on the Jordà-Schularick-Taylor database. For the model moments, we simulate 10,000 artificial samples from the model's stationary distribution (with a burn-in of 1,200 months), each with 1,740 months. On each artificial sample, we time-aggregate monthly market excess returns and consumption growth into annual observations and implement the exactly same procedures as in Tables 4 and 5. We report the mean, 5th, 50th, and 95th percentiles across the simulations as well as the p-value that is the fraction of simulations with which a given model moment is higher than its data moment. In all the long-horizon regressions, the log price-to-consumption ratio, $\log(P_t/C_t)$, is standardized to have a mean of zero and a standard deviation of one. The forecast horizon, H, ranges from one year (1y) to five years (5y). The t-values of the slopes are adjusted for heteroscedasticity and autocorrelations of $2(H-1)$ lags. The slopes and R-squares are in percent.

	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
	Panel A: Predicting stock market excess returns														
	Data					Mean					p				
b	−1.52	−3.41	−4.71	−5.74	−6.30	−1.82	−3.45	−4.91	−6.22	−7.40	0.33	0.50	0.47	0.42	0.34
t	−1.22	−1.64	−1.77	−1.95	−2.07	−2.36	−2.86	−3.17	−3.39	−3.57	0.09	0.09	0.08	0.08	0.09
R^2	1.87	4.02	5.69	7.39	9.01	3.86	6.83	9.40	11.59	13.52	0.77	0.76	0.78	0.77	0.76
	5th					50th					95th				
b	−2.96	−5.40	−7.61	−9.66	−11.55	−1.81	−3.41	−4.84	−6.13	−7.29	−0.74	−1.64	−2.41	−3.10	−3.64
t	−3.87	−4.49	−4.99	−5.33	−5.68	−2.33	−2.81	−3.10	−3.31	−3.48	−0.93	−1.36	−1.59	−1.71	−1.79
R^2	0.58	1.67	2.62	3.55	4.31	3.44	6.35	8.86	11.00	13.09	8.64	13.70	17.94	21.34	24.49
	Panel B: Predicting consumption growth														
	Data					Mean					p				
b	−0.34	−0.80	−1.22	−1.62	−1.99	−1.27	−1.86	−2.44	−3.00	−3.52	0.01	0.06	0.09	0.12	0.13
t	−0.64	−0.82	−0.93	−1.02	−1.09	−2.69	−2.41	−2.49	−2.64	−2.79	0.01	0.07	0.10	0.13	0.13
R^2	2.51	3.32	4.12	4.95	5.77	7.34	7.20	8.44	9.86	11.27	0.88	0.74	0.70	0.68	0.68
	5th					50th					95th				
b	−2.02	−3.06	−4.09	−5.07	−6.03	−1.24	−1.83	−2.41	−2.95	−3.47	−0.61	−0.71	−0.83	−0.99	−1.12
t	−4.47	−4.57	−4.96	−5.41	−5.74	−2.63	−2.29	−2.32	−2.44	−2.60	−1.15	−0.69	−0.55	−0.53	−0.51
R^2	1.36	0.65	0.53	0.54	0.58	6.68	6.11	6.95	8.25	9.55	15.51	17.63	21.02	24.69	27.92
	Panel C: Predicting excess return volatilities														
	Data					Mean					p				
b	−17.43	−17.77	−17.26	−16.57	−16.16	−15.94	−13.55	−12.03	−11.01	−10.15	0.55	0.68	0.75	0.78	0.81
t	−1.90	−2.07	−1.80	−1.59	−1.44	−1.48	−1.73	−1.84	−1.89	−1.88	0.64	0.62	0.49	0.41	0.37
R^2	6.32	12.27	15.84	17.87	19.02	2.12	3.30	4.54	5.61	6.34	0.06	0.02	0.02	0.03	0.04
	5th					50th					95th				
b	−36.81	−28.96	−25.35	−23.21	−21.78	−15.80	−13.46	−12.02	−10.95	−10.04	4.60	1.70	0.99	0.68	0.91
t	−3.43	−3.69	−3.90	−4.03	−4.06	−1.46	−1.72	−1.82	−1.85	−1.84	0.42	0.20	0.15	0.11	0.17
R^2	0.02	0.04	0.06	0.07	0.08	1.37	2.37	3.38	4.24	4.80	6.73	9.73	13.01	15.98	18.01
	Panel D: Predicting consumption growth volatilities														
	Data					Mean					p				
b	17.49	17.72	18.36	19.20	19.73	−34.67	−32.89	−31.47	−30.14	−28.85	0.00	0.00	0.00	0.00	0.00
t	1.61	1.78	1.84	1.95	2.00	−3.36	−3.98	−4.03	−3.95	−3.82	0.00	0.00	0.00	0.00	0.00
R^2	6.08	10.51	13.40	15.15	16.34	7.69	13.16	15.89	17.19	17.73	0.58	0.62	0.60	0.57	0.54
	5th					50th					95th				
b	−56.53	−51.99	−49.84	−47.99	−46.64	−35.58	−34.16	−32.69	−31.32	−29.94	−8.95	−9.24	−8.75	−7.93	−7.04
t	−5.81	−6.88	−7.22	−7.23	−7.15	−3.39	−4.02	−4.01	−3.88	−3.72	−0.77	−0.98	−0.98	−0.96	−0.91
R^2	0.58	1.31	1.80	1.94	1.88	7.06	12.80	15.58	16.84	17.38	16.72	26.19	30.99	33.55	34.79

Table 7 : Comparative Statics

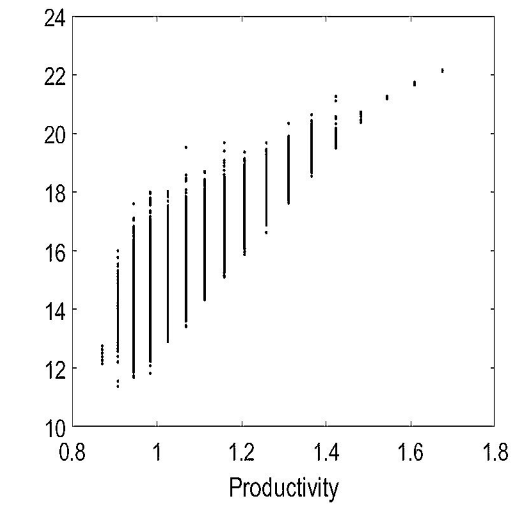
The first column of numbers shows the model moments from the benchmark calibration. The remaining columns show the model moments from 14 comparative statics. γ is relative risk aversion; ψ the intertemporal elasticity of substitution; b the flow value of unemployment; η the bargaining weight for workers; s the separation rate; ι the curvature of the matching function; κ the unit cost of vacancy posting; ν the adjustment cost parameter; δ the capital depreciation rate; $1/(1-\omega)$ the elasticity of capital-labor substitution; and α the distribution parameter. In each experiment, all the other parameters are identical to those in the benchmark calibration. For the model moments, σ_C is the consumption growth volatility per annum, ρ_{C1} the first-order autocorrelation of consumption growth, and Prob_C, Size_C, and Dur_C the probability, size, and duration of consumption disasters with a cumulative decline hurdle rate of 10%. σ_Y is the output growth volatility, ρ_{Y1} the first-order autocorrelation of output growth, and Prob_Y, Size_Y, and Dur_Y the probability, size, and duration of output disasters with a cumulative decline hurdle rate of 10%. σ_I is the investment growth volatility, ρ_{I1} the first-order autocorrelation of investment growth. The consumption, output, and investment volatilities, and the probability and size of consumption and output disasters are in percent. Their durations are in years. $E[U]$ is mean unemployment rate, σ_U, σ_V, and σ_θ the quarterly volatilities of unemployment, vacancy, and labor market tightness, respectively, ρ_{UV} the cross-correlation of unemployment and vacancy, and $e_{w,y/n}$ the wage elasticity to labor productivity. Finally, $E[r_S - r_f]$ is the average equity premium, $E[r_f]$ the average interest rate, σ_S stock market volatility, and σ_f the interest rate volatility, all of which are in annual percent.

	Benchmark	γ	γ	ψ	ψ	γ, ψ	b	η	s	ι	κ	ν	δ	ω	α
		7.5	5	1.5	1	1	0.85	0.025	0.0325	1.35	0.025	1.5	0.01	−1	0.3
σ_C	5.13	4.24	3.94	4.89	4.51	3.83	2.62	5.19	5.17	5.09	5.24	4.98	4.71	5.78	4.26
ρ_{C1}	0.21	0.18	0.15	0.20	0.19	0.16	0.14	0.22	0.21	0.21	0.22	0.23	0.17	0.19	0.21
Prob_C	5.83	4.28	3.82	5.40	4.77	3.54	2.36	6.42	5.89	5.74	5.93	5.46	5.26	6.31	5.10
Size_C	23.41	20.69	19.36	22.68	21.68	19.70	13.80	22.65	23.36	23.23	23.70	23.68	21.34	25.04	20.27
Dur_C	4.10	4.46	4.46	4.16	4.29	4.52	4.98	4.12	4.12	4.11	4.10	4.21	4.12	3.95	4.34
σ_Y	6.43	5.58	5.17	6.23	5.91	5.21	4.11	6.37	6.45	6.40	6.52	6.45	5.98	6.97	5.62
ρ_{Y1}	0.20	0.18	0.16	0.20	0.19	0.17	0.15	0.21	0.21	0.20	0.21	0.21	0.17	0.20	0.20
Prob_Y	10.90	9.37	8.61	10.47	9.99	8.66	7.44	10.91	10.85	10.80	10.99	10.76	10.17	11.31	9.99
Size_Y	22.31	20.03	18.94	21.76	20.91	19.13	16.00	22.12	22.32	22.15	22.50	22.44	20.84	23.38	20.27
Dur_Y	3.73	3.84	3.88	3.74	3.78	3.88	4.00	3.73	3.73	3.73	3.72	3.75	3.72	3.66	3.81
σ_I	8.59	6.27	4.56	8.13	7.36	5.32	2.55	8.45	8.67	8.54	8.66	9.41	7.30	8.91	6.71
ρ_{I1}	0.15	0.13	0.11	0.15	0.14	0.11	0.09	0.16	0.16	0.15	0.16	0.15	0.14	0.16	0.15
$E[U]$	8.63	5.71	4.63	7.90	6.87	4.90	3.45	8.81	8.51	8.50	8.90	8.54	6.86	9.06	7.20
σ_U	0.32	0.35	0.35	0.33	0.34	0.34	0.07	0.31	0.33	0.32	0.32	0.32	0.35	0.36	0.30
σ_V	0.34	0.27	0.24	0.32	0.30	0.24	0.16	0.34	0.34	0.34	0.33	0.33	0.30	0.35	0.31
σ_θ	0.34	0.27	0.25	0.32	0.30	0.24	0.16	0.34	0.34	0.34	0.34	0.33	0.31	0.35	0.31
ρ_{UV}	−0.07	−0.08	−0.09	−0.08	−0.08	−0.09	−0.30	−0.07	−0.07	−0.07	−0.08	−0.07	−0.08	−0.08	−0.08
$e_{w,y/n}$	0.26	0.26	0.26	0.26	0.26	0.26	0.27	0.37	0.26	0.26	0.26	0.26	0.26	0.25	0.26
$E[r_S - r_f]$	4.26	1.55	0.54	3.82	3.17	0.53	0.45	3.98	4.41	4.30	4.02	4.03	2.57	4.72	2.27
$E[r_f]$	1.59	2.45	2.75	1.58	1.54	2.68	2.82	1.67	1.49	1.51	1.83	1.62	2.26	1.38	2.29
σ_S	11.77	9.50	7.99	11.32	10.61	8.68	7.33	11.13	11.91	11.72	11.79	11.05	10.01	12.13	9.15
σ_f	3.13	2.27	1.78	3.74	4.60	3.32	0.64	2.95	3.09	3.25	2.81	3.11	2.36	3.46	2.23

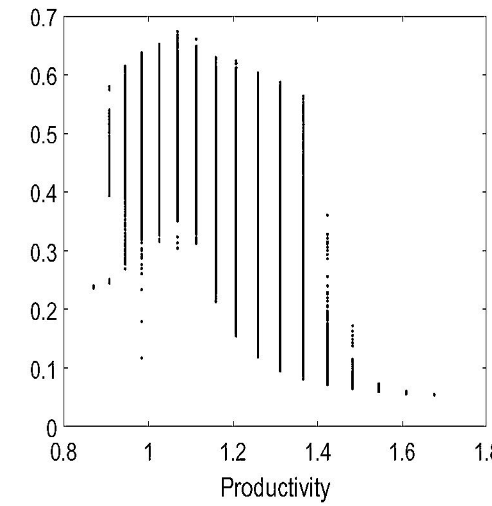
Figure 1 : Scatterplots of Key Moments Against Productivity

From the model's stationary distribution with the benchmark calibration (after a burn-in period of 1,200 monthly periods), we simulate a long sample path with one million months. The equity premium, stock market volatility, and consumption volatility are in percent.

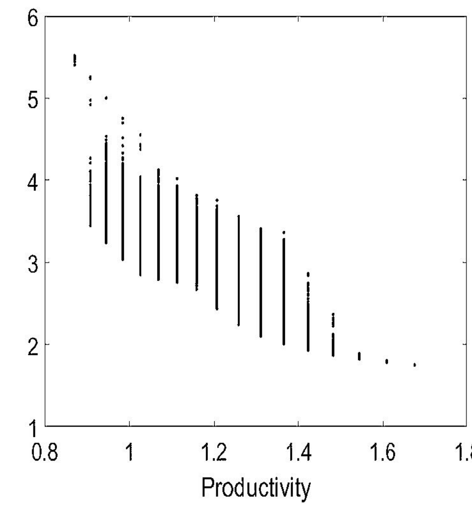
Panel A: Price-to-consumption, P_t/C_t



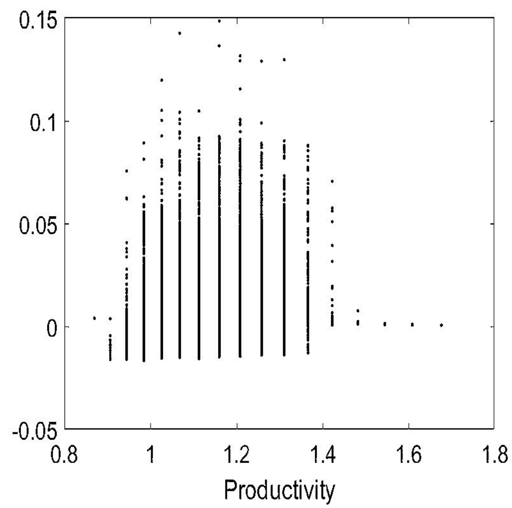
Panel B: The equity premium, $E_t[r_{St+1} - r_{ft+1}]$



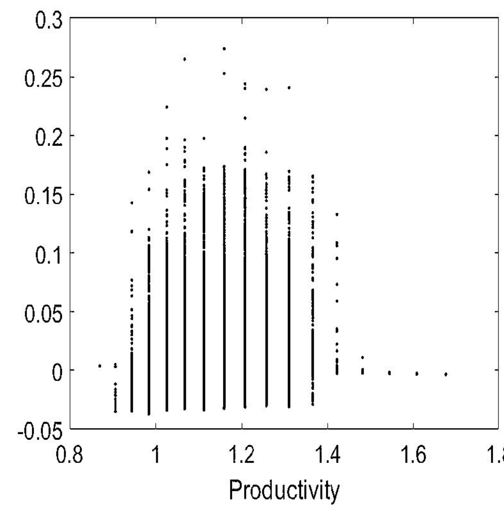
Panel C: Stock market volatility, σ_{St}



Panel D: The risk free rate, r_{ft+1}



Panel E: Expected consumption growth, $E_t[g_{Ct+1}]$



Panel F: Consumption volatility, σ_{Ct}

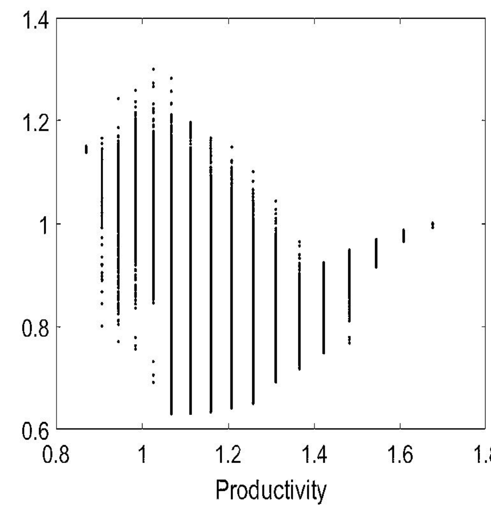


Figure 2 : The Term Structure of the Equity Premium

From the model's stationary distribution with the benchmark calibration (after a burn-in period of 1,200 monthly periods), we simulate a long sample path with one million months. Risk premiums and volatilities are in annualized percentage. Maturity is in years.

Panel A: Risk premium, dividend strip | Panel B: Volatility, dividend strip | Panel C: Sharpe ratio, dividend strip

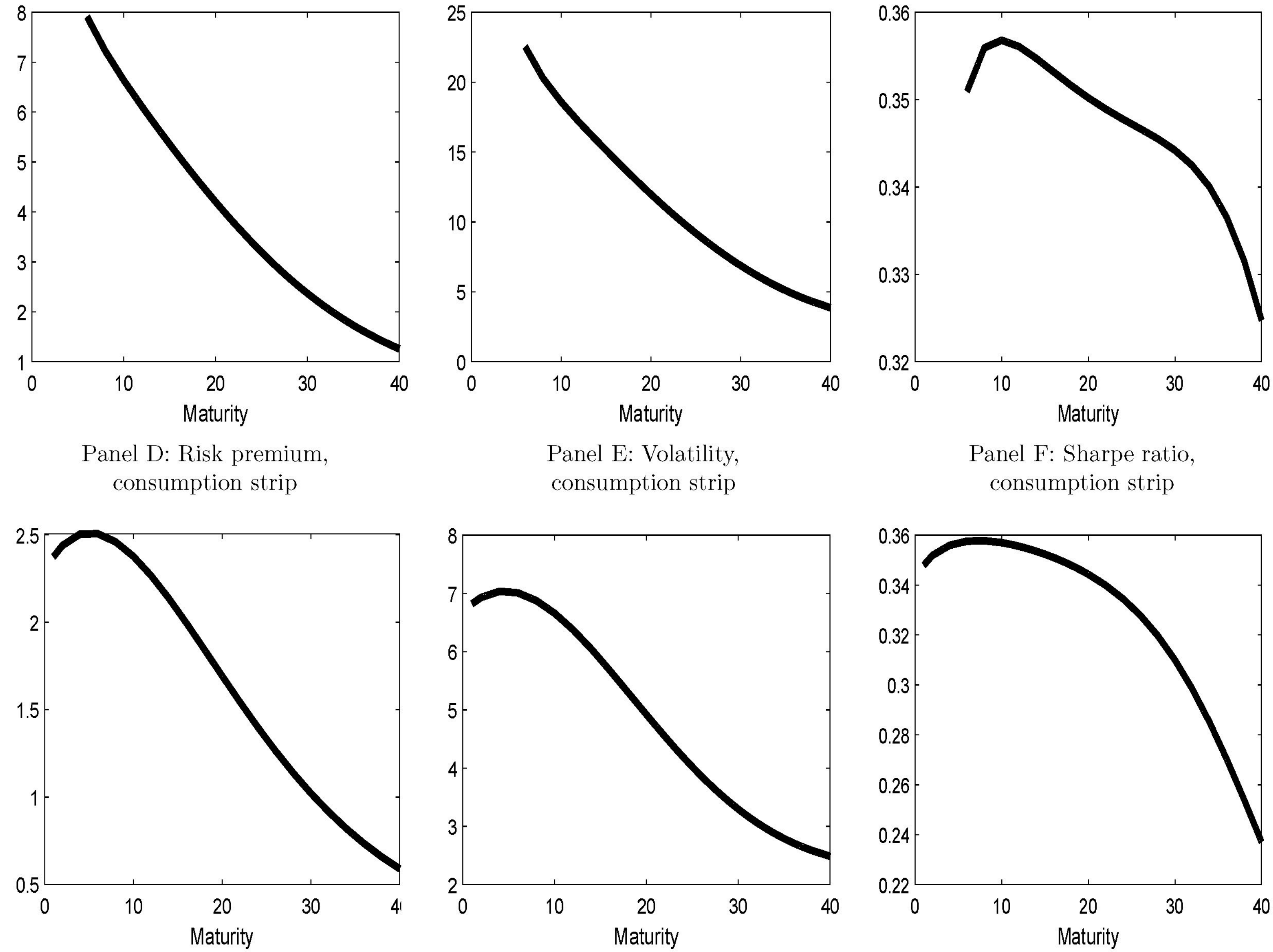
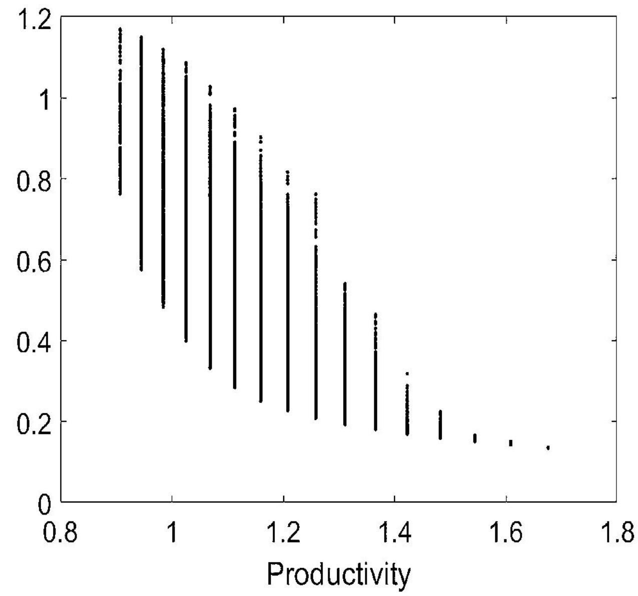


Figure 3 : Scatterplot of the Welfare Cost Against Productivity

From the model's stationary distribution with the benchmark calibration (after a burn-in period of 1,200 monthly periods), we simulate a long sample path with one million months. The vertical axis is the welfare cost, χ_t, and the horizonal axis is the productivity, $\exp(x_t)$.



Internet Appendix (for Online Publication Only): "Searching for the Equity Premium"

A Data

For each country, we construct its dividend index based on three series in the Jordà-Schularick-Taylor macrohistory database, including capital gain (P_t/P_{t-1}), in which P_t is the nominal price level of a stock market index; dividend-to-price (D_t/P_t), in which D_t is nominal dividends delivered by the index; and consumer price index. We first back out the P_t series by cumulating the capital gain series and then construct the D_t series by multiplying P_t with the dividend-to-price series. We scale nominal dividends by consumer price index to yield real dividends. The total number of nonmissing dividends between 1870 and 2015 in the Jordà-Schularick-Taylor dataset is 2,034. Three countries have in total seven dividend observations that equal zero, Germany, Portugal, and Spain. For Switzerland, the capital gain series runs from 1900 to 2015, with 1926–1959 missing. As such, its constructed dividends series starts in 1960. For Netherlands, both its capital gain and dividend-to-price series are missing from 1918 to 1949. As such, its dividends series starts in 1950.

In predicting market excess returns, consumption growth, and their volatilities, we drop Canada from Jordà-Schularick-Taylor macrohistory database. The reason is that its capital gain series (required to construct the price-to-consumption ratio) is incompatible with its total return series from the Dimson-Marsh-Staunton (2002) database. The implied dividend series are frequently negative, unlike the other countries, all of which have nonnegative dividends.

B Derivations

B.1 The Stock Return

Equation (4) implies that the marginal products of capital and labor are given by, respectively:

$$\frac{\partial Y_t}{\partial K_t} = \frac{Y_t}{K_t}\frac{\alpha\,(K_t/K_0)^{\omega}}{\alpha\,(K_t/K_0)^{\omega}+(1-\alpha)N_t^{\omega}}, \tag{S1}$$

$$\frac{\partial Y_t}{\partial N_t} = \frac{Y_t}{N_t}\frac{(1-\alpha)N_t^{\omega}}{\alpha\,(K_t/K_0)^{\omega}+(1-\alpha)N_t^{\omega}}, \tag{S2}$$

As such, Y_t is of constant returns to scale, i.e., $K_t\partial Y_t/\partial K_t + N_t\partial Y_t/\partial N_t = Y_t$. From equation (9):

$$\frac{\partial \Phi_t}{\partial I_t} = a_2\left(\frac{I_t}{K_t}\right)^{-\frac{1}{\nu}} \tag{S3}$$

$$\frac{\partial \Phi_t}{\partial K_t} = a_1 + \frac{a_2}{\nu-1}\left(\frac{I_t}{K_t}\right)^{1-\frac{1}{\nu}} \tag{S4}$$

It follows that $\Phi(I_t, K_t)$ is of constant returns to scale, i.e., $I_t\partial\Phi_t/\partial I_t + K_t\partial\Phi_t/\partial K_t = \Phi_t$.

The Lagrangian for the firm's problem is:

$$\begin{aligned}\mathcal{L} &= \cdots + Y_t - W_t N_t - \kappa V_t - I_t - \mu_{Nt}[N_{t+1} - (1-s)N_t - q(\theta_t)V_t] - \mu_{Kt}[K_{t+1} - (1-s)K_t - \Phi(I_t, K_t)] \\ &+\lambda_t q(\theta_t) V_t + E_t\left[M_{t+1}\left(Y_{t+1} - W_{t+1}N_{t+1} - \kappa V_{t+1} - I_{t+1} - \mu_{Nt+1}[N_{t+2} - (1-s)N_{t+1} - q(\theta_{t+1})V_{t+1}]\right.\right. \\ &\left.\left.-\mu_{Kt+1}[K_{t+2} - (1-s)K_{t+1} - \Phi(I_{t+1}, K_{t+1})] + \lambda_{t+1} q(\theta_{t+1})V_{t+1} + \cdots\right)\right] \qquad \text{(S5)}\end{aligned}$$

The first-order conditions with respect to V_t and N_{t+1} are given by, respectively,

$$\mu_{Nt} = \frac{\kappa}{q(\theta_t)} - \lambda_t \qquad \text{(S6)}$$

$$\mu_{Nt} = E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial N_{t+1}} - W_{t+1} + (1-s)\mu_{Nt+1}\right]\right] \qquad \text{(S7)}$$

Combining the two equations yields the intertemporal job creation condition in equation (14). The first-order conditions with respect to I_t and K_{t+1} are given by, respectively,

$$\mu_{Kt} = \frac{1}{\partial \Phi_t / \partial I_t} \qquad \text{(S8)}$$

$$\mu_{Kt} = E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + \left(1 - \delta + \frac{\partial \Phi_{t+1}}{\partial K_{t+1}}\right)\frac{1}{\partial \Phi_{t+1}/\partial I_{t+1}}\right]\right] \qquad \text{(S9)}$$

Combining equations (S3)–(S9) yields equation (12).

We first show $P_t = \mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}$, in which $P_t = S_t - D_t$ is ex-dividend equity value, with a guess-and-verify approach (Goncalves, Xue, and Zhang 2020). We first assume it holds for $t+1$: $P_{t+1} = \mu_{Kt+1}K_{t+2} + \mu_{Nt+1}N_{t+2}$. We then show it also holds for t. It then follows that the equation must hold for all periods. We start with recursively formulating equation (11): $P_t = E_t[M_{t+1}(P_{t+1} + D_{t+1})]$. Using $P_{t+1} = \mu_{Kt+1}K_{t+2} + \mu_{Nt+1}N_{t+2}$ to rewrite the right-hand side yields:

$$\begin{aligned}P_t &= E_t\left[M_{t+1}\left[\mu_{Kt+1}K_{t+2} + \mu_{Nt+1}N_{t+2} + D_{t+1}\right]\right] \\ &= E_t\left[M_{t+1}\left[\mu_{Kt+1}[(1-\delta)K_{t+1} + \Phi_{t+1}] + \mu_{Nt+1}[(1-s)N_{t+1} + q(\theta_{t+1})V_{t+1}]\right.\right. \\ &\quad \left.\left.+Y_{t+1} - W_{t+1}N_{t+1} - \kappa V_{t+1} - I_{t+1}\right]\right] \\ &= E_t\left[M_{t+1}\left[\mu_{Kt+1}\left[(1-\delta)K_{t+1} + \frac{\partial \Phi_{t+1}}{\partial I_{t+1}}I_{t+1} + \frac{\partial \Phi_{t+1}}{\partial K_{t+1}}K_{t+1}\right] + \mu_{Nt+1}[(1-s)N_{t+1} + q(\theta_{t+1})V_{t+1}]\right.\right. \\ &\quad \left.\left.+\frac{\partial Y_{t+1}}{\partial K_{t+1}}K_{t+1} + \frac{\partial Y_{t+1}}{\partial N_{t+1}}N_{t+1} - W_{t+1}N_{t+1} - \kappa V_{t+1} - I_{t+1}\right]\right] \\ &= K_{t+1}E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + \left(1 - \delta + \frac{\partial \Phi_{t+1}}{\partial K_{t+1}}\right)\mu_{Kt+1}\right]\right] + \mu_{Kt+1}\frac{\partial \Phi_{t+1}}{\partial I_{t+1}}I_{t+1} \\ &\quad +N_{t+1}E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial N_{t+1}} - W_{t+1} + (1-s)\mu_{Nt+1}\right]\right] + \mu_{Nt+1}q(\theta_{t+1})V_{t+1} - \kappa V_{t+1} - I_{t+1} \\ &= \mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}, \qquad \text{(S10)}\end{aligned}$$

in which the third equality follows from constant returns to scale for Y_{t+1} and Φ_{t+1}, and the last equality follows from equations (S6), (S7), (S8), (S9), and the Kuhn-Tucker condition (16).

To prove equation (17),

$$\begin{aligned}
r_{St+1} &= \frac{P_{t+1}+D_{t+1}}{P_t} = \frac{\mu_{Kt+1}K_{t+2}+\mu_{Nt+1}N_{t+2}+D_{t+1}}{\mu_{Kt}K_{t+1}+\mu_{Nt}N_{t+1}} \\
&= \frac{\begin{array}{c}\mu_{Kt+1}\left[(1-\delta)K_{t+1}+\Phi_{t+1}\right]+\mu_{Nt+1}[(1-s)N_{t+1}+q(\theta_{t+1})V_{t+1}] \\ +Y_{t+1}-W_{t+1}N_{t+1}-\kappa V_{t+1}-I_{t+1}\end{array}}{\mu_{Kt}K_{t+1}+\mu_{Nt}N_{t+1}} \\
&= \frac{\begin{array}{c}\mu_{Kt+1}\left[(1-\delta)K_{t+1}+\frac{\partial\Phi_{t+1}}{\partial I_{t+1}}I_{t+1}+\frac{\partial\Phi_{t+1}}{\partial K_{t+1}}K_{t+1}\right]+\mu_{Nt+1}[(1-s)N_{t+1}+q(\theta_{t+1})V_{t+1}] \\ +\frac{\partial Y_{t+1}}{\partial K_{t+1}}K_{t+1}+\frac{\partial Y_{t+1}}{\partial N_{t+1}}N_{t+1}-W_{t+1}N_{t+1}-\kappa V_{t+1}-I_{t+1}\end{array}}{\mu_{Kt}K_{t+1}+\mu_{Nt}N_{t+1}} \\
&= \frac{\left[\frac{\partial Y_{t+1}}{\partial K_{t+1}}+\left(1-\delta+\frac{\partial\Phi_{t+1}}{\partial K_{t+1}}\right)\mu_{Kt+1}\right]K_{t+1}}{\mu_{Kt}K_{t+1}+\mu_{Nt}N_{t+1}}+\frac{\left[\frac{\partial Y_{t+1}}{\partial N_{t+1}}-W_{t+1}+(1-s)\mu_{Nt+1}\right]N_{t+1}}{\mu_{Kt}K_{t+1}+\mu_{Nt}N_{t+1}} \\
&= \frac{\mu_{Kt}K_{t+1}}{\mu_{Kt}K_{t+1}+\mu_{Nt}N_{t+1}}r_{Kt+1}+\frac{\mu_{Nt}N_{t+1}}{\mu_{Kt}K_{t+1}+\mu_{Nt}N_{t+1}}r_{Nt+1}. \qquad \text{(S11)}
\end{aligned}$$

B.2 Wages

We extend the derivation in Petrosky-Nadeau, Zhang, and Kuehn (2018) to our setting with capital accumulation. Let $\partial J_t/\partial N_t$ be the marginal value of an employed worker to the representative household, $\partial J_t/\partial U_t$ the marginal value of an unemployed worker to the household, ϕ_t the marginal utility of the household, $\partial S_t/\partial N_t$ the marginal value of an employed worker to the representative firm, and $\partial S_t/\partial V_t$ the marginal value of an unfilled vacancy to the firm. A worker-firm match turns an unemployed worker into an employed worker for the household as well as an unfilled vacancy into an employed worker for the firm. As such, the total surplus from the Nash bargain is:

$$H_t \equiv \left(\frac{\partial J_t}{\partial N_t}-\frac{\partial J_t}{\partial U_t}\right)/\phi_t+\frac{\partial S_t}{\partial N_t}-\frac{\partial S_t}{\partial V_t}. \qquad \text{(S12)}$$

The equilibrium wage arises from the Nash worker-firm bargain as follows:

$$\max_{\{W_t\}} \quad \left[\left(\frac{\partial J_t}{\partial N_t}-\frac{\partial J_t}{\partial U_t}\right)/\phi_t\right]^{\eta}\left(\frac{\partial S_t}{\partial N_t}-\frac{\partial S_t}{\partial V_t}\right)^{1-\eta}, \qquad \text{(S13)}$$

in which $0<\eta<1$ is the worker's bargaining power. The outcome is the surplus-sharing rule:

$$\left(\frac{\partial J_t}{\partial N_t}-\frac{\partial J_t}{\partial U_t}\right)/\phi_t=\eta H_t=\eta\left[\left(\frac{\partial J_t}{\partial N_t}-\frac{\partial J_t}{\partial U_t}\right)/\phi_t+\frac{\partial S_t}{\partial N_t}-\frac{\partial S_t}{\partial V_t}\right]. \qquad \text{(S14)}$$

As such, the worker receives a fraction of η of the total surplus from the wage bargain.

B.2.1 Workers

Tradeable assets consist of risky shares and a riskfree asset. Let r_{ft+1} denote the risk-free interest rate, ξ_t the household's financial wealth, χ_t the fraction of the household's wealth invested in the risky shares, $r_{\xi t+1} \equiv \chi_t r_{St+1} + (1-\chi_t) r_{ft+1}$ the return on wealth, and T_t the taxes raised by the government. The household's budget constraint is given by:

$$\frac{\xi_{t+1}}{r_{\xi t+1}} = \xi_t - C_t + W_t N_t + U_t b - T_t. \tag{S15}$$

The household's dividends income, D_t, is included in the current financial wealth, ξ_t.

Let ϕ_t denote the Lagrange multiplier for the household's budget constraint (S15). The household's maximization problem is given by:

$$J_t = \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} - \phi_t\left(\frac{\xi_{t+1}}{r_{\xi t+1}} - \xi_t + C_t - W_t N_t - U_t b + T_t\right), \tag{S16}$$

The first-order condition for consumption yields:

$$\phi_t = (1-\beta)C_t^{-\frac{1}{\psi}}\left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1}, \tag{S17}$$

which gives the marginal utility of consumption. Using $N_{t+1} = (1-s)N_t + f(\theta_t)U_t$ and $U_{t+1} = sN_t + (1-f(\theta_t))U_t$, we differentiate J_t in equation (S16) with respect to N_t:

$$\begin{aligned}\frac{\partial J_t}{\partial N_t} &= \phi_t W_t + \frac{1}{1-\frac{1}{\psi}}\left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1} \\ &\quad \times \frac{1-\frac{1}{\psi}}{1-\gamma}\beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}-1} E_t\left[(1-\gamma)J_{t+1}^{-\gamma}\left[(1-s)\frac{\partial J_{t+1}}{\partial N_{t+1}} + s\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]\right].\end{aligned} \tag{S18}$$

Dividing both sides by ϕ_t:

$$\frac{\partial J_t}{\partial N_t}/\phi_t = W_t + \frac{\beta}{(1-\beta)C_t^{-\frac{1}{\psi}}}\left[\frac{1}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma} E_t\left[J_{t+1}^{-\gamma}\left[(1-s)\frac{\partial J_{t+1}}{\partial N_{t+1}} + s\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]\right]. \tag{S19}$$

Dividing and multiplying by ϕ_{t+1}:

$$\begin{aligned}
\frac{\partial J_t}{\partial N_t}/\phi_t &= W_t + E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left[\frac{J_{t+1}}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}\left[(1-s)\frac{\partial J_{t+1}}{\partial N_{t+1}} + s\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]/\phi_{t+1}\right] \\
&= W_t + E_t\left[M_{t+1}\left[(1-s)\frac{\partial J_{t+1}}{\partial N_{t+1}} + s\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]/\phi_{t+1}\right]. \qquad \text{(S20)}
\end{aligned}$$

Similarly, differentiating J_t in equation (S16) with respect to U_t yields:

$$\begin{aligned}
\frac{\partial J_t}{\partial U_t} &= \phi_t b + \frac{1}{1-\frac{1}{\psi}}\left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1} \\
&\times\frac{1-\frac{1}{\psi}}{1-\gamma}\beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}-1} E_t\left[(1-\gamma)J_{t+1}^{-\gamma}\left[f(\theta_t)\frac{\partial J_{t+1}}{\partial N_{t+1}} + (1-f(\theta_t))\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]\right]. \qquad \text{(S21)}
\end{aligned}$$

Dividing both sides by ϕ_t:

$$\frac{\partial J_t}{\partial U_t}/\phi_t = b + \frac{\beta}{(1-\beta)C_t^{-\frac{1}{\psi}}}\left[\frac{1}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma} E_t\left[J_{t+1}^{-\gamma}\left[f(\theta_t)\frac{\partial J_{t+1}}{\partial N_{t+1}} + (1-f(\theta_t))\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]\right]. \qquad \text{(S22)}$$

Dividing and multiplying by ϕ_{t+1}:

$$\begin{aligned}
\frac{\partial J_t}{\partial U_t}/\phi_t &= b + E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left[\frac{J_{t+1}}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}\left[f(\theta_t)\frac{\partial J_{t+1}}{\partial N_{t+1}} + (1-f(\theta_t))\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]/\phi_{t+1}\right] \\
&= b + E_t\left[M_{t+1}\left[f(\theta_t)\frac{\partial J_{t+1}}{\partial N_{t+1}} + (1-f(\theta_t))\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]/\phi_{t+1}\right]. \qquad \text{(S23)}
\end{aligned}$$

B.2.2 The Representative Firm

We start by reformulating the firm's problem recursively as:

$$S_t = Y_t - W_t N_t - \kappa V_t - I_t + \lambda_t q(\theta_t) V_t + E_t[M_{t+1} S_{t+1}], \qquad \text{(S24)}$$

subject to $N_{t+1} = (1-s)N_t + q(\theta_t)V_t$ and $K_{t+1} = (1-\delta)K_t + \Phi(I_t, K_t)$.

The first-order condition with respect to V_t says:

$$\frac{\partial S_t}{\partial V_t} = -\kappa + \lambda_t q(\theta_t) + E_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial N_{t+1}} q(\theta_t)\right] = 0. \qquad \text{(S25)}$$

Equivalently,

$$\frac{\kappa}{q(\theta_t)} - \lambda_t = E_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial N_{t+1}}\right]. \tag{S26}$$

In addition, differentiating S_t with respect to N_t yields:

$$\frac{\partial S_t}{\partial N_t} = \frac{\partial Y_t}{\partial N_t} - W_t + (1-s)E_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial N_{t+1}}\right]. \tag{S27}$$

Combining the last two equations yields the job creation condition.

B.2.3 The Wage Rate

From equations (S20), (S23), and (S27), the total surplus of the worker-firm relationship is:

$$\begin{aligned}
H_t &= W_t + E_t\left[M_{t+1}\left[(1-s)\frac{\partial J_{t+1}}{\partial N_{t+1}} + s\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]/\phi_{t+1}\right] - b \\
&\quad -E_t\left[M_{t+1}\left[f(\theta_t)\frac{\partial J_{t+1}}{\partial N_{t+1}} + (1-f(\theta_t))\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]/\phi_{t+1}\right] + \frac{\partial Y_t}{\partial N_t} - W_t + (1-s)E_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial N_{t+1}}\right] \\
&= \frac{\partial Y_t}{\partial N_t} - b + (1-s)E_t\left[M_{t+1}\left[\left(\frac{\partial J_{t+1}}{\partial N_{t+1}} - \frac{\partial J_{t+1}}{\partial U_{t+1}}\right)/\phi_{t+1} + \frac{\partial S_{t+1}}{\partial N_{t+1}}\right]\right] \\
&\quad -f(\theta_t)E_t\left[M_{t+1}\left(\frac{\partial J_{t+1}}{\partial N_{t+1}} - \frac{\partial J_{t+1}}{\partial U_{t+1}}\right)/\phi_{t+1}\right] \\
&= \frac{\partial Y_t}{\partial N_t} - b + (1-s-\eta f(\theta_t))E_t\left[M_{t+1}H_{t+1}\right].
\end{aligned} \tag{S28}$$

The sharing rule implies $\partial S_t/\partial N_t = (1-\eta)H_t$, which, combined with equation (S27), yields:

$$(1-\eta)H_t = \frac{\partial Y_t}{\partial N_t} - W_t + (1-\eta)(1-s)E_t\left[M_{t+1}H_{t+1}\right]. \tag{S29}$$

Combining equations (S28) and (S29) yields:

$$\begin{aligned}
\frac{\partial Y_t}{\partial N_t} - W_t + (1-\eta)(1-s)E_t\left[M_{t+1}H_{t+1}\right] &= (1-\eta)\left(\frac{\partial Y_t}{\partial N_t} - b\right) + (1-\eta)(1-s)E_t\left[M_{t+1}H_{t+1}\right] \\
&\quad -(1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}H_{t+1}\right] \\
W_t &= \eta\frac{\partial Y_t}{\partial N_t} + (1-\eta)b + (1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}H_{t+1}\right].
\end{aligned}$$

Using equations (S14) and (S26) to simplify further:

$$W_t = \eta\frac{\partial Y_t}{\partial N_t} + (1-\eta)b + \eta f(\theta_t)E_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial N_{t+1}}\right] \tag{S30}$$

$$W_t = \eta\frac{\partial Y_t}{\partial N_t} + (1-\eta)b + \eta f(\theta_t)\left[\frac{\kappa}{q(\theta_t)} - \lambda_t\right]. \tag{S31}$$

If $V_t > 0$, then $\lambda_t = 0$, and equation (S31) reduces to equation (18) because $f(\theta_t) = \theta_t q(\theta_t)$. If $V_t \geq 0$ is binding, $\lambda_t > 0$, but $V_t = 0$ means $\theta_t = 0$ and $f(\theta_t) = 0$. Equation (S31) reduces to $W_t = \eta \partial Y_t / \partial N_t + (1 - \eta) b$. Because $\theta_t = 0$, equation (18) continues to hold.

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Table S1 : Basic Properties of Asset Prices in the Historical Sample, with the Longest Possible Sample for Each Moment

The historical cross-country panel is from the Jordà-Schularick-Taylor macrohistory database, except for Canada's asset prices, which we obtain from the Dimson-Marsh-Staunton (2002) database purchased from Morningstar. All (annual) series end in 2015. $E[\widetilde{r}_S]$, $\widetilde{\sigma}_S$, and $E[\widetilde{r}_S - r_f]$ are the average stock market return, stock market volatility, and the equity premium, respectively, without adjusting for financial leverage. $E[r_S - r_f]$ and σ_S are the equity premium and stock market volatility, respectively, after adjusting for financial leverage. $E[r_f]$ is the mean real interest rate, and σ_f the interest rate volatility. All asset pricing moments are in annual percent. We use the longest possible samples of stocks, bills, and bonds described in the second, third, and fourth column, respectively, to calculate each moment. For example, in Australia, the sample for stock market returns starts in 1871, the sample for real interest rates start in 1871, with missing observations from 1945 to 1947, and the sample for long-term government bonds starts in 1900. Other than Italy, which has missing asset prices from 1872 to 1884, all other missing years are in the 20th century.

	Sample, $\widetilde{r}_S$	Sample, r_f	Sample, r_B	$E[\widetilde{r}_S]$	$\widetilde{\sigma}_S$	$E[r_f]$	σ_f	$E[\widetilde{r}_S - r_f]$	$E[r_S - r_f]$	σ_S
Australia	1871	1871 (45–47)	1900	8.39	15.77	2.02	4.44	6.33	4.49	11.76
Belgium	1871	1871 (15–18)	1871 (14–19)	5.89	21.97	1.68	9.94	5.25	3.73	16.22
Canada	1900	1900	1900	7.01	17.00	1.60	4.79	5.41	3.84	12.26
Denmark	1873	1875	1871 (15)	7.54	16.36	2.98	5.77	4.59	3.26	11.88
Finland	1896	1871	1871	8.83	30.57	0.15	10.50	9.57	6.80	22.98
France	1871	1871 (15–21)	1871	3.21	22.14	−0.47	7.78	4.45	3.16	16.75
Germany	1871	1871 (23, 45–49)	1871 (44–48)	9.44	32.04	−0.23	13.17	9.00	6.39	20.15
Italy	1871	1871 (1872–1884, 15–21)	1871	5.75	26.18	0.58	10.50	6.05	4.29	20.41
Japan	1886 (46–47)	1876	1881	8.86	27.69	−0.41	12.90	8.87	6.29	21.10
Netherlands	1900	1871	1871	6.96	21.44	1.37	5.04	6.19	4.39	15.32
Norway	1881	1871	1871	5.67	19.82	1.10	5.96	4.77	3.39	14.53
Portugal	1871	1880	1871	4.05	25.20	−0.01	9.43	3.82	2.71	19.29
Spain	1900	1871 (36–38)	1900 (37–40)	5.77	21.07	0.70	6.83	6.28	4.46	15.88
Sweden	1871	1871	1871	8.00	19.54	1.77	5.60	6.23	4.42	14.26
Switzerland	1900	1871	1900 (15)	6.50	19.09	1.64	5.88	5.70	4.05	14.04
UK	1871	1871	1871	6.86	17.77	1.16	4.82	5.70	4.05	12.96
USA	1872	1871	1871	8.40	18.68	2.23	4.71	6.23	4.43	13.66
Mean				6.89	21.90	1.05	7.53	6.14	4.36	16.08
Median				6.96	21.07	1.16	5.96	6.05	4.29	15.32

Table S2 : Basic Properties of the Real Consumption, Output, and Investment Growth and Asset Prices, 1950–2015

The historical cross-country panel is from the Jordà-Schularick-Taylor macrohistory database. The only exception is asset prices data for Canada, which we obtain from the Dimson-Marsh-Staunton (2002) database purchased from Morningstar. All (annual) series end in 2015. In Panel A, $\overline{g}_C$, σ_C, S_C, K_C, and $\rho_C^{(i)}$ denote the mean (in percent), volatility (in percent), skewness, kurtosis, and ith-order autocorrelation, for $i = 1, 2, \ldots, 5$, of real per capita consumption growth. In Panel B, $\overline{g}_Y$, σ_Y, S_Y, K_Y, and $\rho_Y^{(i)}$ denote the mean (in percent), volatility (in percent), skewness, kurtosis, and ith-order autocorrelation for real per capita output growth. In Panel C, $\overline{g}_I$, σ_I, S_I, K_I, and $\rho_I^{(i)}$ denote the mean (in percent), volatility (in percent), skewness, kurtosis, and ith-order autocorrelation for real per capita investment growth. Finally, in Panel D, $E[\widetilde{r}_S]$, $\widetilde{\sigma_S}$, and $E[\widetilde{r}_S - r_f]$ are the average stock market return, stock market volatility, and the equity premium, respectively, without adjusting for financial leverage. $E[r_S - r_f]$ and σ_S are the equity premium and stock market volatility, respectively, after adjusting for financial leverage. $E[r^f]$ is the mean real interest rate, and σ_f the interest rate volatility. All asset pricing moments are in annual percent.

	Panel A: Real consumption growth									Panel B: Real output growth								
	$\overline{g}_C$	σ_C	S_C	K_C	$\rho_C^{(1)}$	$\rho_C^{(2)}$	$\rho_C^{(3)}$	$\rho_C^{(4)}$	$\rho_C^{(5)}$	$\overline{g}_Y$	σ_Y	S_Y	K_Y	$\rho_Y^{(1)}$	$\rho_Y^{(2)}$	$\rho_Y^{(3)}$	$\rho_Y^{(4)}$	$\rho_Y^{(5)}$
Australia	1.78	2.02	−0.14	3.55	0.17	−0.24	−0.11	0.19	0.30	1.95	1.86	−0.56	4.19	0.19	−0.03	−0.07	0.09	0.24
Belgium	1.89	1.92	0.20	3.42	0.34	0.21	0.41	0.18	0.21	2.22	2.01	−0.28	2.95	0.28	0.27	0.20	0.23	0.05
Canada	2.01	1.81	−0.61	4.00	0.31	0.07	0.17	−0.07	−0.26	1.94	2.25	−0.73	3.73	0.25	−0.04	0.05	0.05	−0.01
Denmark	1.24	2.43	−0.03	2.95	0.22	0.01	0.03	−0.17	−0.30	1.85	2.33	−0.05	3.88	0.26	0.09	0.16	0.16	0.09
Finland	2.62	3.17	−0.40	3.04	0.40	−0.08	−0.05	−0.05	−0.03	2.54	3.24	−0.93	5.23	0.42	0.01	0.08	0.02	0.03
France	2.34	1.79	0.19	2.18	0.65	0.48	0.40	0.42	0.41	2.37	1.87	−0.27	3.35	0.56	0.41	0.47	0.44	0.40
Germany	2.81	2.46	0.71	2.98	0.73	0.53	0.50	0.51	0.49	2.80	2.69	0.18	3.93	0.48	0.17	0.31	0.48	0.36
Italy	2.51	2.72	−0.30	2.97	0.67	0.46	0.52	0.48	0.41	2.69	2.71	−0.79	3.56	0.51	0.35	0.42	0.39	0.39
Japan	3.90	3.53	0.72	3.00	0.74	0.62	0.69	0.66	0.61	3.80	3.69	0.26	2.72	0.69	0.59	0.61	0.53	0.48
Netherlands	1.92	2.47	−0.16	2.45	0.67	0.32	0.15	0.08	0.13	2.22	2.20	−0.12	3.78	0.39	0.05	0.05	0.13	0.12
Norway	2.39	2.19	0.21	3.76	0.23	−0.02	−0.18	−0.14	−0.13	2.53	1.87	−0.52	2.73	0.51	0.28	0.16	0.20	0.19
Portugal	3.05	3.56	−0.58	4.03	0.36	0.16	0.08	−0.14	−0.18	2.93	3.48	−0.34	3.87	0.51	0.23	0.26	−0.01	0.02
Spain	2.79	3.54	0.08	3.20	0.51	0.25	0.20	0.23	0.23	3.15	3.21	0.07	2.62	0.50	0.32	0.23	0.22	0.14
Sweden	1.55	1.92	−0.59	3.12	0.38	0.18	0.08	−0.09	−0.16	2.09	2.14	−1.13	5.31	0.32	−0.02	0.03	0.12	0.15
Switzerland	1.44	1.42	0.11	2.59	0.61	0.24	0.14	0.10	0.11	1.62	2.29	−0.65	4.06	0.30	−0.04	−0.03	0.08	0.02
UK	1.97	2.09	−0.13	3.11	0.45	0.05	−0.11	−0.11	0.00	1.88	1.90	−0.85	4.89	0.33	−0.13	−0.12	−0.01	0.02
USA	2.08	1.73	−0.21	2.49	0.32	0.03	−0.06	0.02	−0.04	1.91	2.21	−0.43	2.88	0.12	−0.01	−0.15	0.06	−0.07
Mean	2.25	2.40	−0.05	3.11	0.46	0.19	0.17	0.12	0.11	2.38	2.47	−0.42	3.75	0.39	0.15	0.16	0.19	0.15
Median	2.08	2.19	−0.13	3.04	0.40	0.18	0.14	0.08	0.11	2.22	2.25	−0.43	3.78	0.39	0.09	0.16	0.13	0.12

	Panel C: Real investment growth									Panel D: Asset prices						
	$\overline{g}_I$	σ_I	S_I	K_I	$\rho_I^{(1)}$	$\rho_I^{(2)}$	$\rho_I^{(3)}$	$\rho_I^{(4)}$	$\rho_I^{(5)}$	$E[\widetilde{r}_S]$	$\widetilde{\sigma}_S$	$E[r_f]$	σ_f	$E[\widetilde{r}_S - r_f]$	$E[r_S - r_f]$	σ_S
Australia	2.33	5.70	−0.45	2.82	0.09	−0.29	−0.08	0.16	0.07	7.33	20.46	1.44	3.97	5.89	4.18	14.90
Belgium	2.63	7.08	−0.73	3.95	0.05	−0.13	−0.09	−0.01	−0.16	8.07	21.05	1.60	2.91	6.47	4.59	15.02
Canada	2.10	5.65	−0.46	3.15	0.23	0.02	−0.21	−0.20	−0.28	7.47	16.33	1.80	3.12	5.66	4.02	11.51
Denmark	1.32	9.06	−1.34	6.86	0.24	0.07	0.06	−0.13	−0.19	9.60	21.37	2.24	2.85	7.36	5.22	15.19
Finland	2.41	9.01	−0.66	4.25	0.49	0.04	−0.19	−0.20	−0.09	12.17	33.86	0.76	4.50	11.41	8.10	24.47
France	1.86	6.18	−2.56	14.73	0.14	−0.03	−0.18	−0.13	−0.19	6.45	26.13	1.08	3.29	5.38	3.82	18.71
Germany	2.60	6.41	0.28	3.78	0.39	−0.06	−0.08	−0.02	0.03	12.09	27.71	1.72	1.78	10.37	7.36	19.62
Italy	2.37	5.53	−0.63	3.12	0.41	0.11	0.18	0.22	0.10	6.02	25.99	1.23	3.09	4.79	3.40	18.69
Japan	4.11	7.86	0.56	2.84	0.52	0.19	0.30	0.30	0.28	9.58	22.37	1.21	3.40	8.37	5.94	16.04
Netherlands	2.21	6.11	0.10	3.42	0.24	0.00	−0.07	−0.11	−0.27	9.43	21.81	1.15	2.83	8.28	5.88	15.58
Norway	2.18	8.58	0.29	4.50	0.13	−0.14	−0.03	−0.10	−0.22	7.25	25.99	−0.21	3.26	7.46	5.30	18.69
Portugal	2.64	9.58	−0.22	3.08	0.22	0.21	0.06	−0.13	0.08	4.86	33.53	−0.73	4.85	5.59	3.97	24.38
Spain	3.60	9.32	−0.20	3.40	0.45	0.30	−0.07	−0.12	−0.27	7.93	24.53	−0.22	4.43	8.15	5.79	17.91
Sweden	2.49	5.32	−1.41	5.32	0.28	−0.09	−0.13	−0.08	0.03	11.14	24.01	0.82	2.58	10.32	7.33	17.23
Switzerland	2.25	7.93	0.46	5.96	0.35	0.03	−0.04	−0.21	−0.24	8.33	21.41	0.06	2.13	8.27	5.87	15.33
UK	2.61	5.75	−0.76	4.16	0.38	0.02	−0.03	0.02	0.05	9.13	22.94	1.21	3.63	7.92	5.62	16.27
USA	1.91	4.98	−0.89	4.45	0.27	−0.12	−0.27	−0.21	−0.08	8.56	16.83	1.41	2.25	7.15	5.08	12.03
Mean	2.45	7.06	−0.51	4.69	0.29	0.01	−0.05	−0.06	−0.08	8.55	23.90	0.97	3.23	7.58	5.38	17.15
Median	2.37	6.41	−0.46	3.95	0.27	0.02	−0.07	−0.11	−0.09	8.33	22.94	1.21	3.12	7.46	5.30	16.27

Table S3 : Gollin's (2002) Labor Share Calculations

For the 12 countries that are in both Gollin (2002) and Jordà-Schularick-Taylor macrohistory database, this table reports the labor shares reported in Gollin's Table 2. The three columns correspond to the last three columns labeled "Adjustment 1," "Adjustment 2," and "Adjustment 3," respectively, in Gollin's table.

	Method 1	Method 2	Method 3
Australia	0.719	0.669	0.676
Belgium	0.791	0.743	0.740
Finland	0.765	0.734	0.680
France	0.764	0.717	0.681
Italy	0.804	0.717	0.707
Japan	0.727	0.692	0.725
Netherlands	0.721	0.680	0.643
Norway	0.678	0.643	0.569
Portugal	0.825	0.748	0.602
Sweden	0.800	0.774	0.723
UK	0.815	0.782	0.719
US	0.773	0.743	0.664
Mean	0.765	0.720	0.677
Median	0.769	0.726	0.681

Table S4 : Dividend Dynamics in the Historical and Post-1950 Samples

Real output is from the Jordà-Schularick-Taylor macrohistory database. Appendix A describes our construction of dividends from their database. We use two detrending methods for real dividends and output. "Prop. dev." means the HP-filtered proportional deviations from the mean, and "Log dev." means log deviations from the HP-trend. ρ_{DY} is the correlation between the cyclical components of dividends and output, and σ_D/σ_Y the volatility of the cyclical component of dividends divided by that of output. We examine three frequencies, annual, 3-year, and 5-year. For the 3-year frequency, we sum up the three annual observations within a given 3-year interval. The 3-year intervals are nonoverlapping. The 5-year series are constructed analogously. The HP smoothing parameters for the 1-, 3-, and 5-year series are $1600/4^4 = 6.25$, $1600/12^4 = 0.077$, and $1600/20^4 = 0.01$, respectively, all of which correspond to 1,600 in the quarterly frequency. in Panel A, the column "Sample" indicates the starting year of a country. For Japan, the annual observations from 1946 and 1947 are missing. In Panel B, all countries start their samples in 1950, except for Switzerland, which starts in 1960. In calculating log deviations, the zero-dividend observations are removed.

Panel A: The historical sample

		1-year frequency				3-year frequency				5-year frequency			
		Prop. dev.		Log dev.		Prop. dev.		Log dev.		Prop. dev.		Log dev.	
	Sample	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y
Australia	1870	0.109	6.943	0.121	3.430	0.231	5.144	0.295	2.565	0.551	4.596	0.633	2.593
Belgium	1870	0.183	8.501	0.498	4.706	0.419	9.797	0.825	5.718	0.765	5.622	0.923	2.830
Denmark	1872	0.191	14.885	0.182	6.836	0.263	12.065	0.226	6.513	0.027	10.580	0.092	6.031
Finland	1912	0.083	10.143	0.308	6.883	0.670	8.464	0.504	5.754	0.815	4.225	0.437	5.477
France	1870	0.169	5.590	0.119	2.658	0.234	5.311	−0.029	3.564	0.479	4.443	−0.204	3.234
Germany	1870	0.018	6.052	0.211	20.321	0.114	3.821	0.552	2.924	0.241	1.952	0.894	3.652
Italy	1870	0.035	5.097	0.396	6.373	0.262	5.925	0.847	10.566	0.571	5.603	0.764	7.340
Japan	1886 (46–47)	0.027	10.673	0.612	8.949	0.058	6.347	0.806	10.513	0.110	7.107	0.882	7.470
Netherlands	1950	−0.001	16.807	0.203	14.904	0.548	13.768	0.390	13.042	0.369	18.404	0.258	13.533
Norway	1880	0.216	10.520	0.214	8.517	0.348	5.638	0.440	6.574	0.342	5.413	0.656	5.580
Portugal	1870	−0.021	3.062	0.007	7.762	0.043	1.572	0.597	12.343	0.108	1.570	0.454	14.630
Spain	1899	0.035	11.598	0.269	8.865	0.176	6.008	0.562	11.541	0.266	4.142	0.625	5.039
Sweden	1871	−0.019	9.309	0.152	5.664	0.091	5.756	0.440	5.112	0.565	4.616	0.701	5.631
Switzerland	1960	0.026	11.061	0.051	13.165	0.429	8.658	0.342	7.725	0.034	7.987	0.014	9.190
UK	1871	0.272	4.314	0.094	5.072	0.570	3.001	0.399	2.788	0.114	2.735	0.044	2.538
USA	1871	0.472	3.176	0.415	2.626	0.527	3.367	0.419	2.682	0.307	2.099	0.458	2.118
Mean		0.112	8.608	0.241	7.921	0.312	6.540	0.476	6.870	0.354	5.693	0.477	6.055
Median		0.059	8.905	0.207	6.860	0.262	5.840	0.440	6.133	0.325	4.606	0.542	5.529

Panel B: The post-1950 sample												
	1-year frequency				3-year frequency				5-year frequency			
	Prop. dev.		Log dev.		Prop. dev.		Log dev.		Prop. dev.		Log dev.	
	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y
Australia	0.166	10.847	0.206	9.315	0.220	10.371	0.217	8.048	0.667	6.390	0.732	4.997
Belgium	−0.037	15.171	−0.020	11.548	−0.064	13.081	0.000	9.053	0.263	11.680	0.368	9.798
Denmark	0.200	37.696	0.127	15.578	0.336	29.277	0.351	14.641	0.369	14.957	0.406	8.189
Finland	0.054	12.063	0.182	11.121	0.573	8.375	0.874	9.303	0.795	5.395	0.712	7.865
France	−0.113	9.507	−0.060	10.510	0.082	9.709	0.105	9.099	0.096	5.515	0.277	5.893
Germany	−0.232	10.350	0.059	10.676	0.126	9.745	0.257	12.919	−0.326	12.597	0.338	11.169
Italy	0.019	8.131	−0.058	10.659	−0.009	11.848	0.157	11.149	0.268	10.223	0.362	14.628
Japan	0.287	4.826	0.393	5.584	0.198	5.750	0.192	4.645	0.489	4.724	0.556	4.506
Netherlands	−0.001	16.807	0.203	14.904	0.548	13.768	0.390	13.042	0.369	18.404	0.258	13.533
Norway	0.188	33.166	0.076	23.009	0.085	19.021	0.070	16.454	0.574	4.214	0.287	6.402
Portugal	−0.243	6.067	0.073	16.699	0.156	5.767	0.720	26.052	0.510	2.865	0.812	20.826
Spain	−0.052	16.567	0.027	12.367	0.044	6.939	0.055	5.929	0.196	5.250	0.109	4.727
Sweden	−0.033	11.772	0.182	8.797	0.615	9.721	0.816	9.321	0.436	3.884	0.513	5.503
Switzerland	0.026	11.061	0.051	13.165	0.429	8.658	0.342	7.725	0.034	7.987	0.014	9.190
UK	0.634	3.837	0.639	3.695	0.734	4.251	0.735	4.165	0.411	3.593	0.482	3.336
USA	0.645	3.801	0.497	2.962	0.710	4.377	0.646	3.508	0.371	3.302	0.506	2.671
Mean	0.094	13.229	0.161	11.287	0.299	10.666	0.370	10.316	0.345	7.561	0.421	8.327
Median	0.022	10.954	0.101	10.898	0.209	9.715	0.299	9.201	0.370	5.455	0.387	7.134

Table S5 : Predicting Excess Returns and Consumption Growth with Log Price-to-consumption in the post-1950 Sample

The historical cross-country panel is from the Jordà-Schularick-Taylor macrohistory database, except for Canada. The annuals series start in 1950 and end in 2015. Panel A performs predictive regressions of stock market excess returns on log price-to-consumption, $\sum_{h=1}^{H}[\log(r_{St+h}) - \log(r_{ft+h})] = a + b\log(P_t/C_t) + u_{t+h}$, in which H is the forecast horizon, r_{St+1} real stock market return, r_{ft+1} real interest rate, P_t real market index, and C_t real consumption. r_{St+1} and r_{ft+1} are over the course of period t, and P_t and C_t are at the beginning of t. Excess returns are adjusted for a financial leverage ratio of 0.29. Panel B performs long-horizon predictive regressions of log consumption growth on $\log(P_t/C_t)$, $\sum_{h=1}^{H}\log(C_{t+h}/C_t) = c + d\log(P_t/C_t) + v_{t+h}$. In both regressions, $\log(P_t/C_t)$ is standardized to have a mean of zero and a standard deviation of one. H ranges from one year (1y) to five years (5y). The t-values of the slopes are adjusted for heteroscedasticity and autocorrelations of $2(H-1)$ lags. The slopes and R-squares are in percent.

	Slopes					t-values of slopes					R-squares				
	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
	Panel A: Predicting stock market excess returns														
Australia	−4.79	−7.74	−8.35	−9.52	−9.70	−3.03	−4.13	−4.19	−3.06	−2.46	12.17	19.86	21.17	22.51	21.26
Belgium	−2.39	−5.00	−6.91	−9.44	−10.86	−1.45	−1.57	−1.61	−1.89	−2.39	2.46	5.64	8.16	11.80	15.00
Denmark	−0.43	−1.76	−2.32	−3.05	−3.08	−0.17	−0.41	−0.46	−0.67	−0.76	0.08	0.61	0.79	1.13	1.13
Finland	−3.76	−9.48	−14.03	−17.33	−19.08	−1.36	−2.46	−4.08	−5.30	−5.33	3.68	9.66	14.50	18.23	20.25
France	−1.85	−4.05	−5.95	−8.59	−11.47	−0.97	−1.17	−1.09	−1.21	−1.48	1.26	3.10	4.85	7.24	11.90
Germany	−6.24	−11.41	−14.93	−18.14	−19.06	−2.78	−3.21	−3.15	−3.20	−3.40	12.48	20.22	24.99	29.08	29.57
Italy	−0.98	−2.51	−4.20	−5.61	−6.34	−0.52	−0.63	−0.77	−0.80	−0.76	0.32	0.93	1.76	2.40	2.76
Japan	−4.00	−9.60	−13.90	−17.98	−21.83	−2.30	−2.96	−4.35	−5.80	−5.96	8.19	18.14	25.40	31.70	36.39
Netherlands	−3.04	−6.48	−8.91	−11.12	−13.51	−1.68	−1.87	−2.09	−2.46	−3.06	4.13	8.98	12.71	16.31	20.65
Norway	−3.89	−7.14	−8.74	−9.80	−11.69	−1.99	−2.68	−2.70	−2.59	−2.87	4.99	9.69	12.52	14.56	18.57
Portugal	−2.16	−8.22	−14.17	−17.85	−17.39	−0.48	−0.94	−1.30	−1.51	−1.66	0.77	3.93	6.64	7.60	5.75
Spain	−0.32	−2.18	−4.83	−7.32	−9.22	−0.17	−0.54	−0.86	−1.17	−1.32	0.04	0.78	2.22	3.64	4.76
Sweden	−1.57	−3.12	−4.06	−5.13	−6.09	−0.75	−0.84	−0.91	−1.10	−1.24	0.95	1.88	2.46	3.28	4.10
Switzerland	−3.09	−6.51	−8.50	−10.67	−12.95	−1.70	−2.30	−2.85	−3.89	−4.17	4.02	8.50	11.76	15.72	20.05
UK	−6.50	−11.41	−13.92	−14.44	−16.54	−3.01	−4.32	−4.54	−5.95	−6.68	17.37	30.67	38.71	42.28	49.39
USA	−2.89	−5.59	−7.18	−9.65	−12.36	−2.18	−2.27	−2.24	−2.47	−2.79	5.83	10.67	13.61	18.59	23.90
Mean	−2.99	−6.39	−8.81	−10.98	−12.57	−1.53	−2.02	−2.32	−2.69	−2.90	4.92	9.58	12.64	15.38	17.84
Median	−2.96	−6.49	−8.42	−9.73	−12.02	−1.56	−2.07	−2.16	−2.47	−2.63	3.85	8.74	12.14	15.14	19.31
	Panel B: Predicting consumption growth														
Australia	0.40	0.41	0.36	0.81	1.20	1.79	0.93	0.58	1.35	1.85	4.04	1.78	1.08	5.54	10.21
Belgium	0.09	0.09	0.21	0.41	0.54	0.42	0.25	0.43	0.68	0.76	0.24	0.09	0.27	0.62	0.78
Denmark	−0.08	−0.42	−0.69	−1.05	−1.38	−0.27	−0.55	−0.61	−0.79	−0.94	0.10	1.04	1.61	2.57	3.51
Finland	0.31	0.06	−0.40	−0.73	−0.90	0.95	0.08	−0.39	−0.63	−0.72	0.97	0.01	0.37	0.89	1.10
France	0.95	1.81	2.68	3.51	4.37	4.45	3.67	3.84	4.18	4.69	28.51	32.88	37.06	40.26	43.62
Germany	−0.10	−0.47	−1.05	−1.43	−1.84	−0.29	−0.51	−0.69	−0.72	−0.73	0.15	1.07	2.65	3.20	3.74
Italy	1.58	3.04	4.43	5.68	6.84	5.69	4.47	4.07	3.74	3.51	33.87	37.49	40.15	40.67	40.62
Japan	0.51	0.86	1.44	1.86	2.17	1.48	0.89	0.90	0.80	0.71	2.12	1.79	2.65	2.63	2.30
Netherlands	0.67	1.12	1.46	1.87	2.35	2.43	1.49	1.24	1.17	1.14	7.43	6.49	5.94	6.42	7.60
Norway	0.23	0.38	0.56	0.73	1.00	0.78	0.65	0.77	0.95	1.29	1.16	1.26	1.78	2.36	3.78
Portugal	0.19	0.05	0.13	0.62	1.54	0.36	0.04	0.08	0.38	0.98	0.26	0.01	0.03	0.53	2.66
Spain	1.75	3.04	4.02	4.90	5.62	4.78	3.94	3.36	2.99	2.72	24.75	26.39	25.77	24.87	23.66
Sweden	0.00	−0.22	−0.39	−0.56	−0.74	−0.01	−0.44	−0.46	−0.48	−0.53	0.00	0.45	0.74	1.01	1.30
Switzerland	0.22	0.31	0.36	0.35	0.34	1.32	0.84	0.61	0.43	0.33	2.52	1.40	1.00	0.64	0.44
UK	0.45	0.46	0.45	0.23	−0.12	2.14	1.14	0.80	0.30	−0.13	4.78	1.73	0.99	0.19	0.04
USA	0.28	0.16	0.11	0.19	0.22	1.34	0.31	0.15	0.20	0.20	2.69	0.30	0.09	0.19	0.20
Mean	0.47	0.67	0.86	1.09	1.33	1.71	1.07	0.92	0.91	0.95	7.10	7.14	7.64	8.29	9.10
Median	0.30	0.34	0.36	0.51	0.77	1.33	0.74	0.59	0.56	0.73	2.32	1.33	1.35	2.46	3.08

Table S6 : Predicting Volatilities of Stock Market Excess Returns and Consumption Growth with Log Price-to-consumption in the Post-1950 Sample

The historical cross-country panel is from the Jordà-Schularick-Taylor macrohistory database, except for Canada. The annuals series start in 1950 and end in 2015. For a given horizon, H, we measure excess return volatility as $\sigma_{St,t+H-1} = \sum_{h=0}^{H-1} |\epsilon_{St+h}|$, in which ϵ_{St+h} is the h-period-ahead residual from the first-order autoregression of excess returns, $\log(r_{St+1}) - \log(r_{ft+1})$ (adjusted for a financial leverage ratio of 0.29). Panel A performs long-horizon predictive regressions of excess return volatilities, $\log \sigma_{St+1,t+H} = a + b \log(P_t/C_t) + u^{\sigma}_{t+H}$. For a given H, consumption growth volatility is $\sigma_{Ct,t+H-1} = \sum_{h=0}^{H-1} |\epsilon_{Ct+h}|$, in which ϵ_{Ct+h} is the h-period-ahead residual from the first-order autoregression of log consumption growth, $\log(C_{t+1}/C_t)$. Panel B performs long-horizon predictive regressions of consumption growth volatilities, $\log \sigma_{Ct+1,t+H} = c + d \log(P_t/C_t) + v^{\sigma}_{t+H}$. $\log(P_t/C_t)$ is standardized to have a mean of zero and a standard deviation of one. H ranges from one year (1y) to five years (5y). The t-values are adjusted for heteroscedasticity and autocorrelations of $2(H-1)$ lags. The slopes and R-squares are in percent.

	Slopes					t-values of slopes					R-squares				
	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
	Panel A: Predicting stock market volatility														
Australia	1.71	6.76	3.98	4.74	2.83	0.11	0.55	0.31	0.36	0.21	0.01	0.59	0.36	0.62	0.27
Belgium	1.96	2.38	1.97	1.08	−0.63	0.20	0.32	0.28	0.15	−0.10	0.04	0.17	0.19	0.08	0.03
Denmark	13.67	11.12	11.79	11.24	10.74	1.14	0.91	0.82	0.73	0.66	1.95	2.30	3.25	3.28	3.18
Finland	19.65	14.49	14.23	11.41	8.21	1.48	1.48	1.85	2.04	1.72	2.78	5.01	6.96	5.39	3.28
France	−9.75	−10.93	−9.49	−10.00	−10.89	−0.74	−1.43	−1.64	−2.47	−3.97	0.97	3.74	6.17	10.88	16.47
Germany	17.77	15.12	15.12	15.11	13.28	1.07	1.76	1.92	1.93	1.74	1.56	5.04	10.80	16.58	16.29
Italy	−33.16	−28.63	−23.29	−19.12	−18.45	−1.75	−2.07	−2.52	−2.80	−3.46	4.55	10.16	15.51	20.36	27.24
Japan	6.13	12.73	12.65	11.56	10.48	0.41	1.24	1.25	1.22	1.16	0.33	4.57	6.30	6.41	6.37
Netherlands	6.06	8.47	11.33	11.06	8.98	0.42	0.67	1.04	1.20	1.16	0.32	1.44	4.74	5.73	5.26
Norway	−34.27	−29.37	−25.24	−25.28	−26.05	−2.43	−4.10	−3.70	−3.45	−3.36	3.56	12.38	22.44	28.85	35.65
Portugal	−42.17	−42.76	−43.85	−46.49	−46.10	−2.14	−2.42	−2.93	−3.25	−2.89	11.85	22.46	28.69	34.92	31.02
Spain	−18.04	−22.59	−19.14	−18.62	−17.79	−1.41	−2.40	−2.09	−2.05	−1.94	2.42	9.73	11.80	15.50	17.10
Sweden	15.21	17.32	19.29	18.48	19.27	1.48	1.88	2.37	2.46	2.73	3.54	9.61	19.61	22.61	28.04
Switzerland	7.05	11.57	9.51	11.11	11.03	0.39	0.87	0.90	1.18	1.30	0.27	2.01	3.01	5.79	7.64
UK	1.05	6.22	11.23	14.44	16.17	0.07	0.56	1.26	2.11	2.62	0.01	0.88	3.89	7.43	11.03
USA	12.24	10.13	11.34	12.42	13.72	0.83	1.17	1.93	2.60	3.07	1.42	2.42	5.21	9.98	17.62
Mean	−2.18	−1.12	0.09	0.20	−0.32	−0.06	−0.06	0.07	0.12	0.04	2.23	5.78	9.31	12.15	14.16
Median	4.01	7.62	10.37	11.08	8.59	0.30	0.62	0.86	0.95	0.91	1.49	4.15	6.23	8.71	13.66
	Panel B: Predicting consumption growth volatility														
Australia	−4.80	12.99	14.07	13.44	12.20	−0.20	1.60	1.88	2.03	2.09	0.17	4.54	8.03	9.83	9.27
Belgium	−4.34	0.39	5.59	10.58	12.26	−0.33	0.04	0.65	1.26	1.63	0.24	0.00	1.16	5.51	8.86
Denmark	−23.77	−22.00	−16.49	−14.52	−15.41	−1.83	−2.23	−1.65	−1.55	−1.85	3.69	7.15	5.85	6.89	12.03
Finland	−25.16	−14.09	−8.95	−5.96	−5.41	−1.84	−1.03	−0.63	−0.42	−0.40	4.60	3.10	1.74	0.91	0.89
France	16.54	17.63	16.51	16.50	16.37	1.28	1.95	2.11	2.74	3.44	2.07	6.56	9.91	13.57	18.08
Germany	−6.27	−1.99	−0.64	1.33	4.44	−0.47	−0.18	−0.07	0.17	0.57	0.21	0.07	0.01	0.07	1.11
Italy	8.31	5.18	5.95	7.19	9.02	0.73	0.56	0.65	0.93	1.56	0.73	0.79	1.33	2.60	5.04
Japan	1.35	−8.36	−6.69	−7.02	−8.53	0.06	−0.67	−0.55	−0.59	−0.73	0.01	1.09	1.05	1.36	2.26
Netherlands	6.40	9.28	11.58	11.02	10.08	0.54	0.80	0.96	1.02	1.05	0.42	2.10	4.59	5.09	5.23
Norway	−22.89	−23.25	−24.00	−21.67	−19.13	−1.86	−1.76	−2.01	−2.28	−2.66	3.70	8.06	11.96	12.72	12.60
Portugal	−18.51	−12.71	−10.28	−9.25	−9.76	−2.00	−1.38	−1.02	−0.82	−0.77	5.10	3.87	3.57	3.38	3.57
Spain	41.05	34.10	31.91	30.02	29.87	2.88	2.55	2.50	2.32	2.22	11.21	14.65	18.82	23.51	28.32
Sweden	−12.16	−20.44	−17.17	−13.44	−12.42	−0.78	−1.43	−1.41	−1.44	−1.60	0.91	6.08	6.43	5.21	6.27
Switzerland	−13.49	−13.67	−13.37	−9.64	−6.97	−1.01	−1.06	−1.11	−0.84	−0.69	1.40	2.71	3.78	2.63	1.61
UK	−24.73	−16.02	−16.09	−16.80	−16.53	−1.91	−1.85	−2.24	−2.79	−2.77	3.67	4.52	7.87	11.74	14.83
USA	−4.93	−13.20	−10.05	−10.31	−10.96	−0.31	−1.14	−0.98	−1.02	−1.07	0.12	2.32	2.44	3.14	4.42
Mean	−5.46	−4.14	−2.38	−1.16	−0.68	−0.44	−0.33	−0.18	−0.08	0.00	2.39	4.23	5.53	6.76	8.40
Median	−5.60	−10.54	−7.82	−6.49	−6.19	−0.40	−0.85	−0.59	−0.50	−0.55	1.16	3.48	4.19	5.15	5.75

Do Investors Recognize Biases in Securities Analysts' Forecasts?

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Abstract

This study presents direct evidence on the question whether investors recognize the widely documented biases in securities analysts' earnings forecasts. The internal rate of return implied by current stock price and consensus earnings forecasts is found to be correlated with indicators of bias in a manner consistent with investors discounting optimistic earnings forecasts at higher rates of return and less optimistic forecasts at lower rates of return. In a departure from studies of excess returns, the evidence in implied returns indicates that investors recognize the biases in analysts' earnings forecasts.

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Abstract

This study presents direct evidence on the question whether investors recognize the widely documented biases in securities analysts' earnings forecasts. The internal rate of return implied by current stock price and consensus earnings forecasts is found to be correlated with indicators of bias in a manner consistent with investors discounting optimistic earnings forecasts at higher rates of return and less optimistic forecasts at lower rates of return. In a departure from studies of excess returns, the evidence in implied returns indicates that investors recognize the biases in analysts' earnings forecasts.

1. Introduction

A substantial literature investigating analysts' earnings forecasts supports the conclusion that they are biased. A more recent and growing body of research asserts that because investors fail to optimally process available information, they overweight analyst forecasts resulting in substantial mispricing of common stock. This assertion is based on evidence purporting to show the existence of profitable trading strategies formed on indicators of bias. However, on the question whether investors fail to recognize analyst bias, the evidence from realized returns is circumstantial and open to varying interpretation. By now, analyst biases have been extensively documented. Thus, without a compelling explanation of investors' inability to account for them in valuing common stock, the attribution of seemingly profitable trading strategies to deficiencies in investor judgment must be considered tenuous and needing additional corroborating evidence. The present study takes a new approach to the question whether investors fail to recognize analyst forecast bias and investigates the determinants of expected return in a recent cross section of U.S. public companies.

Clearly, from the perspective of financial market efficiency, the inability of investors to recognize analyst bias is troubling. But, is it true? If investors are able to recognize biases in analyst earnings forecasts, then in valuing stocks they will apply higher discount rates to forecasts they believe are biased upward (i.e., optimistic) and lower rates to those they believe are biased downward (pessimistic). It should be the case, then, that stock price relative to the consensus earnings forecast is correlated with indicators of bias. That is, for a given consensus forecast, stock price will be lower (higher) to the extent investors perceive the forecast to be optimistic (pessimistic). If investors are unable to recognize analyst bias (or, equivalently, if they believe analyst forecasts are unbiased), then stock price relative to the consensus forecast will be uncorrelated with indicators of bias. In this study, the relation of stock price to consensus forecast is measured by reverse engineering an equity valuation model to obtain the internal rate of return implied by current stock price and the consensus forecast. The implied return is found to be strongly correlated with indicators of bias in a fashion consistent with investors discounting optimistic (pessimistic) consensus forecasts at higher (lower) rates of return. Hence, in contrast to assertions made in previous studies, the results presented here support the view that equity investors are indeed capable of recognizing and adjusting for analyst bias. As a preliminary indication of this, the sample median implied return of stocks rated by analysts as Buy, Hold and Sell are 10.7%, 8.6% and 7.6%, respectively. Differences among them are highly statistically significant.

The rest of the paper is organized as follows. Section 2 reviews the literature on analyst earnings forecasts as well as attempts to model earnings forecast error and to profit therefrom. Against this backdrop, the contribution of the present study is articulated. The empirical methodology and data are described in section 3. Section 4 presents and discusses the findings, and section 5 summarizes and concludes.

2. Review of Literature

The literature on analysts' earnings and stock price forecasts indicates that long-range forecasts are optimistic, short-range forecasts are pessimistic, and forecasts generally do not fully reflect available information. Companies report earnings that on average fall short of consensus long-range forecasts (e.g., Abarbanell & Lehavy, 2003; Agrawal & Chen, 2006; Bradshaw et al., 2006; Brous, 1992; Brous & Kini, 1993; Butler & Lange, 1991; Dreman & Berry, 1995; Easterwood & Nutt, 1999; Francis & Philbrick, 1993; Fried & Givoly, 1982; Kang et al., 1994; Richardson et al., 2004), and stock prices tend not to reach analysts' long-range price targets (e.g., Cowen et al., 2006; Szakmary et al., 2008). Researchers attempting to understand the factors driving these biases have considered analysts' relationships with their employers, with the firms they cover, and with their investor clients. Forecast optimism has been attributed to the investment banking and trading activities of analysts' sell-side employers, to the tendency of analysts to cover firms about which they are optimistic, and to analysts' desire to appease company executives in order to maintain access to valuable information. Management guidance and analysts' desire to establish and maintain credibility with investor clients act to dampen analyst optimism (Cowen et al., 2006; Dugar & Nathan, 1995; Francis & Philbrick, 1993; Lin & McNichols, 1998; Ljungqvist et al., 2007; Michaely & Womack, 1999; Raedy et al., 2006; Richardson et al., 2004).

Apart from being biased, consensus earnings forecasts do not fully incorporate available information and are therefore inefficient. Forecast errors are correlated with prior forecast errors, past stock returns, and past earnings changes (Ali et al., 1992; Abarbanell and Bernard, 1992; Shane and Brous, 2001), and Cohen and Lys (2003) report that analysts underreact to prior information. Attempts to explain these inefficiencies have relied on the existence of defects in analysts' judgment. Conservatism bias, for example, is alleged to cloud analyst judgment. However, Raedy et al. (2006) provide a rational explanation for underreaction in terms of analyst credibility. For a forecast error of given magnitude, credibility is damaged when later information causes a forecast revision in the direction opposite the analyst's previous revision. Hence, analyst forecast inefficiency could arise from rational incentives as opposed to defective judgment.

A related stream of research seeks to model earnings forecast error in order to improve earnings forecasts. Laroque (2013), for example, models earnings forecast error in terms of lagged forecast error, lagged abnormal stock return, and lagged equity market value. Mohanram and Gode (2013) model forecast error in terms of lagged accruals, lagged sales growth, lagged analyst forecasts of long-term growth, lagged change in property, plant and equipment, lagged change in other total assets, lagged stock return, and the revision in analyst forecasts from the prior year. Easton and Monahan (2016) conclude that while these methods are effective in removing errors in earnings forecast levels, they are less effective in removing errors in forecasts of earnings changes.

Another related research stream seeks to identify profitable trading strategies by exploiting predictable earnings forecast error. Kothari et al. (2016) survey the literature on analysts' forecasts and asset pricing and conclude that investors only partially unravel the biases in analysts' forecasts resulting in predictable stock prices. Their conclusion is based on research demonstrating the ability to generate seemingly profitable trading strategies from predictable forecast error. So (2013), for example, addresses the question whether investors overweight analyst forecasts by examining the excess returns of portfolios constructed on the basis of predictable earnings forecast error. He models earnings on the basis of lagged company characteristics and sorts portfolios on the basis of the disparity between characteristic forecasts and analyst consensus forecasts. He finds that excess returns can be earned by purchasing stocks for which consensus forecasts are low vis-à-vis characteristic forecasts and selling those for which the opposite is true. Based on this, So (2013) concludes that "investors fail to fully undo predictable biases in analyst forecasts" (p.636). Consequently, investors overweight analyst forecasts resulting in "substantial valuation errors" (p.616). They pay too much for stocks for which analysts are optimistic and too little for those for which analysts are pessimistic. Da and Warachka (2011) report evidence of market inefficiency with respect to analysts' long-term growth forecasts. When the long-term forecast is well above the short-term forecast, both subsequent returns and subsequent revisions to long-run forecasts tend to be negative. The reverse is true when the long-run forecast is well below the short-run forecast. Da and Warachka (2011) conclude that the growth disparity is a "robust predictor" of abnormal return that can be attributed to "investor inattention" to long-term growth prospects.

The evidence from excess returns on the soundness of investor judgment is not conclusive. To begin, not all research finds that profitable strategies can be implemented from analyst bias. For example, Hughes, et al. (2008) conclude that "trading strategies based directly on the predictable component of forecast errors are not profitable" (p. 266). Moreover, Barber, et al. (2001) document high levels of trading required to capture the excess returns of strategies based on analyst recommendations. It is not clear from Da and Warachka (2011) and So (2013) that the returns they document are sufficient to cover transactions costs, which, in addition to bid-ask spreads, include the expense of borrowing securities and the uncertainties of maintaining short positions for extended periods. It is also not clear that these excess returns are sufficiently reliable to warrant devoting large sums to capturing them. Figure 3 in So (2013), for example, shows extended periods in which the strategy he documents would have generated low or negative excess returns, and Table 5 shows that the explanatory power of cross sectional regressions for excess return is quite low. Hence, it is not clear that these excess returns can be exploited at meaningful scale, and even if they can be, it is not clear that these inefficiencies continue to exist.

The present study takes a different approach to the question whether investors fail to recognize bias in analyst forecasts. The logic is straightforward. For a given consensus forecast, stock price will be lower to the extent investors perceive the consensus to be biased upward, and it will be higher to the extent investors perceive the consensus to be biased downward. As described in the following section, the relation between stock price and consensus forecast is expressed in terms of an implied rate of return, so that for a given consensus forecast, higher stock price implies lower rate of return. If investors fail to recognize analyst bias (or, equivalently, if they believe the consensus forecast to be unbiased) then there should be no correlation between implied return and indicators of bias. This study seeks to determine whether any such correlation exists in the data.

3. Methodology and Data

The research question concerns whether investors fail to unravel the widely documented biases and inefficiencies in analyst forecasts. Direct evidence is to be found in the internal rate of return implied by current stock price and analysts' consensus earnings forecasts (hereafter, implied return). Easton and Monahan (2016) explain that the justification for using the implied return to estimate expected return is twofold; realized returns are not reliable measures of expected return, and risk factors are either unknown or cannot be reliably forecast. Analysts' consensus earnings forecast is biased when it differs from the market's. It is biased upward (i.e., optimistic) when it exceeds the market forecast and biased downward (pessimistic) when it is below the market forecast. A stock's market price embeds the market forecast. To the extent that the consensus forecast differs from the market forecast, implied return differs from expected return. For the present study it is important to note that implied return embeds the difference, if any, between the market forecast and consensus forecast. If investors fail to recognize bias in analyst forecasts then implied return equals expected return and is uncorrelated with indicators of analyst bias. Alternatively, if investors do recognize analyst bias then implied return is correlated with indicators of bias. The null hypothesis that investors fail to recognize bias is tested in a cross-sectional regression analysis of the correlates of implied return.

Implied returns for individual stocks are computed by the procedure described in Easton (2004). The procedure reverse engineers a valuation model to compute the internal rate of return implied by current stock price and consensus EPS forecasts. It starts from a one-period valuation model:

$$p_0 = (1 + r)^{-1}(p_1 + d_1)$$

where p_0 is current stock price, p_1 and d_1 are the expected stock price and dividend one year hence, and r is the rate at which investors discount future equity cash flows. By recursive substitution for future stock price:

$$p_0 = \frac{e_1}{r} + r^{-1}\sum_{t=1}^{\infty}(1+r)^{-t}agr_t$$

where $agr_t = e_{t+1} + rd_t - (1+r)e_t$, and e_t is earnings per share for year t that is forecast at $t = 0$. agr_t can be shown to equal the forecast change in residual income from t to $t+1$. Assuming constant growth in agr after a two-year forecast horizon,

$$p_0 = \frac{e_1}{r} + \frac{agr_1}{r(r - \Delta agr)}$$

where $\Delta agr = (agr_{t+1}/agr_t) - 1$ denotes the constant perpetual growth rate of agr after t = 2. Assuming further that $\Delta agr = 0$, then $p_0 = (e_2 + rd_2 - e_1)/r^2$, and **implied return (r)** is the solution to equation (1):

$$r^2 - r(d_2/p_0) - (e_2 - e_1)/p_0 = 0 \quad (1)$$

In equation (1), e_1 and e_2 denote consensus EPS forecasts for one year and two years ahead, and $d_2 = d_0(1 + r')^2$. d_0 denotes trailing one-year dividend per share, and r' is the solution to (1) assuming dividends equal zero. Easton and Monahan (2016) review the literature employing this model.

For the sake of assessing the robustness of empirical results, implied return is also computed by the method in Peasnell, et al. (2018), which incorporates analysts' consensus long-term growth forecast in the implied return equation. In Peasnell et al. (2018), implied return (r) is the solution to the following:[1]

[1] This is equation (2) on p. 221 of Peasnell et al. (2018).

$$p_0 = \frac{e_1}{r} + \frac{agr_1}{r}\left[1 - \frac{\left(\frac{1 + LTG}{1 + r}\right)^4}{r - LTG}\right] + \left(\frac{agr_1(1 + LTG)^4}{r^2(1 + r)^5}\right)$$

where LTG is the consensus long-term (i.e., 3-5 year) growth forecast, and other variables are as defined previously.

In valuing stock, investors will discount the consensus earnings forecast at a higher (lower) rate when the consensus is thought to be optimistic (pessimistic). Hence, for a given consensus forecast, stock price will be lower (and implied return higher) when the consensus is biased up, and stock price will be higher (and implied return lower) when the consensus is biased down. To test the hypothesis that investors fail to recognize bias in analyst forecasts, determinants of implied return are investigated in cross-sectional regression. The base regression is given by equation (2):

$$r_i = \alpha_0 + \alpha_1 Rating_i + \alpha_2 Beta_i + \alpha_3 Vol_i + \alpha_4 Debt_i + \alpha_5 Size_i + \alpha_6 Own_i + \varepsilon_i \qquad (2)$$

Rating is analysts' consensus investment rating for each stock on a scale from 1 (strong sell) to 5 (strong buy). Peasnell, et al. (2018) show that investment rating and analysts' earnings forecasts are positively correlated. Hence, biases in earnings forecasts can be expected to translate into investment rating, so that, conversely, Rating can be assumed to be positively correlated with analyst bias. If investors fail to recognize analyst bias, the coefficient on Rating is not statistically different from zero. Under the alternative by which investors recognize and make adjustments for biased forecasts, the coefficient on Rating is positive.

To assess the robustness of results from equation (2), Rating is operationalized alternatively in discrete form as dummy variables Buy and Sell indicating, respectively, the top and bottom quartiles of the Rating distribution. Rating is further operationalized in terms of the residual from a regression model for investment rating motivated by Peasnell, et al. (2018). Consensus forecast bias is also measured in terms of analysts' long-term growth forecasts. These measures are described further below.

Remaining variables in equation (2) are control variables that on theoretical and/or empirical grounds appear to influence the cross section of stock returns. Beta measures the stock's sensitivity to a market index. As a measure of undiversifiable risk, its coefficient is expected to be positive. Vol denotes target price return volatility. It is calculated as the natural log of the ratio of analysts' highest target stock price to their lowest target price. It represents uncertainty about fundamental value, and its coefficient is expected to be positive. Debt is calculated as the ratio of net debt to trailing one-year EBITDA. As a measure of financial risk, its coefficient is expected to be positive. Size is measured as the natural log of market capitalization. As an inverse measure of risk, its coefficient is expected to be negative. Own is the proportion of shares outstanding held by institutions. Cowen et al. (2006) and Ljungqvist et al. (2007) find that analyst bias is tempered in the presence of institutional investors. Hence, the coefficient on Own is expected to be negative.

All data were downloaded from a Bloomberg terminal[2] on the following dates: October 23, 2018, November 2, 2018, November 8, 2018, December 10, 2018, January 16, 2019, February 4, 2019, February 11, 2019, February 13, 2019, February 20, 2019, February 25, 2019, March 11, 2019, March 18, 2019, April 1, 2019, April 8, 2019, and April 15, 2019. Because the main results of this study are confirmed for each of these dates in separate analyses, only the

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[2] As of this writing, more than 300 universities integrate Bloomberg into their curricula: https://www.bloomberg.com/professional/expertise/universities/.

results from April 15, 2019 are reported. Results for non-reported dates are available on request.

The initial sample of 2,865 is the result of a screen for all firms that are members of the Russell 3000 index with a stock price at least equal to $1 per share and a ratio of stock price to book value greater than zero. After eliminating stocks with fiscal year end other than December 31, 2018 (555), firms in financial services (731), and firms lacking sufficient data for analysis (742), the sample size is reduced to 837. Summary statistics are presented in Table 1. Table 1 shows that the median firm has an implied return of 8.6%, a market cap of $4.3 billion, and is followed by 12 analysts with a consensus rating of 4.0 on a scale of 1 (strong sell) to 5 (strong buy). Hence, the median firm is recommended by analysts for addition to investment portfolios. The distribution of Rating is consistent with the widely recognized dearth of sell ratings issued by sell-side analysts. In subsequent analysis, the variable **Buy** takes a value of 1 for stocks with Rating of 4.4 or higher (the upper quartile) and 0 otherwise. The variable **Sell** takes a value of 1 for stocks with a Rating of 3.4 or lower (the lower quartile) and 0 otherwise.[3]

Table 1 here

Table 2 reports sample characteristics by Rating. Characteristics of the median Buy-rated firm show that analysts are optimistic about the prospects of the companies they recommend. For Buys, the consensus target price exceeds current stock price by 21.5%, EPS is forecast to increase by 21% from one year to two years in the future, and forecast long-term growth is 15%. These forecasts substantially exceed comparable figures for Holds and Sells. Investors, however, appear somewhat dubious. They are paying a median 16 times forward

[3] Results are robust to alternative percentile values used in the definitions of Buy and Sell.

EPS for Buys versus 18.2 for Holds and 19.2 for Sells. The median ratio of forward P/E to short-term growth (PEG) is 0.9 for Buys versus 1.4 and 1.8 for Holds and Sells, respectively. The median implied return for Buys exceeds that for Holds by 2.1 percentage points and that for Sells by 3.1 percentage points. In sum, the preliminary results reported in Table 2 suggest that investors consider analyst bias in their investment decisions.

Table 2 here

4. Empirical Results

Regression results for implied return as a function of analysts' consensus investment rating are presented in Table 3. In model (1), Rating is analysts' consensus investment rating on a scale from 1 (strong sell) to 5 (strong buy). In (2), Buy and Sell are dummy variables indicating the top and bottom quartiles, respectively, of the Rating distribution. In (3) and (4), Rating, Buy and Sell are defined as in (1) and (2) for the residuals of a regression model for Rating. This is described further below. All regressions include dummy variables indicating membership among 40 different industry groups. In each model, the null hypothesis can be rejected with a high degree of confidence. In model (1), the coefficient on Rating is positive and highly statistically significant at better than the .01 level. Hence, holding all other stock and firm characteristics constant, stocks with higher investment ratings have lower market prices and higher implied returns, which is consistent with the view that investors are able to distinguish analyst bias and factor it into their investment decisions. Consistent with this interpretation, model (2) shows that the coefficient on Buy is positive and that on Sell is negative, and both are highly statistically significant at better than .01. As evidence of the models' overall reliability, coefficients on all control variables in (1) and (2) are statistically significant with anticipated signs, and adjusted R^2 indicate good explanatory power for implied return. Hence, the effect of analyst bias on implied return is apparent even while controlling for a range of firm characteristics that are known on theoretical and/or empirical grounds to impact the cross section of return.

Estimated coefficients on Buy and Sell in model (2) indicate the average implied return of a buy-rated stock is 1.6 percentage points above that of a hold-rated stock, and the average implied return of a sell-rated stock is 1.3 percentage points below that of a hold-rated stock. Hence, the difference in implied return between buy-rated and sell-rated stocks is nearly 3 percentage points. At sample medians for consensus forecasts, the implied price of a buy-rated stock is more than $13 per share less than that of a sell-rated stock with the same dividend and forecast earnings. Clearly, investors appear to unravel analyst biases by applying higher discount rates to increasingly optimistic forecasts.

To assess the robustness of these inferences on Rating, regression analyses are repeated using the residuals from a regression model for Rating as a function of consensus short-term and long-term earnings growth forecasts, book return on equity, the ratio of book to market value, prior earnings forecast error, beta, stock price volatility, and market capitalization.[4] Columns (3) and (4) of Table 3 present these results. In (3) and (4), Rating, Buy and Sell are defined as in (1) and (2) for the residuals of a regression model for Rating. Confirming the above inference, coefficients on Rating, Buy and Sell are all highly significant with signs consistent with the view that investors unravel analyst biases.

[4] See equation (6a), page 227, in Peasnell, et al. (2018). These results are available on request.

Table 3 here

Analyst forecast bias is measured further in terms of long-term growth forecasts. Da and Warachka (2011) show that when the long-term growth forecast is well above the short-term forecast, both subsequent returns and subsequent revisions to long-term forecasts tend to be negative. The reverse is true when the long-term forecast is well below the short-term forecast. Consequently, it is reasonable to conclude that extreme values of the distribution of consensus long-term growth forecast might indicate analyst forecast bias. To assess this possibility, the base regression model equation (2) is augmented to include consensus long-term growth forecast to investigate the extent to which it is correlated with implied return. If forecast bias is reflected in analysts' long-term growth forecasts, then the coefficient on long-term growth will be positive. For each firm, long-term growth forecast is normalized by industry mean and standard deviation.[5] These results are presented in Table 4 where $\mathbf{LTG}$ denotes industry-normalized long-term growth.

Model (1) in Table 4 shows that implied return is positively correlated with industry-normalized long-term growth and that the correlation is highly statistically significant at the .01 level. In Model (2), LTG is discretized in dummy variables $\mathrm{LTG\ High}$ and $\mathrm{LTG\ Low}$ indicating, respectively, the top and bottom quartiles of the distribution of LTG. The coefficient on $\mathrm{LTG\ High}$ is positive and statistically significant at the .01 level, and the coefficient on $\mathrm{LTG\ Low}$ is negative and significant at the .05 level. Model (3) shows that the coefficients of LTG and Rating retain their signs and statistical significance when both are included as explanatory

[5]Results are robust to alternative transformations and to no transformation.

variables. Hence, it appears that long-term growth and investment rating capture independent dimensions of analyst bias. Model (4) shows that implied return is increasingly sensitive to investment rating in the context of extreme long-term growth forecasts. The coefficient on Rating x LTG High is positive and significant at the .01 level, which shows that investors increasingly discount analysts' short-term (i.e., one- and two-year ahead) earnings forecasts in the context of strong buy recommendation coupled with optimistic long-term growth forecast. The coefficient on Rating x LTG Low is negative and significant at the .10 level, which shows that investors discount analysts' short-term earnings forecasts at a lower rate when analysts' long-term growth forecasts are relatively conservative and when analysts are not pounding the table for investors to buy the shares. These results provide additional confirming evidence that investors recognize analyst forecast bias.

Table 4 Here

5. Conclusion

Although the biases and inefficiencies in securities analysts' forecasts are widely documented, a body of research asserts that investors do not take account of them in their valuation of common stock. The evidence presented in support of this claim is the apparent ability to generate profitable trading strategies based on indicators of analyst bias. This evidence, however, is not unequivocal. The abnormal returns that are said to be available from various trading strategies provide, at best, indirect evidence of defects in investor decision making. Additionally, it is not clear whether they continue to exist after their existence has been published. It is not clear that they are of sufficient magnitude to exceed the transactions costs that must be incurred to capture them or that they are sufficiently persistent to justify allocating meaningful sums to exploit them at scale. Apart from these issues concerning the quality of the evidence, a compelling theory of ongoing defects in investor decision making is lacking. Hence, the claim that investors do not consider analyst forecast biases in their decision making seems counterintuitive especially in light of the hunger for alpha that is evident in the investment community.

The present study takes a new approach to the question whether investors recognize biases in analyst forecasts. It investigates the determinants of the rate of return of common stock that is implied by current stock price and analysts' consensus earnings forecasts. If investors fail to recognize biases in consensus forecasts, then implied return will be uncorrelated with indicators of bias. The study finds that implied return is in fact highly correlated with indicators of bias in a manner consistent with investors discounting optimistic earnings forecasts at higher rates of return and pessimistic forecasts at lower rates of return. Hence, in contrast to the inferences reached in studies of realized returns, the present study concludes that investors do indeed appear to recognize the biases in analysts' forecasts.

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Table 1
Summary Statistics

The table reports summary statistics for N = 837 observations with non-missing data and fiscal year end December 31, 2018. *Implied return* is computed from equation (1). *Rating* is derived from analysts' investment recommendations measured on a scale from 1 (strong sell) to 5 (strong buy). *Number of analysts* is the count of analysts issuing forecasts. *Beta* is the slope coefficient from a regression of a stock's return to the S&P 500 index return . *Target price volatility* is the log ratio of analysts' highest target price to lowest target price. *Debt* is the ratio of net debt to trailing one-year EBITDA. *Institutional ownership* is the proportion of shares outstanding owned by institutions. *Market cap* is stock price times shares outstanding, in millions of dollars. *Price to forward EPS* is the ratio of current stock price to consensus EPS one year ahead. *Short-term EPS growth* is the forecast growth of EPS from year 1 to year 2: $e_2/e_1 - 1$. *Long-term growth* is analysts' consensus forecast of 3-5 year ahead growth.

	Mean	**Min**	**Q1**	**Median**	**Q3**	**Max**
Implied return	0.101	0.011	0.070	0.086	0.117	0.429
Rating	3.9	1.0	3.4	4.0	4.4	5.0
Number of analysts	14.0	1.0	7.0	12.0	19.0	50.0
Beta	1.03	0.24	0.89	1.03	1.17	2.05
Target price volatility	0.41	0.00	0.25	0.36	0.51	2.38
Debt	1.86	-175.0	0.40	2.01	3.59	63.82
Institutional ownership	0.62	0.09	0.55	0.64	0.72	0.94
Market cap ($ millions)	19,673	62	1,634	4,290	13,151	906,884
Price-to-forward EPS	25.6	2.8	13.0	18.2	25.6	573.2
Short-term EPS growth	0.29	0.00	0.09	0.13	0.24	21.10
Long-term growth	0.14	-0.27	0.07	0.11	0.16	1.82

Source: Bloomberg Finance L.P., accessed April 15, 2019.

Table 2
Sample Characteristics by Consensus Rating

The table reports median values by analysts' consensus rating. Buy- (Sell-) rated stocks are those in the top (bottom) quartile of the distribution of consensus ratings. *Implied return* is computed from equation (1). *Target price implied return* is the log ratio of consensus target price to current stock price. *Target price volatility* is the log ratio of analysts' highest target price to lowest target price. *Stock price volatility, 1 year trailing* is the standard deviation of daily proportionate stock price changes during the prior 260 trading days, annualized. *Stock price growth, 1 year trailing* is the log ratio of current stock price to stock price one year prior. *Market cap* is stock price times shares outstanding. *Institutional ownership* is the proportion of shares outstanding owned by institutions. *Number of analysts* is the count of analysts issuing forecasts. *Price to forward EPS* is the ratio of current stock price to consensus EPS one year ahead. *Short-term EPS growth* is the forecast growth of EPS from year 1 to year 2: $e_2/e_1 - 1$. *PEG* is 100 times the ratio of the forward PE to short-term EPS growth. *Long-term growth* is analysts' consensus 3-5 year ahead growth rate forecast.

		Rating	
	Sell	**Hold**	**Buy**
Implied return	0.076	0.086	0.107
Target price implied return	0.010	0.103	0.215
Target price volatility	0.382	0.373	0.310
Stock price volatility, 1 year trailing	0.311	0.329	0.381
Stock price growth, 1 year trailing	0.046	0.047	0.001
Market cap ($ millions)	3,555	5,490	2,906
Institutional ownership	0.63	0.64	0.63
Number of analysts	11	13	9
Price to forward EPS	19.2	18.2	16.0
Short-term EPS growth	0.10	0.14	0.21
PEG	1.8	1.4	0.9
Long-term growth	0.09	0.13	0.15
Number of firms	255	459	123

Source: Bloomberg Finance L.P., accessed April 15, 2019.

Table 3

Regression Results for Implied Return as a Function of Investment Rating

$$\mathbf{r_i = \alpha_0 + \alpha_1 Rating_i + \alpha_2 Beta_i + \alpha_3 Vol_i + \alpha_4 Debt_i + \alpha_5 Size_i + \alpha_6 Own_i + \varepsilon_i}$$

In model (1), Rating is analysts' consensus investment rating on a scale from 1 (strong sell) to 5 (strong buy). In model (2), Buy and Sell are dummy variables for the top and bottom quartiles, respectively, of the Rating distribution. In models (3) and (4), Rating, Buy and Sell are defined as in (1) and (2) for the residuals of a regression model for Rating. Beta is the slope coefficient from a regression of a stock's return to the S&P 500 index return . Vol is the log ratio of analysts' highest target price to lowest target price. Debt is the ratio of net debt to trailing one-year EBITDA. Size is the natural log of market capitalization in millions of dollars. Own is the proportion of shares outstanding owned by institutions. All regressions include dummy variables indicating membership among 40 different industry groups. Heteroscedasticity consistent standard errors in parentheses. ***,**,* indicate significance at the .01, .05, .10 levels in two-tailed tests.

	(1)	(2)	(3)	(4)
Constant	0.303***	0.351***	0.278***	0.279***
	(0.027)	(0.029)	(0.027)	(0.028)
Rating	0.015***		0.009***	
	(0.003)		(0.002)	
Buy		0.016***		0.007**
		(0.004)		(0.003)
Sell		-0.013***		-0.008***
		(0.003)		(0.003)
Beta	0.016*	0.017*	0.022***	0.022***
	(0.010)	(0.010)	(0.008)	(0.008)
Vol	0.034***	0.034***	0.024***	0.024***
	(0.007)	(0.007)	(0.006)	(0.006)
Debt	0.001*	0.001*	0.000	0.000
	(0.000)	(0.000)	(0.000)	(0.000)
Size	-0.012***	-0.011***	-0.008***	-0.008***
	(0.001)	(0.001)	(0.001)	(0.001)
Own	-0.034***	-0.032**	-0.032***	-0.033***
	(0.013)	(0.013)	(0.010)	(0.010)
N	837	837	672	672
adjusted R^2	.454	0.448	.432	.432

Table 4
Regression Results for Implied Return as a Function of Rating and Long-term Growth

$$\mathbf{r_i = \alpha_0 + \alpha_1 Rating_i + \alpha_2 LTG_i + \alpha_3 Beta_i + \alpha_4 Vol_i + \alpha_5 Debt_i + \alpha_6 Size_i + \alpha_7 Own_i + \varepsilon_i}$$

Rating is analysts' consensus investment rating on a scale from 1 (strong sell) to 5 (strong buy). LTG is analysts' consensus long-term earnings growth forecast normalized by industry mean and standard deviation. LTG High and LTG Low are dummy variables constructed on the basis of the distribution of industry-normalized LTG. Beta is the slope coefficient from a regression of a stock's return to the S&P 500 index return. Vol is the log ratio of analysts' highest target price to lowest target price. Debt is the ratio of net debt to trailing one-year EBITDA. Size is the natural log of market capitalization in millions of dollars. Own is the proportion of shares outstanding owned by institutions. All regressions include dummy variables indicating membership among 40 different industry groups. Heteroscedasticity consistent standard errors in parentheses. ***,**,* indicate significance at the .01, .05, .10 levels in two-tailed tests.

	(1)	(2)	(3)	(4)
Constant	0.279*** (0.027)	0.280*** (0.027)	0.263*** (0.027)	0.265*** (0.027)
Rating			0.008*** (0.002)	0.007*** (0.002)
LTG	0.007*** (0.002)		0.006*** (0.002)	
LTG High		0.012*** (0.003)		
LTG Low		-0.007** (0.003)		
Rating x LTG High				0.0024*** (0.0008)
Rating x LTG Low				-0.0014* (0.0008)
Beta	0.026*** (0.009)	0.025*** (0.008)	0.021** (0.009)	0.020** (0.009)
Vol	0.023*** (0.006)	0.023*** (0.006)	0.028*** (0.006)	0.028*** (0.006)
Debt	0.0004 (0.0003)	0.0004 (0.0003)	0.0004 (0.0003)	0.0004 (0.0003)
Size	-0.008*** (0.001)	-0.008*** (0.001)	-0.009*** (0.001)	-0.009*** (0.001)
Own	-0.031*** (0.010)	-0.032*** (0.010)	-0.031*** (0.010)	-0.031*** (0.010)
N	687	687	687	687
adjusted R^2	.471	.474	.482	.483

Do Investors Recognize Biases in Securities Analysts' Forecasts?

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Abstract

This study presents direct evidence on the question whether investors recognize the widely documented biases in securities analysts' earnings forecasts. The internal rate of return implied by current stock price and consensus earnings forecasts is found to be correlated with indicators of bias in a manner consistent with investors discounting optimistic earnings forecasts at higher rates of return and less optimistic forecasts at lower rates of return. In a departure from studies of excess returns, the evidence in implied returns indicates that investors recognize the biases in analysts' earnings forecasts.

JEL Codes: G11, G12, G14, G41
Keywords: analyst forecast bias, behavioral bias, market efficiency, earnings

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Abstract

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1. Introduction

A substantial literature investigating analysts' earnings forecasts supports the conclusion that they are biased. A more recent and growing body of research asserts that because investors fail to optimally process available information, they overweight analyst forecasts resulting in substantial mispricing of common stock. This assertion is based on evidence purporting to show the existence of profitable trading strategies formed on indicators of bias. However, on the question whether investors fail to recognize analyst bias, the evidence from realized returns is circumstantial and open to varying interpretation. By now, analyst biases have been extensively documented. Thus, without a compelling explanation of investors' inability to account for them in valuing common stock, the attribution of seemingly profitable trading strategies to deficiencies in investor judgment must be considered tenuous and needing additional corroborating evidence. The present study takes a new approach to the question whether investors fail to recognize analyst forecast bias and investigates the determinants of expected return in a recent cross section of U.S. public companies.

Clearly, from the perspective of financial market efficiency, the inability of investors to recognize analyst bias is troubling. But, is it true? If investors are able to recognize biases in analyst earnings forecasts, then in valuing stocks they will apply higher discount rates to forecasts they believe are biased upward (i.e., optimistic) and lower rates to those they believe are biased downward (pessimistic). It should be the case, then, that stock price relative to the consensus earnings forecast is correlated with indicators of bias. That is, for a given consensus forecast, stock price will be lower (higher) to the extent investors perceive the forecast to be optimistic (pessimistic). If investors are unable to recognize analyst bias (or, equivalently, if they believe analyst forecasts are unbiased), then stock price relative to the consensus forecast will be uncorrelated with indicators of bias. In this study, the relation of stock price to consensus forecast is measured by reverse engineering an equity valuation model to obtain the internal rate of return implied by current stock price and the consensus forecast. The implied return is found to be strongly correlated with indicators of bias in a fashion consistent with investors discounting optimistic (pessimistic) consensus forecasts at higher (lower) rates of return. Hence, in contrast to assertions made in previous studies, the results presented here support the view that equity investors are indeed capable of recognizing and adjusting for analyst bias. As a preliminary indication of this, the sample median implied return of stocks rated by analysts as Buy, Hold and Sell are 10.7%, 8.6% and 7.6%, respectively. Differences among them are highly statistically significant.

The rest of the paper is organized as follows. Section 2 reviews the literature on analyst earnings forecasts as well as attempts to model earnings forecast error and to profit therefrom. Against this backdrop, the contribution of the present study is articulated. The empirical