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STATEMENT OF ACTUARIAL OPINION

It is my opinion that: (1) the techniques and methodology used herein to evaluate the actuarial status of the Federal Old-Age and Survivors Insurance and Disability Insurance Trust Funds are based upon sound principles of actuarial practice and are generally accepted within the actuarial profession; and (2) the assumptions used and the resulting actuarial estimates are, individually and in the aggregate, reasonable for the purpose of evaluating the actuarial status of the trust funds, taking into consideration the past experience and future expectations for the population, the economy, and the program. I am a member of the American Academy of Actuaries and I meet the Qualification Standards of the American Academy of Actuaries to render this actuarial opinion.

Stephen C. Goss

Associate, Society of Actuaries
Member, American Academy of Actuaries
Chief Actuary, Social Security Administration

CRESSET.

https://cressetcapital.com/post/understanding-wage-inflations-role-and-outlook/

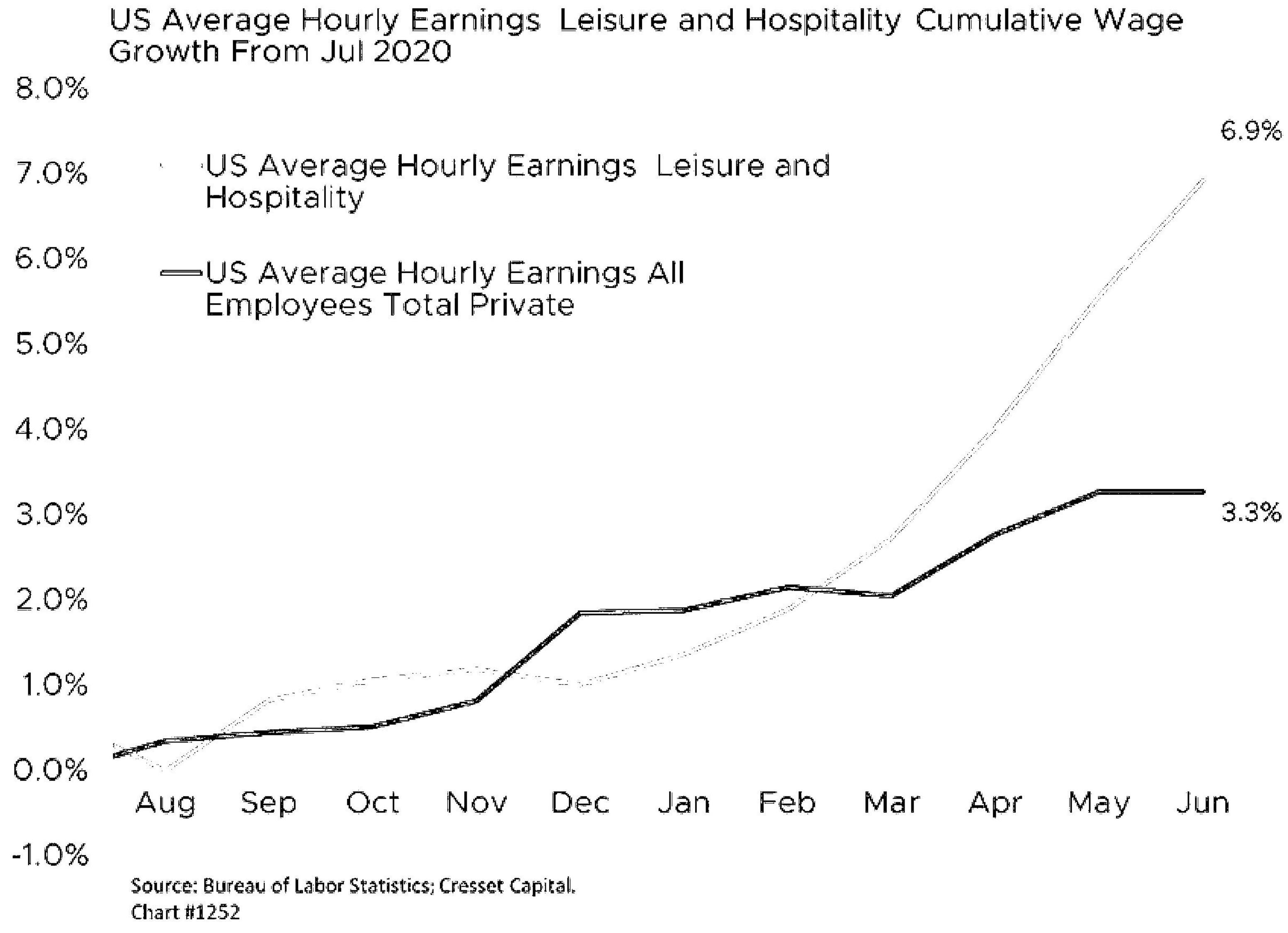
UNDERSTANDING WAGE INFLATION'S ROLE AND OUTLOOK

MARKET COMMENTARY | JACK ABLIN | 6/23/21

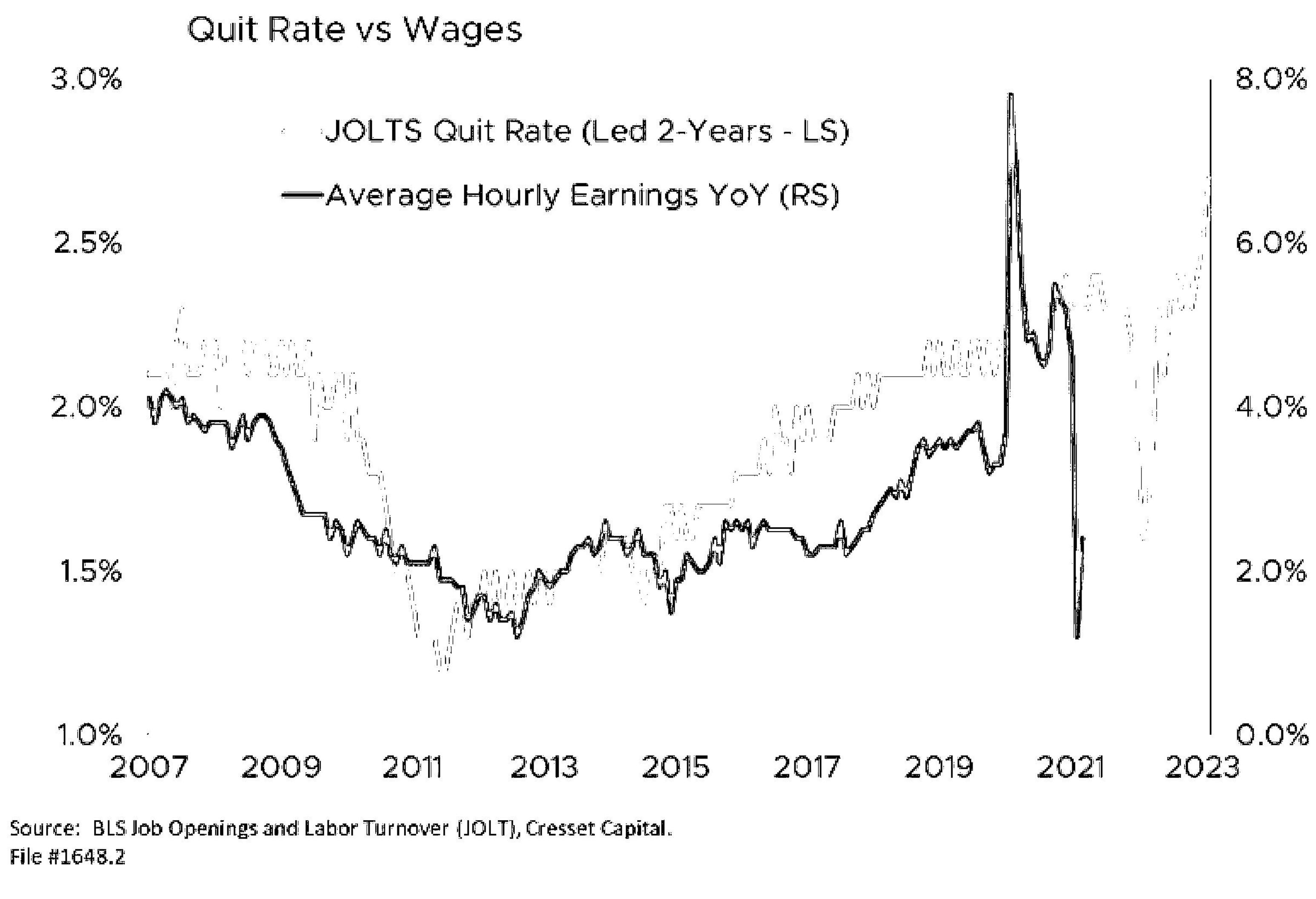
Fed governors flinched last week by accelerating their rate hike timetable. Federal Open Market Committee (FOMC) members, surprised by this spring's inflation spike, are penciling in the possibility of persistently elevated pricing. Many of the recent price hike offenders appear to be transitory, like used cars and hotel rooms. If history is a guide, sustained inflation cannot occur without a wage spiral. That's why understanding the labor market is the key to determining the future path of prices.

FOMC officials believe that maximum employment is a critical condition for raising short-term interest rates. That's a higher bar than full employment, their previous objective. While current labor market conditions suggest inflation could be lifting off, we need to keep in mind that there are still more than 8.8 million unemployed Americans. We are watching several factors in today's labor market that could help determine what happens to prices and the monetary policy response.

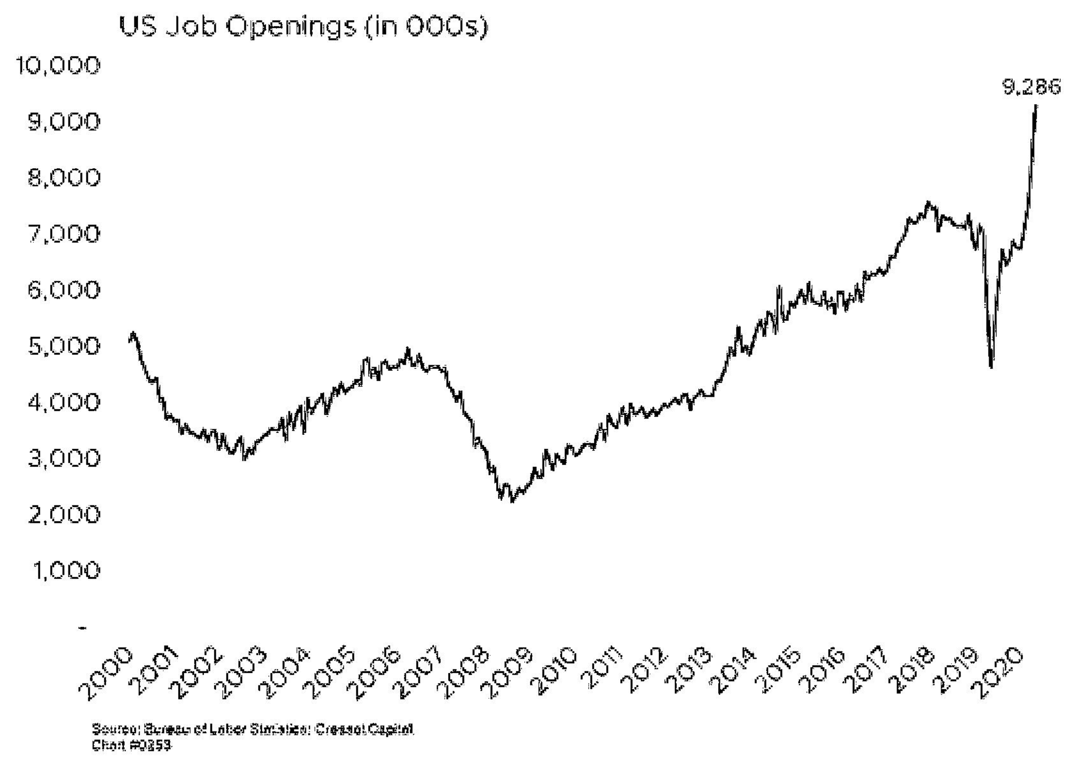
Prompted by tight labor market conditions, Americans are voluntarily leaving their jobs at a rate not seen in decades. This is a very interesting trend to watch. The "quit rate", at 2.7 per cent, is at its highest level in over 20 years, as emboldened workers rethink their professional lives. Low-wage workers, particularly those in the leisure and hospitality industries, are resisting low pay and long hours in sectors that required them to commute and mingle with potentially infectious customers. As a result, employers are being compelled to up the ante. Wages among leisure and hospitality employees, which include restaurant workers, have risen nearly 7 per cent since last July, according to the Labor Department. That dwarfs the 3.3 per cent wage gain among private sector workers in general. McDonalds and other restaurant giants are offering signing bonuses to would-be workers.



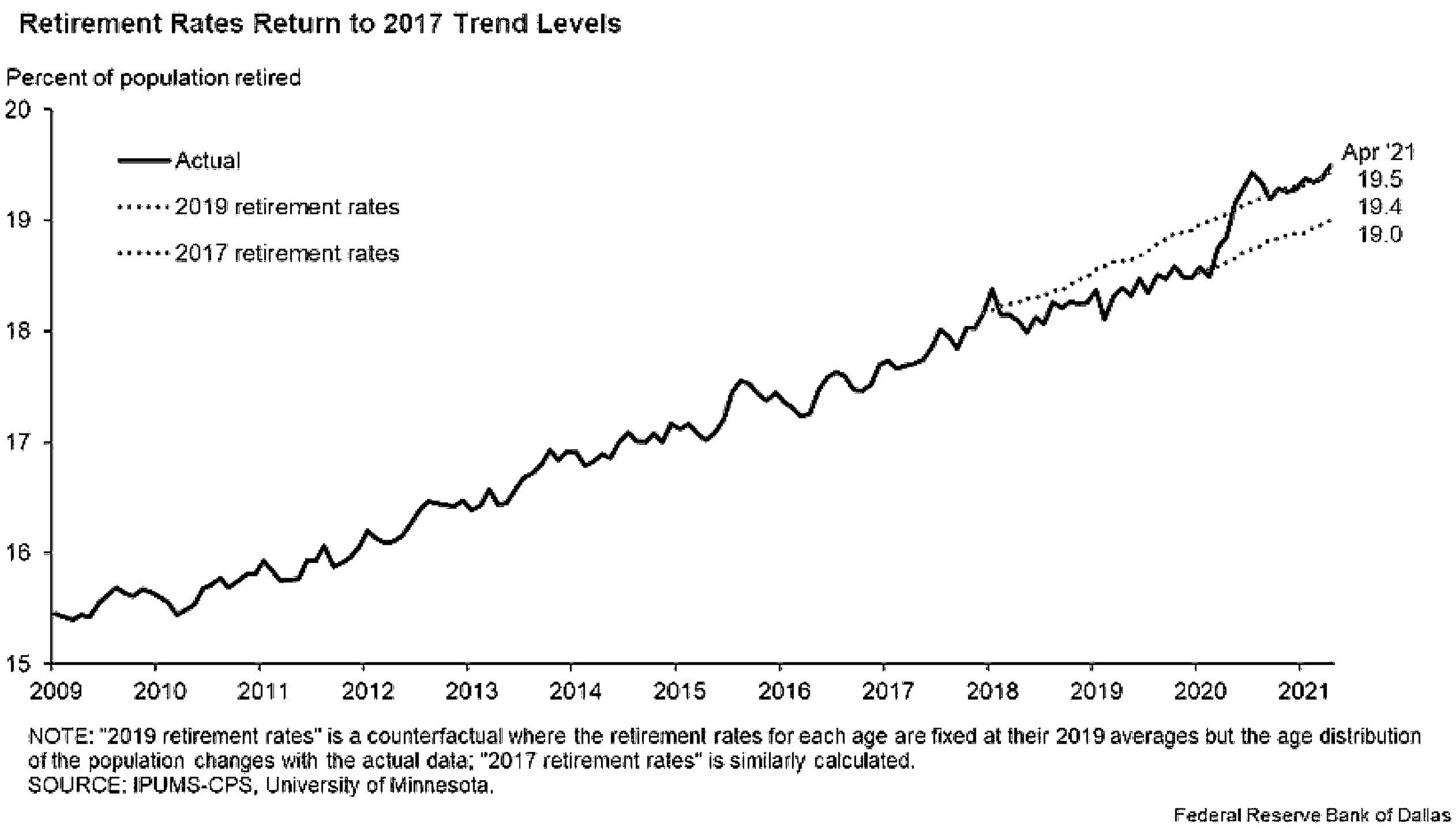
The idea of working from home is appealing to the low-wage service workers who often toil in hot kitchens or tidy up after messy hotel guests. Professional services and technology companies are using remote work as a recruiting tool, an attractive alternative to long commutes or living in cramped apartments in downtown business districts. Most of those positions, however, require a higher skill level than leisure sector work. Historical data show the quit rate tends to lead hourly wage growth by about two years. It should also be noted that the quit rate tends to ebb and flow with current labor market conditions and could easily slip if job opportunities become less abundant.



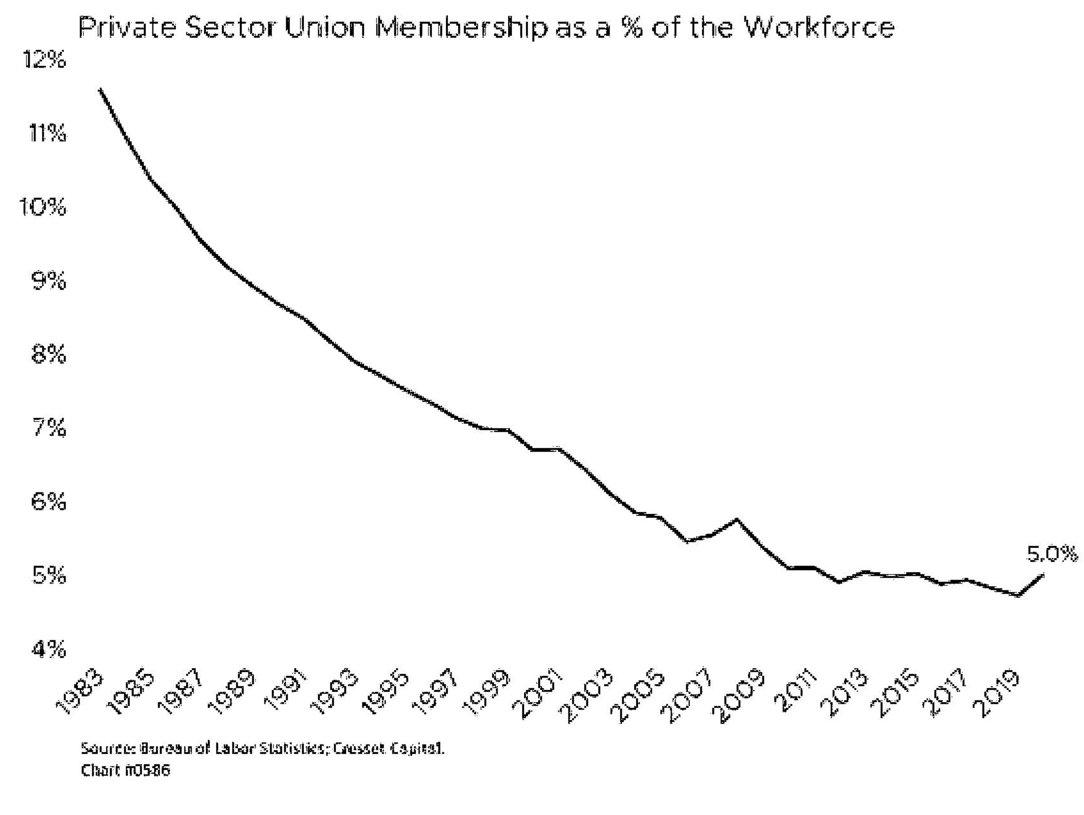
Another factor influencing the direction of the labor market is the availability of work. As of April there are more than 9 million job openings – an increase of more than 3 million open positions from pre-pandemic levels in February 2020 and the highest number of job openings on record. More than 1.5 million jobs are available in leisure, 1.4 million in education, 1.5 million in professional and business services and 1.6 million jobs in trade, transportation and utilities. Putting it another way, there is currently one job opening for every out-of-work American. Because of the skills mismatch between sidelined workers and current job requirements, the unemployment rate will never reach zero. Job openings are generally cyclical, suggesting a reversal will occur as business conditions slow. However, it should be noted that, with the exception of 2020, job openings have been consistently expanding for most of the last decade.



The pandemic pushed many older Americans into retirement, shrinking the availability of experienced labor. According to a recent study by the Dallas Fed, the hot labor market in 2018 and 2019 likely prompted many older workers to delay retirement, causing the share of the population in retirement to increase more slowly than the rate of aging would have implied. During 2020 and early 2021, the rate of retirement returned to its 2017 trend. Since the pandemic, about 2.6 million Americans reported that they retired, and an additional 1.3 million left the workforce to be caregivers (individuals whose primary activity is taking care of a household or family). It's unlikely to expect the those 2.6 million to return to the workforce. The share of the population in retirement increased from 18.5 per cent in February 2020 to 19.5 per cent in April 2021.



In our view, it's unlikely rising consumer prices will spill over into persistent wage acceleration. Several secular factors weigh in favor of a cyclical wage blip. First is the long-term trend downtrend in organized labor. Labor unions have historically boosted worker compensation, yet union membership, particularly in the private sector, has steadily declined. As of 2019, 5 per cent of the private sector workforce was unionized, down from nearly 12 per cent in 1983. Last year, Amazon fulfillment center workers in Bessemer, Alabama rejected unionization in favor of the company's pay and benefits package. Workers at large employers of unskilled labor, like McDonalds and Walmart, have also failed to organize. It's unlikely unskilled wages will morph into persistent inflation if union participation is declining. CPI escalator clauses built into union wage contracts were probably the key ingredient that transformed the one-time price shock of the 1973 OPEC oil embargo into persistent double-digit inflation that lasted through the end of that decade.

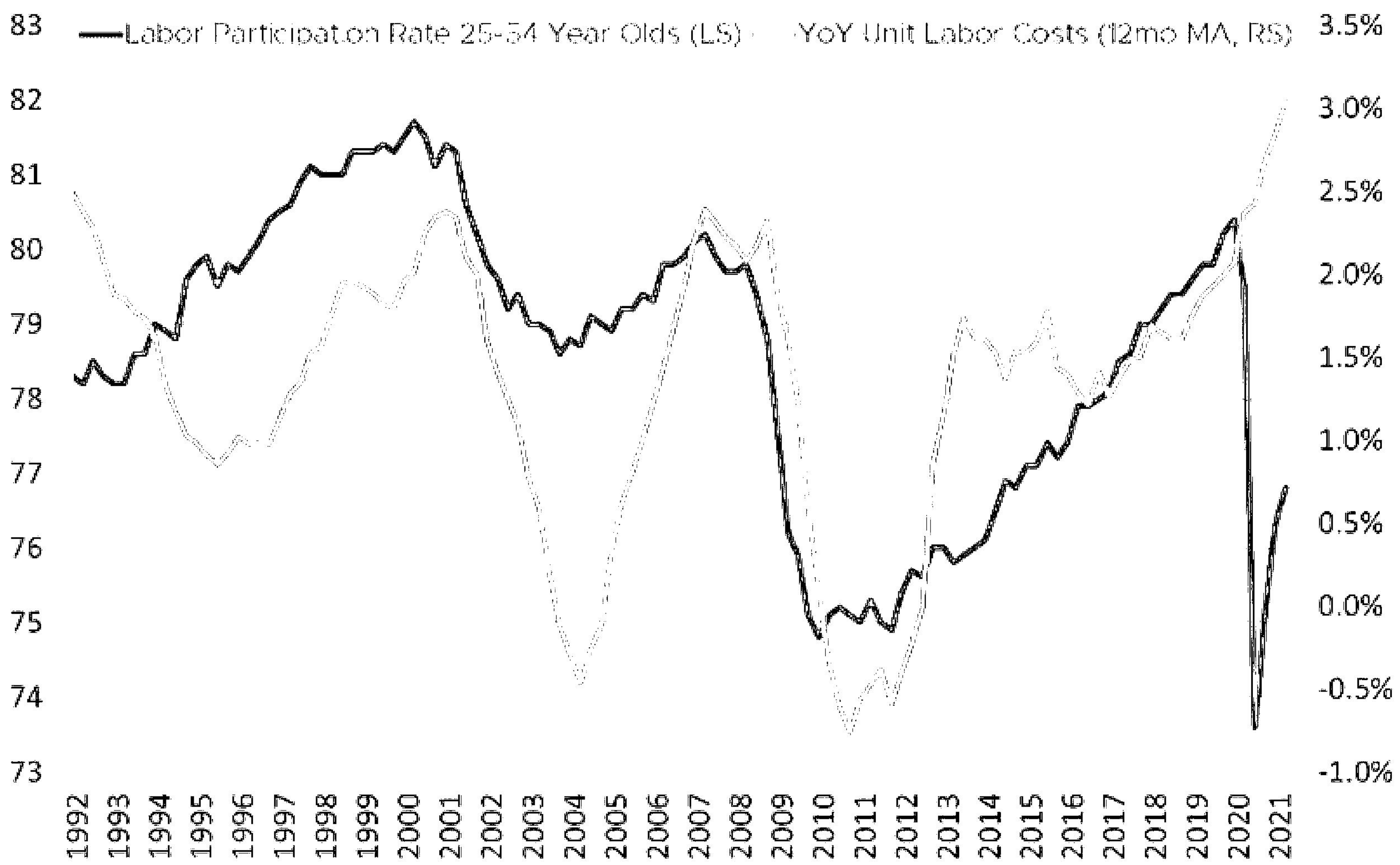


Second, the intransigence among sidelined workers is likely attributable to federal unemployment benefits that were granted as part of the January relief package. State unemployment benefits typically last up to 26 weeks, and amounts vary by state. The US average is $347/week, according to January data from the Labor Department. Under the Pandemic Unemployment Assistance Program, the federal data from the Labor Department. Under the Pandemic Unemployment Assistance Program, the federal government kicked in an additional $300/week to those who qualify. Unemployment benefits, which are

contributing to would-be workers staying home, are set to run out and will likely prod people back into the job market. Nearly 15 million workers claimed unemployment benefits in May, up from about 2 million before the pandemic, according to *The Wall Street Journal*. It's argued that the combination of state and federal unemployment benefits gives workers more than they would likely earn by having a job. Enhanced unemployment benefits are set to expire in September.

Third, productivity gains driven by investments in technology and innovation have tended to offset wage growth historically. Employers facing higher labor costs have made capital investments in cost-saving technology, like software and capital equipment, to raise output per employee. Thanks to technological advances, employers no longer rely on receptionists, stenographers or elevator operators. The Internet has supplanted middlemen, as many producers now sell directly to consumers. The unit labor cost metric is one way the Bureau of Labor Statistics gauges the impact of productivity on labor costs. Unit labor costs, which adjust wage gains by productivity, grew 3 per cent over the last 12 months as wage growth outpaced productivity gains. Unit labor costs tend to respond to labor market conditions, as costs rise in tandem with the labor participation rate. Tight labor markets tend to push labor costs higher, and vice versa. Nowadays, unit labor costs are rising even though labor participation has been slack. Under normal conditions that would be disturbing, but the sudden fall and rebound in labor activity could be distorting the numbers.

Labor Participation vs Unit Labor Costs



Source: Bureau of Labor Statistics; Cresset Capital.
Chart #1011

Wages are one of the three critical factors of goods and service production, along with commodity prices and capital. We have observed that these three factors tend to offset one another. Much like the air in a balloon, if one input (like interest rates) is pushed or held below fair value, the other two costs (like labor and commodities), have room to expand. We expect financing costs will rise as interest rates trend back to fair value – between 2.8-3.8 per cent – and will crowd out wage growth and commodity prices. However, as long as interest rates remain below fair value, wages have room to rise.

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2023 Edition

Long-Term Asset Class Assumptions

AIMCo

Table of Contents

Joint CIO and CFMO Message

Last year was challenging for global financial markets. The persistent rise in inflation caught many central banks off guard, forcing them to undertake the most aggressive interest rate hiking cycle seen in decades. This resulted in a significant repricing of risk, which saw bond yields rise and global equity values fall. The silver lining is that valuations across many public market asset classes have become more attractive.

Dr. Marlene Puffer
Chief Investment Officer

Amit Prakash
Chief Fiduciary Management Officer

These starting points are important drivers that feed into our forward-looking expectations for financial market performance and are reflected in our 2023 Long-Term Asset Class Assumptions, which we are pleased to present in this report. This year we have again included a forecasted range of outcomes to complement our baseline estimates. Considering a range of expectations is always a prudent practice and reflects the fact that the path ahead remains highly uncertain. We must consider the possibility that the world may be facing a regime shift away from what we've become accustomed to over the past ten years: lengthy economic cycles, minimal geopolitical risk and low inflation that contributed to an environment of easy monetary policy and muted volatility. For instance, an environment where both inflation and interest rates are somewhat higher versus history may also be contemplated. For each asset class, we have decomposed our return forecast into the building blocks that have proven to be reliable estimates of forward-looking risk and return.

In 2022, higher discount rates and greater economic uncertainty caused global equity prices to experience their largest annual decline since the Global Financial Crisis (GFC). As the threat of stagflation and recession continues to cloud global growth outlooks in the short term, the compression in valuation multiples makes for a more favourable outlook over the longer term. Yet there is still some disparity across markets. Valuations in the United States for example, continue to be priced at a premium relative to Europe, Canada, and Emerging Markets. Emerging markets are also expected to benefit from higher nominal growth relative to developed markets over the next decade.

Interest rates across most of the world are significantly higher than they were at the end of 2021, which has improved future return expectations for fixed income investments. As we enter 2023, nominal rates may be near a cyclical peak as most major central banks have signalled that they are nearing a pause in their rate hiking cycle, but interest rates and inflation may remain higher than we have been accustomed to over the past decade, at least in the medium term. Against this backdrop, the expected income that can be generated throughout the life of a bond has increased, which now accounts for the bulk of fixed income returns over the forecast horizon.

We're pleased to present a more in-depth examination of the outlook for short-term interest rates, which is our spotlight topic this year. As always, we look forward to continuing the dialogue around this forecast and exploring how we can further enhance our clients' portfolios as a result.

AIMCo 2023 Long-Term Assumptions

The long-term capital markets assumptions have become a staple deliverable for our clients at the beginning of each calendar year. At its heart, it provides long-term return and risk forecasts for 17 asset classes, broadly split into fixed income, public equities and illiquid assets. By forecasting various macroeconomic variables and economic scenarios along with modelling different components or building blocks, we arrive at reasonable risk and return expectations over a 10-year time horizon.

While we broadly follow the same proven process every year, we are constantly evolving our processes and methodologies to improve our forecasting ability. Introduced in last year's forecast, and presented again this year, we have included a range of forecasted returns for each asset class to supply our readers with a more robust picture of return expectations. These values can be found in the Long-Term Forecasted Return Ranges.

Return Assumptions

Our 2023 forecast is based, similar to last year, on expectations of a challenging global macroeconomic backdrop as we enter a period of materially slower growth in the near term. In 2023, we expect many countries to experience above-target inflation and tight monetary policy from central banks. We anticipate inflation decline this year, albeit potentially remaining higher than levels witnessed prior to the pandemic. Government bond yields, over the forecast horizon, are expected to remain relatively high, at levels closer to the ones seen before the Global Financial Crisis.

Given market developments over 2022, the outlook is more attractive for public equities over the next decade in comparison to last year's forecast. On the valuation side, price-to-earnings (P/E) ratios have improved from last year, more in line with their long-term average, if not lower in some cases (i.e. there is room for multiple expansion). Earnings growth is expected to remain healthy due to a combination of increased support from the inflation pass-through and robust demand globally.

Illiquid assets continue to exhibit high return-to-risk ratios. Overall, current private asset valuations are less expensive in comparison to last year, resulting in a reduced drag on returns. Illiquid real assets can be significantly impacted by macroeconomic factors such as inflation and real yields. However, they can also provide a degree of protection against inflation as these investments can pass through a portion of inflation to end-users.

Risk Assumptions

The risk forecast is based on the same methodology (VAR-GARCH-DCC[1] statistical model) we have utilized in previous years. The model allows us to forecast asset class volatilities and correlations simultaneously to provide consistent, forward-looking estimates. This forecast incorporates a more accurate market return distribution to capture the so-called "fat tails". Additional details can be found in the Risk Assumptions Methodology section.

[1] Vector Autoregressive - Generalized Autoregressive Conditional Heteroscedasticity - Dynamic Conditional Correlation.

Overview

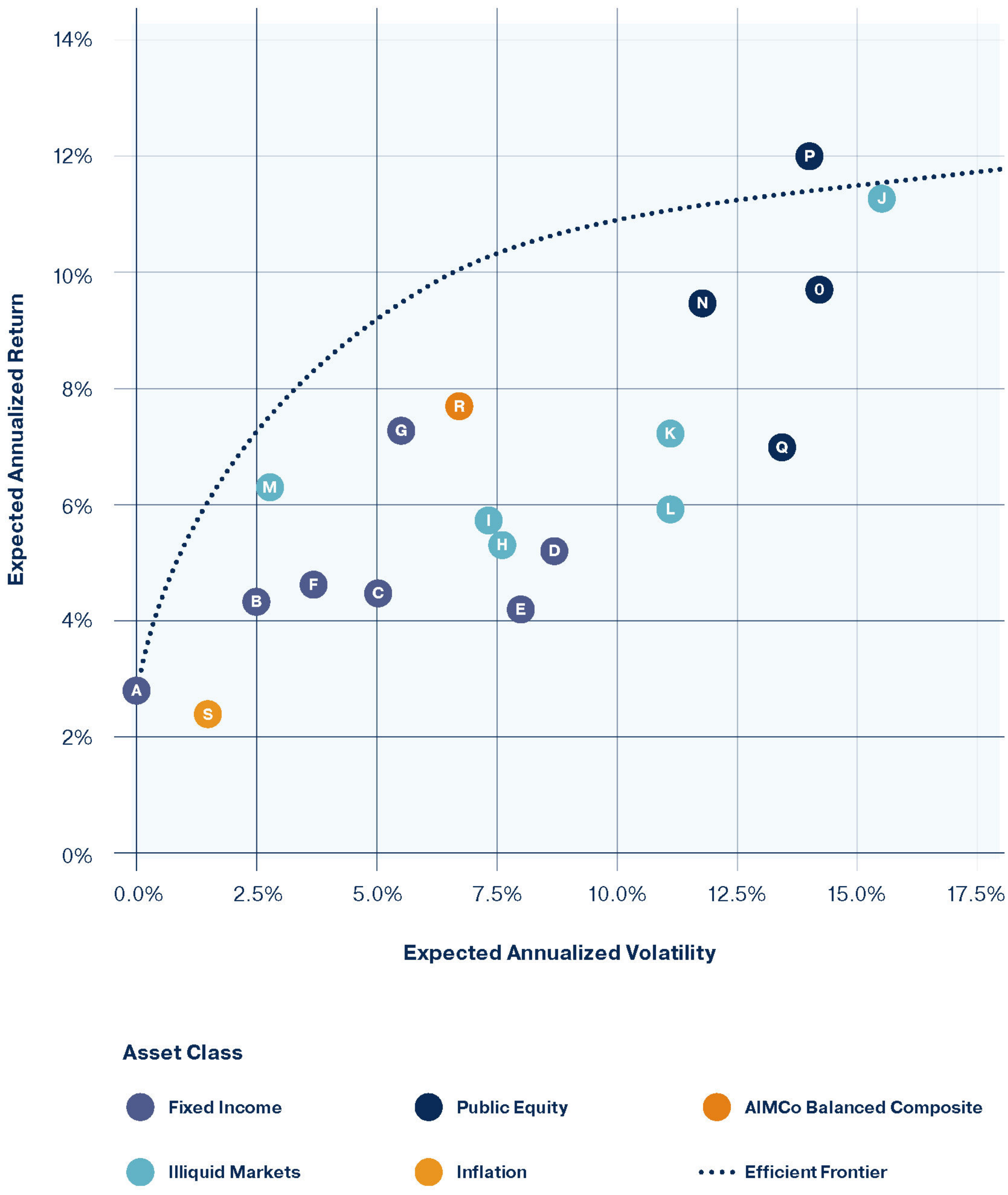
By taking into account both long-term return and risk expectations, we can envision an efficient frontier that incorporates all our forecasts. Putting it all together, a balanced portfolio (represented by AIMCo's aggregate balanced fund), is expected to achieve a 7.7%[2] annualized return over the next decade.

Table 1: Forecasted Return and Risk 2023-2032

Asset Class	Benchmark	Expected Return	Expected Volatility
Fixed Income			
A Money Markets	FTSE Canada 30-Day T-Bill Index	2.8%	0.1%
B Short-Term Bonds	FTSE Canada Short-Term All Government Bond Total Return Index	4.3%	2.5%
C Universe Bonds	FTSE Canada Universe Bond Total Return Index	4.5%	5.0%
D Long Bonds	FTSE Canada Long-Term All Government Bond Total Return Index	5.2%	8.7%
E Real Return Bonds	FTSE Canada Real Return Bond Total Return Index	4.2%	8.0%
F Private Mortgages	60% FTSE Short-Term Overall Bond Index and 40% FTSE Canada Mid-Term Overall Bond Index + 50 bps	4.6%	3.7%
G Private Debt & Loan	40% S&P/LSTA Leveraged Loan Index + 40% S&P European Leveraged Loan Index + 90bps	7.3%	5.5%
Illiquid Markets			
H Canadian Real Estate	REALpac/IPD Canadian All Property Index – Large Institutional Subset	5.3%	7.6%
I Foreign Real Estate	MSCI Global Region Property Index	5.7%	7.4%
J Private Equity	Total CPI 1 Month Lagged + 650 bps (5-year rolling average)	11.3%	15.5%
K Infrastructure	Total CPI 1 Month Lagged + 450 bps (5-year rolling average)	7.2%	11.1%
L Renewable Resources	Total CPI 1 Month Lagged + 450 bps (5-year rolling average)	5.9%	11.1%
M Absolute Return	Money Markets + 350 bps	6.3%	2.8%
Public Equity			
N Global Equities	MSCI World Net Total Return Index	9.5%	11.8%
O Canadian Equities	S&P/TSX Composite Total Return Index	9.7%	14.2%
P Emerging Markets Equities	MSCI Emerging Markets Net Total Return Index	12.0%	14.0%
Q Global Small Cap Equities	MSCI World Small Cap Total Return Index	7.0%	13.4%
R AIMCo Balanced Composite	AIMCo Composite: 38% Equity, 26% Fixed Income, 36% Illiquid Assets	7.7%	6.7%
S Inflation	Canadian Consumer Price Index	2.4%	1.5%

[2] This AIMCo balanced composite representation aggregates the asset class weights of our balanced fund clients. As such, it constitutes an illustrative return as it encompasses clients who may have very different portfolio objectives. This explains why such a weighted average of differing portfolios does not sit on the efficient frontier found in Chart 1.

Chart 1: Efficient Frontier and Asset Class Forecast[2]



[2] This AIMCo balanced composite representation aggregates the asset class weights of our balanced fund clients. As such, it constitutes an illustrative return as it encompasses clients who may have very different portfolio objectives. This explains why such a weighted average of differing portfolios does not sit on the efficient frontier found in Chart 1.

Long-Term Forecasted Return Ranges

Range Forecast

In last year's edition, we introduced forecast ranges to provide readers with a better understanding of the embedded variation within our central forecasts. We are pleased to present the analysis again this year. We have simulated scenarios around the major economic inputs to the various asset classes' building blocks to derive ranges for the expected returns.

Chart 2: Forecasted Return Ranges

Global Economic Scenario

U.S.

The U.S. enters 2023 with, generally, positive albeit waning cyclical momentum as outlook surveys such as PMIs and ISM point to declining economic activity. The ratio of job openings to unemployed people, or labour market tightness, stands high in historical terms but appears to be plateauing. This suggests that unemployment could rise this year alongside wage growth stalling. Private spending will continue to be boosted by a mix of excess savings and falling unemployment. Inflation is expected to continue moderating as base effects and lower housing inflation start having more significant effects. However, inflation is likely to remain uncomfortably above the Federal Reserve's 2% target. Overall, we project 2023 U.S. economic growth between 0% and 1% but downside risks to that scenario exist.

In the long term, as in past years, themes contributing to lower U.S. potential growth remain such as weaker demographics and Baby Boomers retiring leading to a shrinking working-age population, deglobalization, increased regulation and the plight of elevated public debt. However, more immigration-friendly policies introduced by the current U.S. administration may cause a rise in labour force growth and productivity. Monetary policy is expected to be kept less accommodative than before the pandemic, with benchmark policy interest rates remaining around current levels over time. Overall, we forecast the U.S. economy to grow around the 2% level on an annual basis for most of the next decade.

Canada

Canada's economy is expected to stall in 2023, as higher interest rates and a global slowdown flow through domestically. Consumer spending may ebb downward as excess accumulated savings during the pandemic wear off and the housing market decelerates further. This could lead to growth around 0% in 2023. Canadian inflation for 2023 will average, between 4% and 4.5%, with inflation from the services sector driving much of the elevated price pressures. This set of assumptions is also subject to downside risk.

Canadian structural potential growth could increase in the long term. It remains to be seen whether prospects for strong immigration gains could offset the aging population impact of reducing labour force growth. However, global opportunities related to the energy transition theme and reshoring may lead to a demand increase for Canadian goods and services over the long haul. In summary, Canada's growth is forecasted to average slightly above 2%, annually, in the next 10 years. In addition, we believe that various factors could lead to inflation sustainably above the 2% Bank of Canada's target (see "Topic of the Year" section for more) and, as such, interest rates could remain close to current levels over the forecast horizon.

China

We forecast China to grow at a faster pace in 2023 due to the reopening of its domestic economy. Growth could be in the range of 4% to 5% this year. China's continued property sector woes constitute a risk to the outlook.

In the long term, China's adoption of the "common prosperity" policy to reach a high-income status nation by 2025 and targeted industries for investment should support potential growth. Economic growth is expected to hover between 3.5% and 4.5% per year in the next decade as an ageing population crimps labour force growth. Monetary policy should continue to target measures to contain economy-wide imbalances.

Eurozone

The euro area is facing challenging conditions in 2023. Relatively high energy costs should continue to impact businesses and consumers, albeit not as dramatically forecasted in the fall of 2022. But a global economic slowdown may cause a more protracted decline in economic activity given the region's sensitivity to global trade dynamics. Lastly, with higher wage agreements taking hold by the second half of this year, inflation could be persistently above the 2% target of the European Central Bank.

Since the eurozone is the world's most trade-oriented economic block, it stands to be the most affected by the process of reshoring the global manufacturing capacity. Labour costs are elevated in that region. Furthermore, the Eurozone ranks second only to Japan in terms of risk of lower trend growth due to its shrinking working-age population. On the positive side, the region's focus on green economic policies and building a resilient energy infrastructure would involve material investments in the related sectors, in support of growth.

Japan

Japan recently experienced green shoots domestically as the country reopened in earnest post-pandemic. In 2023, domestic demand is expected to drive growth compared to external trade. Inflation is anticipated to continue being in line with the Bank of Japan's medium-term targets thanks to higher wage agreements.

In the long term, however, its potential growth remains hampered by an aging population. Conversely, Japan's industries could benefit from secular trends such as an increase in local manufacturing capacity, a rise in its share of the global electric vehicle market and industry automation product needs. Overall, growth is forecasted to be relatively sluggish and inflation to endure at the lower end of the central bank's expectations.

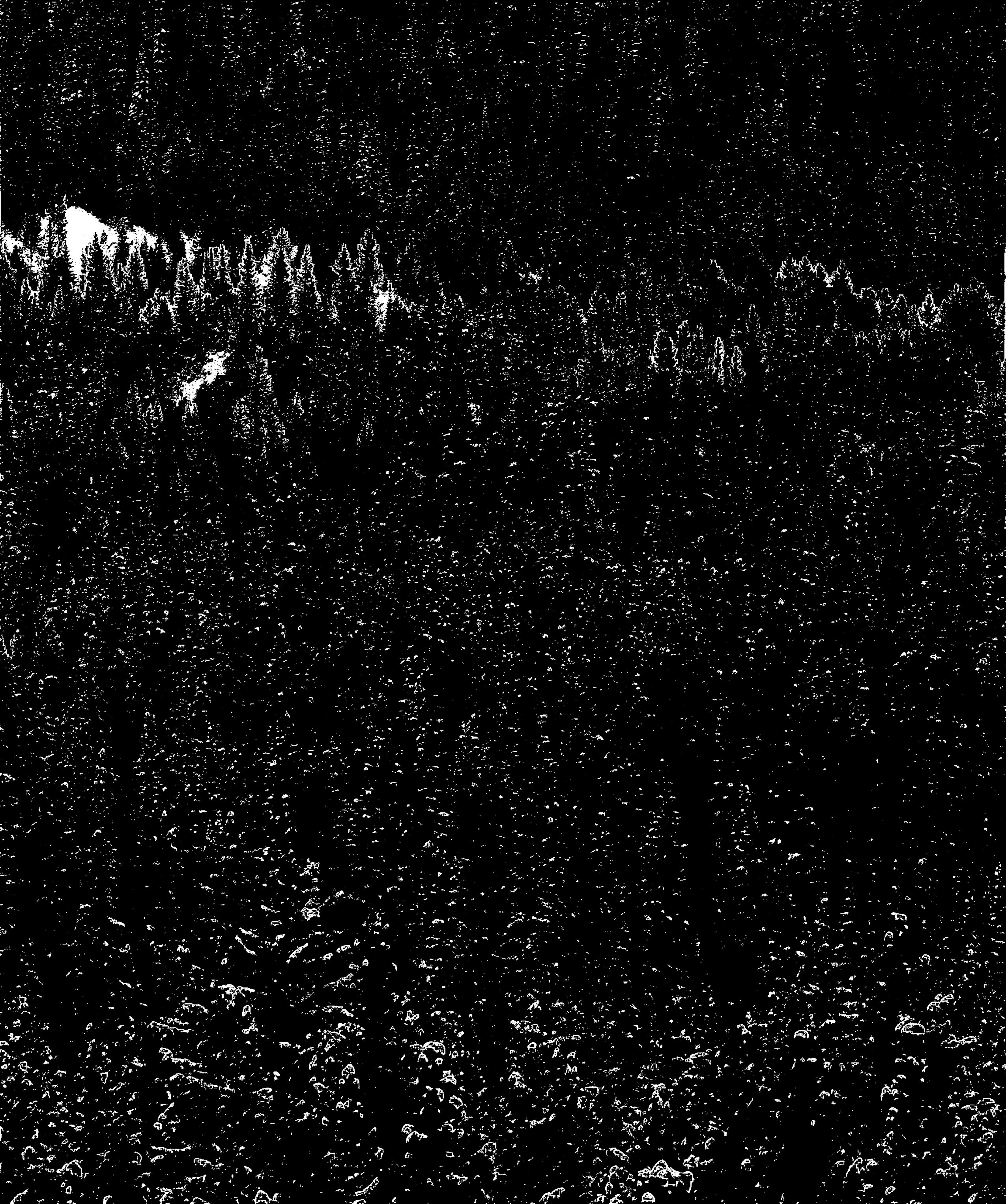
Emerging Markets

Emerging market (EM) countries, excluding China, are slated to enjoy a moderate return to a positive economic growth differential in comparison to developed countries in 2023. Leading economic indicators point to relative outperformance of their manufacturing sector despite the projected global slowdown as China's reopening has positive implications for the EM world. Consumer inflation has likely peaked across emerging markets and most central banks have reached terminal policy rates.

From a structural perspective, emerging markets continue to be less reliant on capital inflows and they continue to benefit from positive demographics and urbanization tailwinds which are supportive long term.

Climate Change

Climate change remains an important subject for AIMCo and our clients. We strive to identify and integrate environmental drivers into our long-term forecasting. Climate change was first introduced in our 2021 long-term asset class forecast and the baseline economic scenario in our 2023 forecast continues to incorporate an assumption of one additional degree of warming to 2050, globally, from levels observed at year-end 2019, in line with a current assessment of credible stated policies, policies susceptible to be implemented over this long horizon.



Topic of the Year

The “Natural” Short-Term Interest Rate

The price of money is one of the primary drivers of our capitalist societies. Interest rates represent the cost of money or the future cost of capital. As long-term investors, calibrating our strategies concerning this hurdle rate and its various impacts is of critical importance. For instance, should interest rates structurally shift higher versus recent history, certain fixed income assets would then become more attractive from a yield-generating standpoint whilst a potentially higher discount rate for cashflows may negatively impact the valuation of riskier assets. Furthermore, a higher cost of capital for most companies would change how they access capital, select capital expenditures and the nature of their cashflow streams. Finally, such an outcome may modify the level of attractiveness for certain investment strategies which, to some degree, use leverage to generate returns in the long term.

Central banks are deemed to lead the way in establishing the basic cost of capital our modern economies are facing via the conduct of monetary policy. They represent the central channel for the price of money. For monetary policy to be successful, central banks need to balance their various mandates which typically encompass aiming for stable prices and maximum employment. The key tool at their disposal in that pursuit is calibrating the interest rate they influence to balance inflation and jobs. Too low interest rates in the economy can discourage savings. Low rates may also lead to wealth inequality as only wealthier households may be able to save for retirement. Lower interest rates can keep unprofitable companies afloat, resulting in unproductive growth and can foster speculation and financial market bubbles. Conversely, too high interest rates can lead to balance sheet stress both for indebted households and corporations, restrictive credit growth, lower trend economic growth and potential market freezes.

The pricing of nominal interest rates has been trending downward since the Global Financial Crisis — hitting not only the zero lower bound but even threading into negative territory for certain jurisdictions up until the COVID crisis. Before the pandemic, extremely low or even negative nominal interest rates were deemed to be in line with the subdued global growth environment we have been experiencing since the early 2010s. The concept to explain such an occurrence is called secular stagnation. It posited that weak productivity and demographic growth would lead countries to be mired in a deflationary environment for an extended period.

The experience of the recent years suggests that we may have begun evading deflation fears for some time. Indeed, in 2022, the combination of pandemic-era spending and lingering supply chain challenges resulting from the coronavirus pandemic drove consumer prices to 40-year highs. In addition, the economic impact of the Russian invasion of Ukraine also led to significant price pressures. Global central banks responded by using their toolset to reduce aggregate demand and inflation via significantly increasing policy interest rates. However, inflation is still well above target at the time of writing.

Whether an economy would remain in a state of secular stagnation or able to reach the escape velocity status in the medium- to long-term could entail a significant impact on the final station in a central bank's interest rate normalization journey. That final monetary policy station is known as the neutral or natural short-term interest rate (R*). In essence, R* represents the "neutral" interest rate that is consistent with stable, non-inflationary growth in the longer term[3]. In other words, it is the interest rate that balances supply and demand when inflation is equal to expected inflation. A recent Bank of England report[4] confirmed that since the 1950s globally, slowing productivity growth and increasing longevity had been the two major factors driving trends in the real neutral rate across the 31 countries studied. Global inflation-adjusted R* rose from the mid-1950s (from 1.25%) to the mid-1970s (to 2.75%), declining since then (reaching -0.25% by 2015).

Are we then heading towards a renewed bout of secular stagnation, reverting to much lower interest rates in the long term once inflation declines in the next year, or is it the opposite situation and the hurdle rate for any financial investment has inched up structurally?

First, other than factors that structurally influence the supply of goods and services such as productivity growth and demographics (the latter for the supply of labour), various studies have highlighted other types of factors explaining the fall of R* through time. They can further be categorized under factors that influence the demand for goods and services (e.g. fiscal policy, too high savings rates resulting in low levels of investment, etc.) and the sensitivity of demand for goods and services to the real interest rate.

Secondly, based on the above findings, there are various arguments in favour of believing that the decline in R* will be persistent versus arguments favouring the view that headwinds are slated to fade. We have summarized such arguments in the table below (in no particular order):

Exhibit A

R* Decline To Remain Persistent	R* To Rise
Population aging leads to a smaller workforce and lower consumption and output growth.	A fall in the global household savings rate as the middle class ranks swell in emerging countries triggering a consumption and output boom.
A slowdown in productivity and output growth is caused by a decline in the pace of technological progress, chronic underinvestment, etc.	Productivity growth could increase over the next decade, perhaps by an incremental 0.5 to 1% versus current trends (1.1% y/y in the 10 years before the pandemic). Robots and investments in new technologies such as artificial intelligence could boost productivity gains.
The risk of too-aggressive tightening, much lower credit growth and expensive government debt forces central banks to remain accommodative at the risk of “zombification” of the economy and lower potential growth.	Inflation expectations could rise boosted by structural public deficits due to income redistribution policies by governments, higher prices for the commodities required for the energy transition or deglobalization.

Source: “The U.S. Economic Outlook and the Implications for Monetary Policy”, Bill Dudley, New York Fed (July 31, 2016), “Three Remarks on the US Treasury Yield Curve”, Olivier Blanchard, Peterson Institute for International Economics (June 22, 2016), M. Saunders, “Why neutral rates have risen and why it matters“, Oxford Economics (January 2023), AIMCo Investment Strategy Research.

[3] According to the Swedish economist Knut Wicksell, the natural rate is the real short-term interest rate consistent with economic output equaling its natural rate (potential) and constant inflation to which it will converge over time. It can help to measure the stance of monetary policy, with policy expansionary (contractionary) if the short-term real interest rate lies below (above) the natural rate. In other words, close to the natural rate of unemployment while inflation is at the central bank target.

[4] Decomposing the drivers of Global R*, BoE (Cesa-Bianchi, Harrison and Sajedi), July 2022.

Overall, we acknowledge the population aging trend as a significant secular force potentially driving a low R* in the long term. That being said, we believe that on balance, the inflation expectations channel might overwhelm the impact of weak demographics as the world becomes more driven by inflationary trends such as policies centred on income redistribution, and deglobalization (or reshoring) alongside the cost of transitioning to a lower carbon economy.

While R* is unobservable, various academic models attempt to measure R* in the U.S., in particular[5]. Such forecasts embed a material degree of uncertainty; however, the Lubik-Matthes model estimated the median U.S. R* at 1.3% as of the end of the third quarter of 2022, for example. To conclude on the prospective range of outcomes for both Canadian and U.S. terminal short-term nominal rates (in nominal not real terms), we compare central bank research forecasts of R* versus estimates based on AIMCo's long-term baseline scenario in the table below:

Exhibit B

	Canada	U.S.
Range of terminal nominal short-rate based on Bank of Canada and Lubik-Matthes for the U.S.	2.0% to 3.0%	1.5% to 5.3%
AIMCo Baseline View vs Central Bank Estimates (as of January 2023)		
AIMCo range of terminal nominal short-rate forecast (2032)	Between 2.0% and 2.5%	Between 2.0% and 2.5%

AIMCo views are similar for both Canada and the U.S. and stand above the zero-lower bound for nominal short-term rates observed in the past, a sign that we are optimistic in averting a return to a so-called secular stagnation era. Moreover, our nominal R* range estimation is in line with the Bank of Canada's equivalent forecast. We note that our upper bound could move higher should Canada's immigration policy target become a reality alongside appropriate measures to provide more housing supply, which could further boost potential growth domestically. In summary, we anticipate Canada's terminal station for short-term rates to sit somewhat higher than what we have grown accustomed to in the last decade. Whether the Bank of Canada sets its base policy rate above or below the nominal R* will dictate if monetary policy is accommodative or restrictive. More importantly, our view implies that the cost of capital for our domestic economy may potentially have increased structurally from the zero-lower bound alongside the real long-term discount rate in risky markets and the hurdle rate for various client investment strategies. A new world?

[5] See Laubach, Thomas and John C Williams (2016): "Measuring the natural rate of interest redux," Business Economics, 51(2), 57-67 for a seminal version.
[6] Faucher et. al., "Potential output and the neutral rate in Canada: 2022 reassessment", Bank of Canada Staff Analytical Note 2022-3.
[7] Thomas A. Lubik and Christian Matthes, "Calculating the Natural Rate of Interest: A Comparison of Two Alternative Approaches," Federal Reserve Bank of Richmond Economic Brief 15-10, October 2015. Assumes the Federal Reserve is successful to bring core PCE inflation to the 2% target. Extracted on January 31, 2023 at www.richmondfed.org/research/national_economy/natural_rate_interest

Forecast by Asset Class

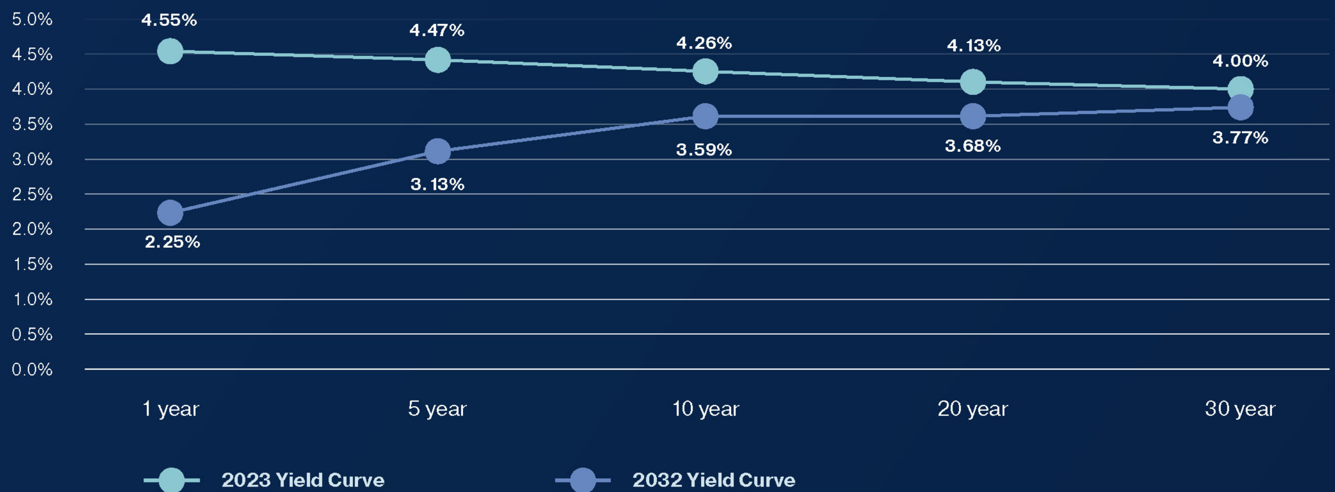
Fixed Income

Overview

Globally, central banks spent the past year aggressively raising rates to fight multi-decade inflation highs. The Bank of Canada (BoC) raised its overnight lending rate by 400 basis points in 2022, the largest and fastest rate hike cycle on record. As rate increases gathered momentum during the year, fixed-income markets grew weary of economic growth prospects leading to inverted yield curves in major economies. While the BoC's policy intends to slow growth, expected fixed income returns will benefit from higher starting yields. As such, our forecast this year highlights materially higher fixed income returns. Over the forecast horizon, we expect Canada's yield curve to moderate and steepen to levels that reflect a more normalized growth and inflation environment. In the short term, while the economic effects of higher rates are only beginning to transmit through the economy, we believe given the BoC's recent rate hike in January 2023, the central bank will now take a wait-and-see approach for most of this year.

Despite increasing chatter of a potential recession in 2023, credit spreads have remained resilient. Our economic growth forecast implies credit spreads will be supported and trade within the lower end of their historical range over the next 10 years. Alternative credit asset classes, namely Private Debt and Loan and Private Mortgages provide opportunities for AIMCo to selectively underwrite unique credit opportunities and are expected to generate a premium over their public market comparables.

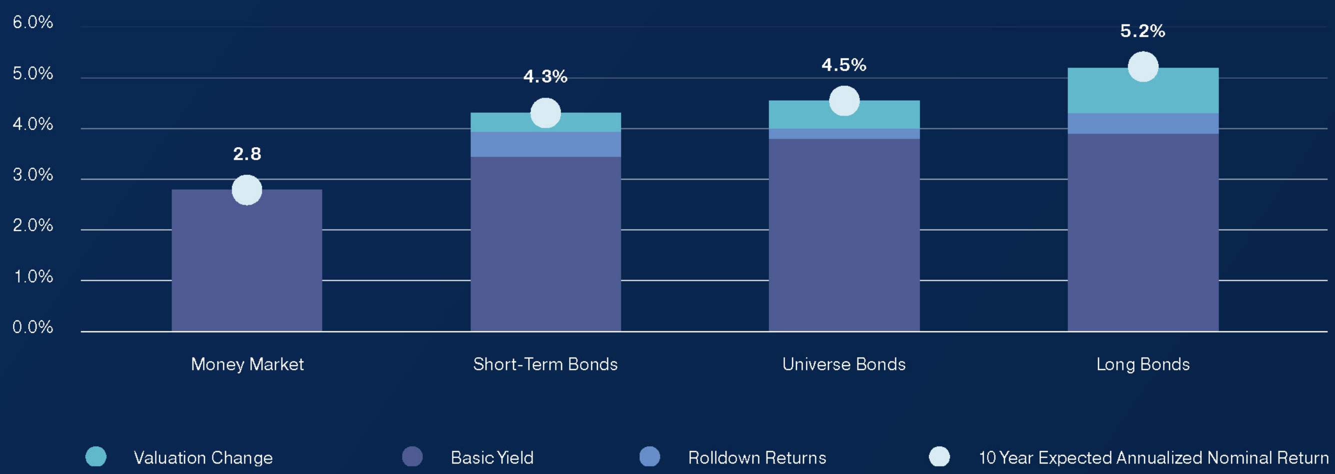
Chart 3: Canadian Sovereign Yield Curves 2023 Economic View



Building Blocks

AIMCo's fixed income capital market assumptions are based on interest rate forecasts, term premiums, roll returns and credit spreads/expected default loss. These underlying components are consistent with our global, long term economic scenario.

Chart 4: Building Blocks for Fixed Income Assets



Sub Asset Classes

Money Market

Fighting inflation has been the focal point of central banks this past year and that will continue in the near term. Inflation-focused rate hikes by the Bank of Canada have created more uncertainty in the magnitude and length of the rate hiking cycle which will continue throughout 2023.

Over the short term, there is increased concern about a deeper slowdown in the general economy as opposed to a mild downturn. That, in combination with the supply-side adjustments, geopolitical concerns and uncertainty over inflation volatility, may create higher uncertainty of rate adjustments. In the mid-to-longer term, the expectation is that inflation will subside and with the combination of mentioned economic pressure, short term rates should moderate from their near-term highs.

Short Term Bonds

Returns for this sector can be expected to be strongly correlated to movements in the overnight rate of the Bank of Canada. After a period of aggressive rate hikes, short-term bonds are coming off a year of negative performance. With the overnight rate ending last year in restrictive territory, the likelihood of short bonds prospectively having a negative year of performance is low.

Our forecast suggests that the returns for short bonds going forward should track the more recent and higher level of central bank rates. Accordingly, the current level of yields in the short-maturity sector is a reasonable expectation of future returns.

Universe Bonds

Following a significant move higher in yields during 2022, the long-term return profile of Universe Bonds is expected to be more stable. At the start of this year, index yields are two times higher than at the same point a year ago. That means the positive asymmetry of bond portfolio returns is a much more tangible benefit going forward. Universe bond yields are expected to fluctuate in a range around current levels as we navigate through the business cycle during the forecast horizon. A return to yield levels prevailing during periods of ultra-accommodative monetary policy is not expected. With a significantly inverted yield curve, there is the possibility that inflation does not return to target in the time frame anticipated by the market in the short term. If elevated inflation becomes persistent, yields at the long end of the bond market are likely to increase from current levels therefore negatively impacting returns.

Long Bonds

After a poor year in 2022, the prospective return for long bonds is more favourable in the years ahead. Long-term Canadian bond yields now reflect a level consistent with a moderate term premium and the implied long-term neutral policy rate of the Bank of Canada. Going forward the return profile for long bonds should be more correlated to the market's assessment of long-term growth and inflation rather than being tethered to moves in the overnight rate of the central bank, which was prevalent for a large part of last year.

Our return forecast for long bonds reflects an extended period of fluctuation around current yields which would be a new era that does not anticipate another drop to pandemic-era levels. The current level of yields depicts a stronger income narrative from a returns perspective while also putting the sector in a better place to act as a total portfolio diversifier going forward when compared to levels from a year ago. A major risk to our forecast is the possibility that economies do not realize the moderation of inflation currently priced by financial markets resulting in longer-dated yields reflecting a less benign long-term inflationary environment.

Real Return Bonds

Despite the emergence of unexpected inflation throughout 2022, the significant rise in real yields resulted in a year of poor absolute returns. As is the case with nominal bonds, the starting yield levels for real return bonds for long-term prospective returns are more favourable compared to last year. Real yields have returned to positive levels, and break-even yields reflect a return to central bank inflation targets from current elevated levels. Accordingly, forecasted returns project a regime consistent with current market conditions and not returning to an ultra-accommodative monetary policy environment. Relative to nominal bonds, real return bonds hold the upside return potential of actual inflation realizing higher than expected inflation. As with nominal bonds, another upward shock in real yields would be a significant headwind to long-term returns.

Private Debt & Loan (PDL)

The shift in monetary policy in 2022 that led central banks to increase interest rates was positive for the long and short-term returns of Private Debt assets. Last year, the positive dynamic of higher rates, given the portfolio's focus on floating rates, was offset by the widening of spreads in response to heightened recessionary fears and decreased market liquidity. Heading into 2023 and beyond, both factors higher rates and wider spreads, will benefit the asset class. While the near-term economic outlook is challenging is challenging, potentially leading to increased defaults. The asset class has historically experienced high recovery rates due to PDL's focus on senior, secured loans. Thus, the high interest rates earned on these assets are expected to absorb the impact of increased defaults in a highly diversified portfolio.

Over the longer term, an environment of higher interest rates and tighter monetary policy will benefit the returns of the asset class, enabling lenders to charge higher rates for borrowing. As borrowing costs have increased, so has the quality of companies looking to secure financing. Thus, investors are benefiting from both higher returns and higher credit quality. The lower-quality borrowers will struggle to receive financing and will have to turn to higher cost means of financing such as equity.

Overall, we expect private debt & loan to show resilience and generate contractual income with stable returns over the long term.

Mortgages

Following years of historically low borrowing costs, commercial mortgage rates more than doubled last year as a result of rising government bond yields and commercial mortgage spreads. The rapid rise in yields during 2022 put downward pressure on mortgage returns in the existing portfolio.

Many Canadian lenders continue to have an appetite for high-quality industrial, logistics and multifamily properties, although, liquidity in the commercial lending market has decreased compared to previous years. Looking ahead, prospective all-in rates for new commercial mortgages are attractive compared to recent years. That's due to higher government bond yields and elevated commercial mortgage spreads, with the latter currently sitting above the long-term average. Although the pace of interest rate hikes is expected to slow, persistently high inflation could lead to further rate increases that are above market expectations. This could negatively impact short-term returns, and at the same time increase long term return expectations.

Forecast by Asset Class

Illiquid Assets

Overview

Illiquid assets are crucial in constructing a well-diversified portfolio, and 2022 was a prime example of that. These asset classes exhibit low correlations to traditional fixed income and public equity markets. Investment opportunities in private asset markets are unique, even within the same asset class category, which helps diversify the risk across the portfolio. One commonality between these asset classes is the longer investment lifespan, which is suitable for long term investors. Furthermore, the illiquid nature of these investments generally means a higher premium and therefore a higher expected return. Over time, investors' appetites for illiquid assets have increased, and our clients continue to allocate more to illiquid asset classes to take advantage of the attractive return-risk characteristics.

Our 2023 forecast paints a favourable picture for all private assets with strong cashflow growth potential. Currently, on behalf of our clients, AIMCo invests in private Canadian real estate domestically and foreign real estate, private equity, infrastructure and renewable resources globally. The building blocks, explaining the sources of return for each private asset class are discussed in their respective sections.

Real Estate

Building Blocks

For real estate, we use forecasts from the Oxford Economics model for Canadian and foreign Real Estate. In this model, income yield, rental income growth and valuation change are the main building blocks. These building blocks are forecasted using the path of macroeconomic variables such as inflation, bond yields along with inputs on market rents, construction activity and relevant demand drivers.

Values for Canadian and Foreign real estate building blocks are shown in the following exhibits. We continue to expect foreign real estate to provide a higher return environment for investments compared to the Canadian market. In both cases, the expected income drives the respective assumptions.

Chart 5: Canadian Real Estate

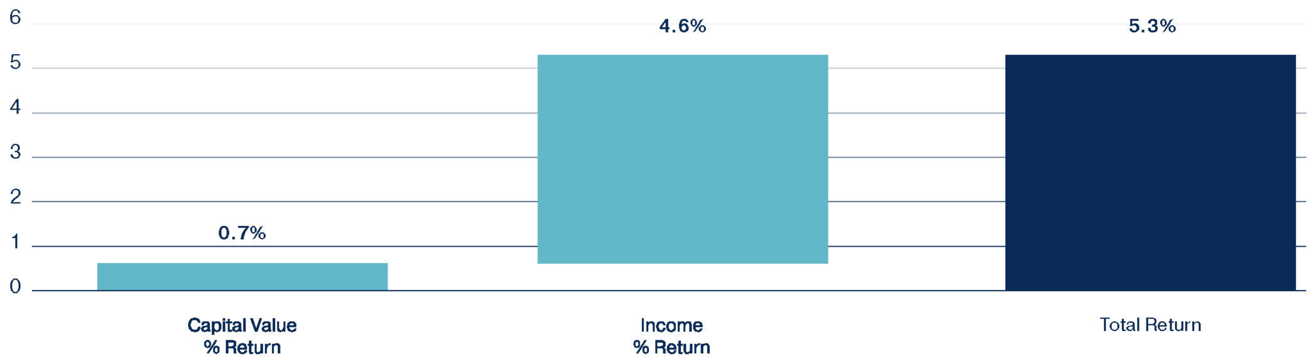


Chart 6: Foreign Real Estate

Market Comments

The overall Real Estate market hit a tipping point in 2022. Equity drawdowns, high inflation, and interest rate hikes began to have an impact on Real Estate pricing, volumes, and debt availability. Cap rates rose across nearly all sectors and geographies, driving up the Canadian national average cap rate figure to its highest level since before the pandemic. Furthermore, investment activity slowed in 2022 as more investors began taking a more cautious approach to capital allocation. However, not all sectors are equal. Namely, industrial, multi-family, and necessity-based retail are continuing to garner interest. Valuations are trending downwards for office properties, particularly in the U.S. and Europe. In addition, cap rates will need to adjust to find an equilibrium when compared with risk-free rates.

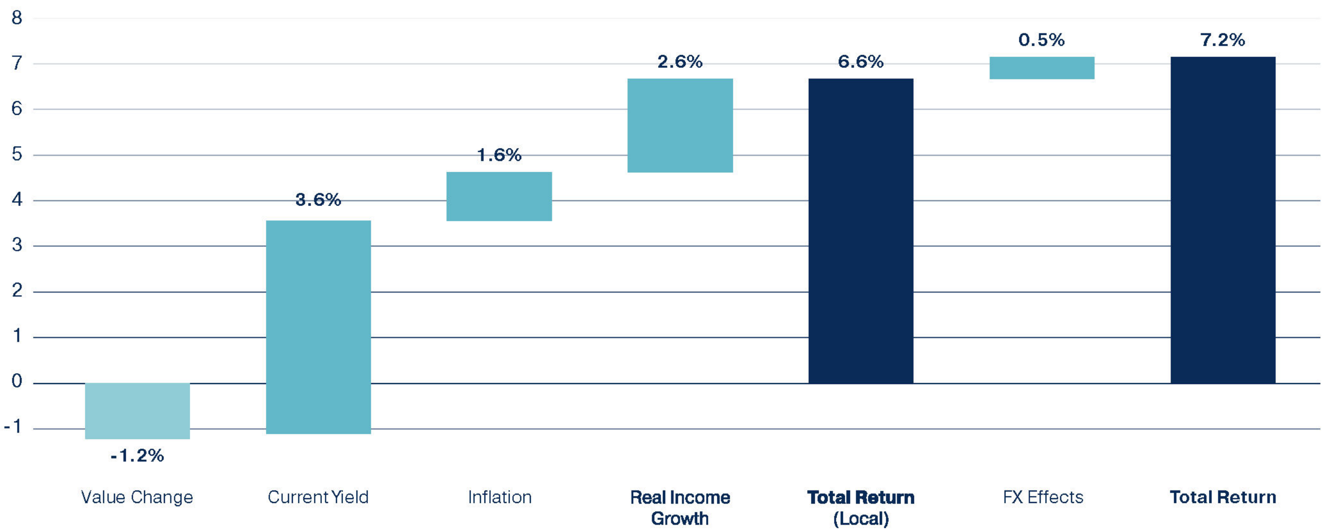
A flight to quality is beginning to take hold within capital markets and spatial markets. Office tenants are prioritizing quality and wellness in their leasing decisions. There is a renewed focus on Core/ Stabilized properties as a better risk-adjusted investment opportunity. Long-term forward-looking returns will benefit from income gains as a result of expanding capitalization rates last year. We also expect a modest gain from valuation changes as rates markets level off over the forecast horizon.

Infrastructure

Building Blocks

Many real assets, including infrastructure, provide both inflation protection and an income stream to investors. While the degree to which a pass-through inflation rate impacts infrastructure return can vary based on specific investments, overall, we expect infrastructure as an asset class will benefit from the higher expected inflation rate over the forecast horizon in this year's economic forecast. The detailed building block values contributing to our return forecast are shown in the following exhibit.

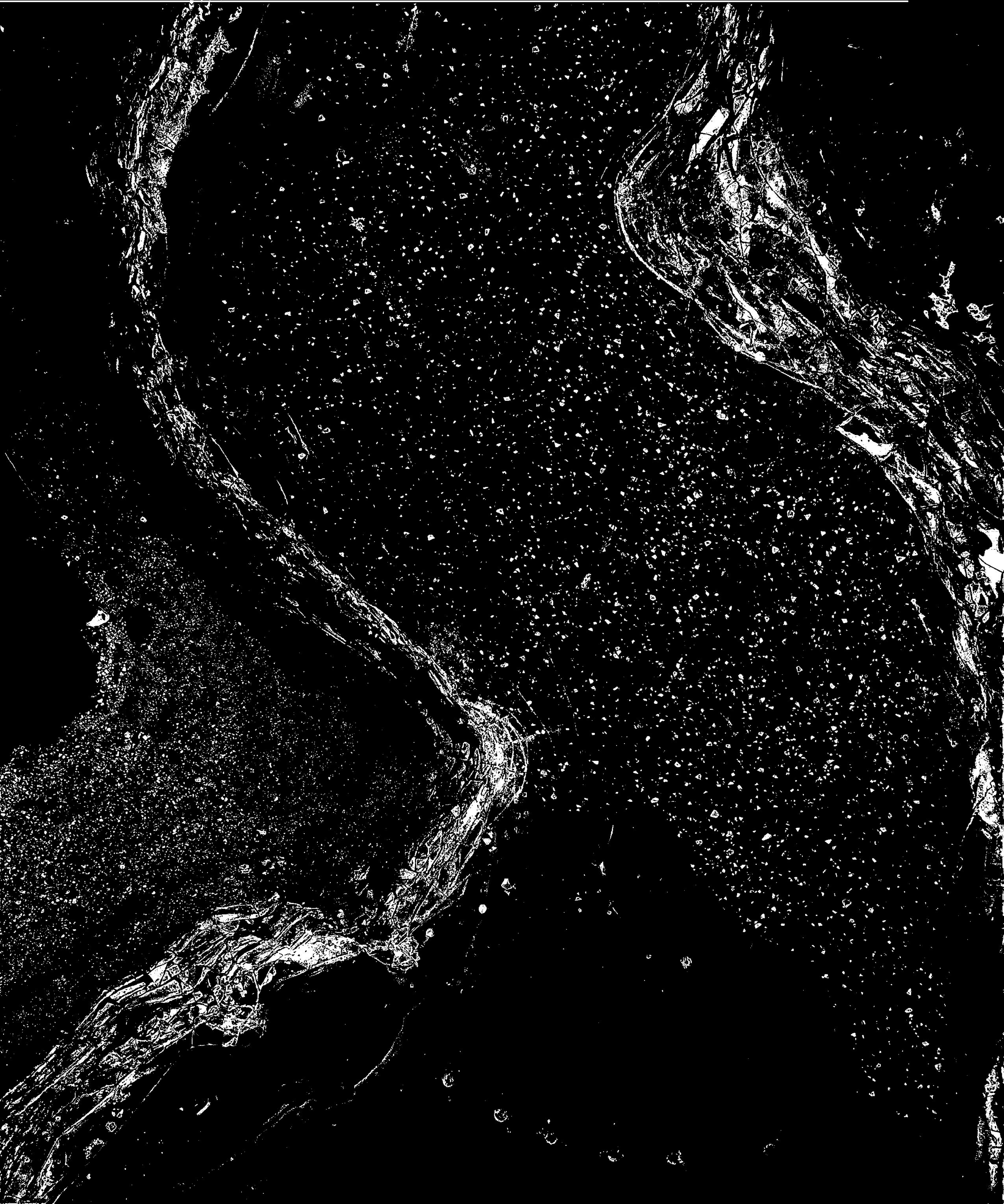
Chart 7: Building Blocks of Infrastructure



Market Comments

Infrastructure sectors performed well in 2022 relative to broader public equities. Underlying business dynamics remained largely resilient and benefitted from demand normalization post-pandemic. While there are still some lingering effects from the pandemic in certain sub-sectors (for example, transport), the shift back to a normalized demand environment continued in 2022 for most countries. Private investment in infrastructure is largely back to pre-pandemic levels and expected to increase on a longer-term basis, benefitting from some key trend drivers such as decarbonization and digitalization. Many government policies in various countries are also expected to be near-term supportive factors for further investment into infrastructure areas that drive decarbonization goals and the reshoring of various supply chains.

Forward-looking long-term returns have largely stayed close to the same levels as last year, with some impacts from valuation multiples reverting to longer-term normalized levels. In addition, the amount of undeployed capital seeking exposure in the space is expected to be a headwind for the asset class. Inflation is expected to be supportive of long-term returns, given the inflation-linked nature of many of the infrastructure sub-sector businesses. While the outlook has improved for several other asset classes, real assets will continue to play an important role as a diversified source of returns.

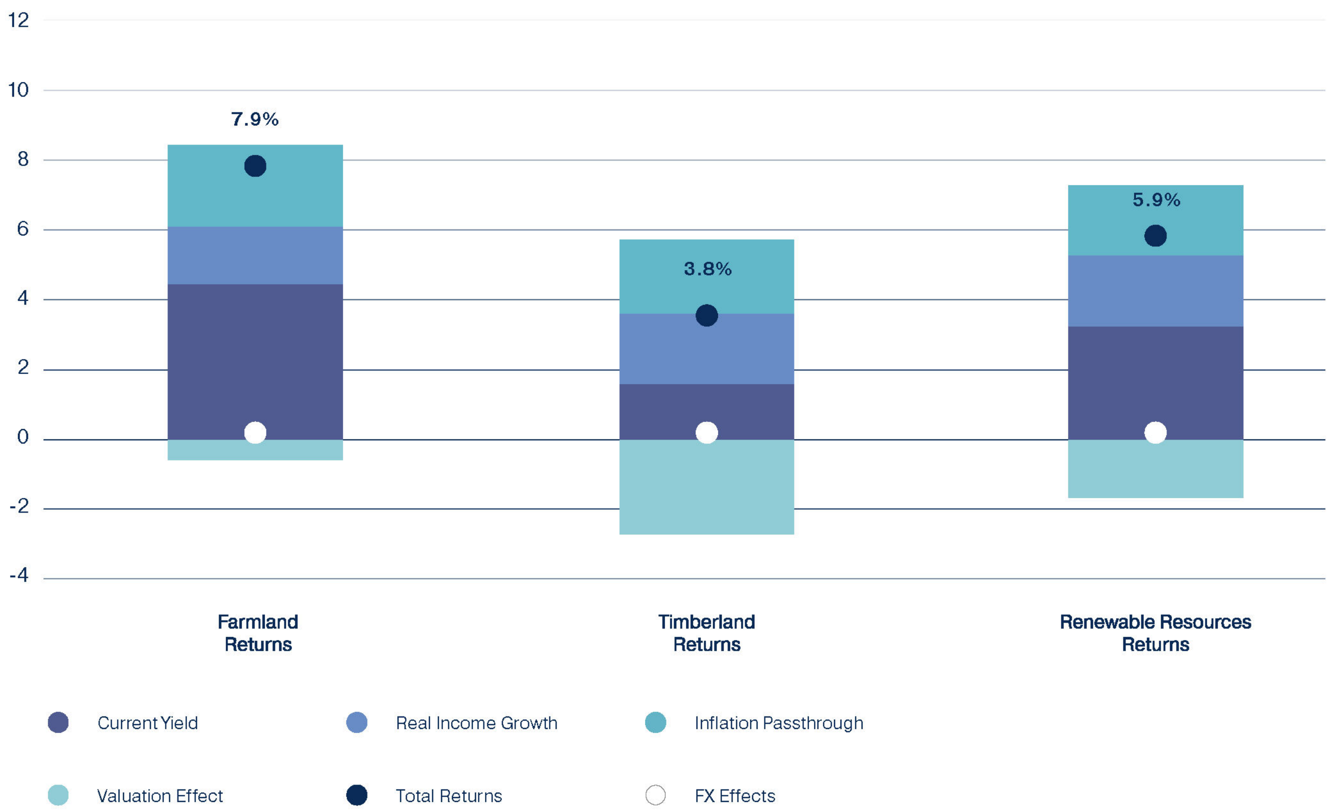


Renewable Resources

Building Blocks

Renewable resources is another asset class which provides a degree of inflation protection for AIMCo clients. We utilized a similar model to last year, incorporating both income generation capacity and valuation growth for this asset class. Our building blocks are highlighted in the following exhibit.

Chart 8: Building Blocks for Renewable Resources



Market Comments

The medium-to-long-term prospects for both timberland and farmland remain positive. The growing global middle class will intensify the demand for forest and agricultural products. At the same time, the supply of arable land to grow trees and crops is finite and decreasing due to competing land uses and climate change.

The demand for timberland and agriculture investments has increased as investors seek low correlations to conventional asset classes, inflation protection, and positive sustainability attributes. Underlying land values are generally correlated with inflation, whereas the income component of returns depends on the extent to which inflation impacts commodity prices/revenues and costs. The impact of inflation on key inputs such as fertilizer has put pressure on farm incomes in 2022, but generally, the increase in costs has been offset by relatively strong commodity prices. Competition has continued to drive valuations higher and compressed expected returns, especially in core timberland regions. Investors continue to seek timberland and agriculture investments due to their positive environmental, social, governance (ESG) characteristics and expected return sources for the asset classes have evolved beyond traditional income with more value being placed on carbon sequestration, conservation, and biodiversity.

Key risks to our Renewable Resources forecasts include sustained high interest rates which could put pressure on timberland and farmland valuations, pressure on farm income due to high input costs, decreased production due to natural disasters or climate change, labour availability, and lower commodity prices.

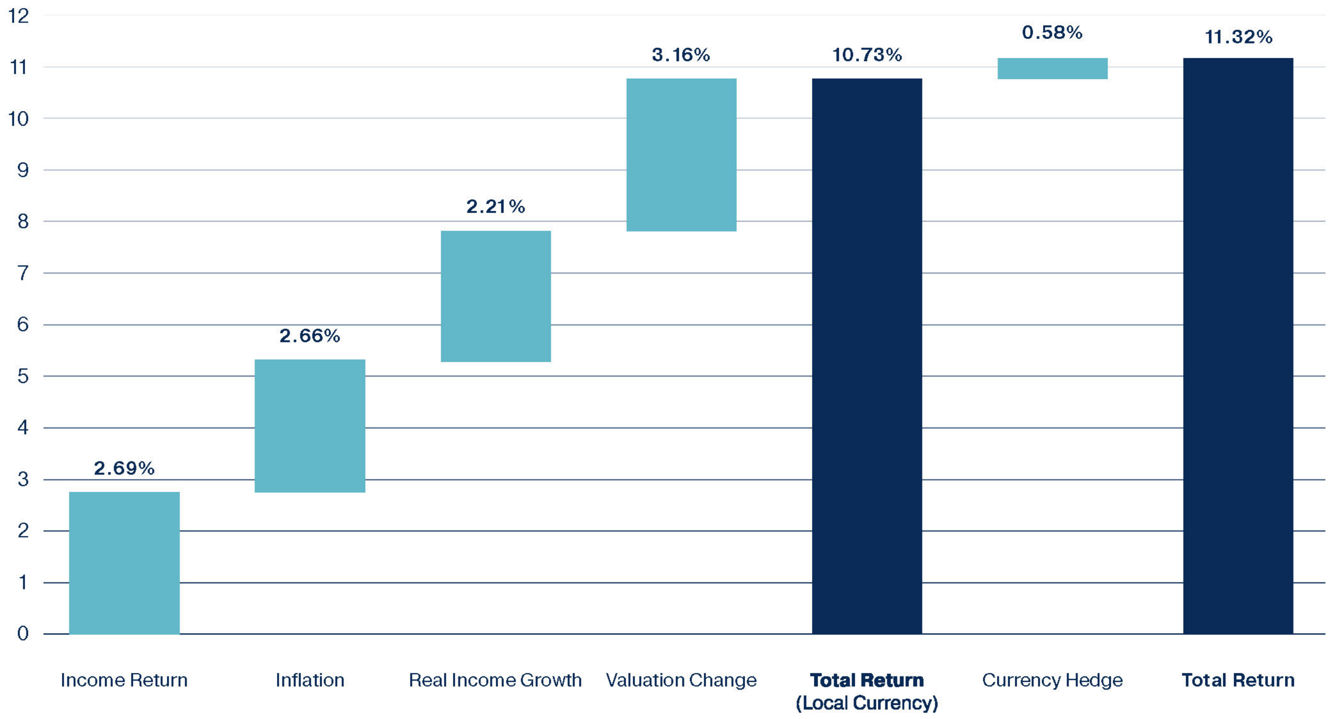
Private Equity

Building Blocks

Our private equity model, based on an AQR paper[8] we introduced in previous editions of our forecast, has similarities to the building blocks we use for public equities. Unlike public equities where we separately attribute income return to dividends and buybacks, for private equity, income return is forecasted using an estimated earnings yield. Our modelling framework uses a public equity comparable, namely MSCI World Index, to estimate the starting earnings yield and its long-term average, translating to an expected earnings yield of 2.7%. Based on our global GDP growth estimate, we assume real growth to be 2.2% for the decade.

Similarly, global inflation is expected to land at 2.7% over the next 10 years, boosting total returns. Valuation is the most challenging parameter to estimate over the long term. We use two metrics from data provider Preqin, "weighted net multiple" and "residual value to paid-in" ratio. By combining the effect of the convergence of the metrics from their current value to their long-term average, we expect a return from valuation gains to be 3.1%. The aggregate effect of all factors including currency translation produces an expected return of 11.3% over the coming decade.

Chart 9: Building Blocks for Private Equity



[8] Source: Demystifying Illiquid Assets: Expected Returns for Private Equity, AQR Whitepaper 1Q19, by Antti Ilmanen, Swati Chandra, Nicholas McQuinn."

Market Comments

Private equity will continue to be an attractive asset class in the long term and play an important role in portfolio diversification. The benefits of having a long-term approach, employing value-creation strategies and attracting best-in-class well-aligned management teams will continue to support attractive risk-adjusted returns. In the short term, valuations and performance have weakened in line with global macro factors including high inflation, supply chain disruption, slowing economic growth and widened bid-ask spreads as private market pricing has lagged depressed public valuations. This dynamic, compounded by a recent slowdown in exits, has led to the "denominator effect" being experienced by many public pension plans that have now reached or exceeded target allocations to the asset class. This, in turn, has contributed to a pullback in fundraising activity, which in aggregate has declined 14% year-over-year as of early December 2022. Nonetheless, there is still a large balance of dry powder already committed to private equity funds ready to invest, estimated to be approximately USD 1.3 trillion.

Private equity deal activity slowed in the second half of 2022 with volumes declining by 22% year over year. Turbulent leveraged financing markets have contributed to this dynamic, effectively limiting the size of deals that firms are currently able to execute. Meanwhile, higher interest rates and global economic uncertainty are also beginning to lower valuations, which had become frothy in recent years. As we enter an environment where leverage is increasingly expensive and valuation multiples may contract, operational value creation will likely drive an increasing share of private equity returns.

Two sectors that could see relative outperformance in this environment are healthcare and enterprise technology. Long-term global secular trends are providing strong tailwinds for the healthcare sector, including (i) demographics—an aging population, higher incidence of chronic diseases, growing middle class seeking better quality, and individualized healthcare; (ii) increased spending—pressure to find affordable products and services, increased spend by government, and fast-track processes for drug development; and (iii) transition towards greater personalization of treatment. The deal environment for the technology sector as well is likely to remain favourable—particularly in an environment with more palatable valuations—given long-term trends of (i) increased digitalization; (ii) greater adoption of recurring revenue models that should help insulate earnings through periods of economic volatility; and (iii) accelerating technological innovation. Rapidly evolving consumer behaviours and demands on businesses are similarly providing emerging opportunities within the consumer products and business services markets as well. Diversity, equity, and inclusion or "DEI" is an ancillary strategy traversing all sectors and represents an additional emerging yet undercapitalized opportunity for private equity investors. Historical performance of firms committed to DEI has proven to be at least comparable to that of nondiverse counterparts and, in certain cases, superior.

As traditional exits via sponsor-to-sponsor sales or IPOs stall in the near term, the continued rise of the secondaries market has helped enhance liquidity optionality in private markets. General Partners (GP) have increasingly embraced the market as a way to monetize stakes in companies that they do not want to exit completely even as they try to satisfy investor commitments. A GP-led secondary via a continuation vehicle creates a way to return capital to existing LPs while enabling the fund to maximize the value of the assets by extending the holding period, which is a trend we may see an increase over the coming years.

Forecast by Asset Class

Public Equities

Overview

Our forecast this year has forward-looking public equity returns increasing compared to a year ago. Lower starting valuations are the main driver of the increase in expected returns, given the retreat of price-to-earnings ratios experienced during 2022. Earnings growth is expected to be bound at the upper end by our GDP forecasts for each market. Rising input prices and cost of debt may manifest as a double-headed risk, should inflation persist over the coming years. Equity markets are, however, real assets and their cash flows are expected to keep pace with inflation over the longer term.

Dividends are reliable sources of return in equity markets. Our models assume dividends will persist based on current trends. In all markets, we expect valuations will help develop a positive contribution to equity returns over the next 10 years. Trailing 12-month Price-to-Earnings (P/E) ratios have dropped since our last forecast, as markets reset to a higher rate environment. We assume valuations will revert to the mean over the forecast horizon.

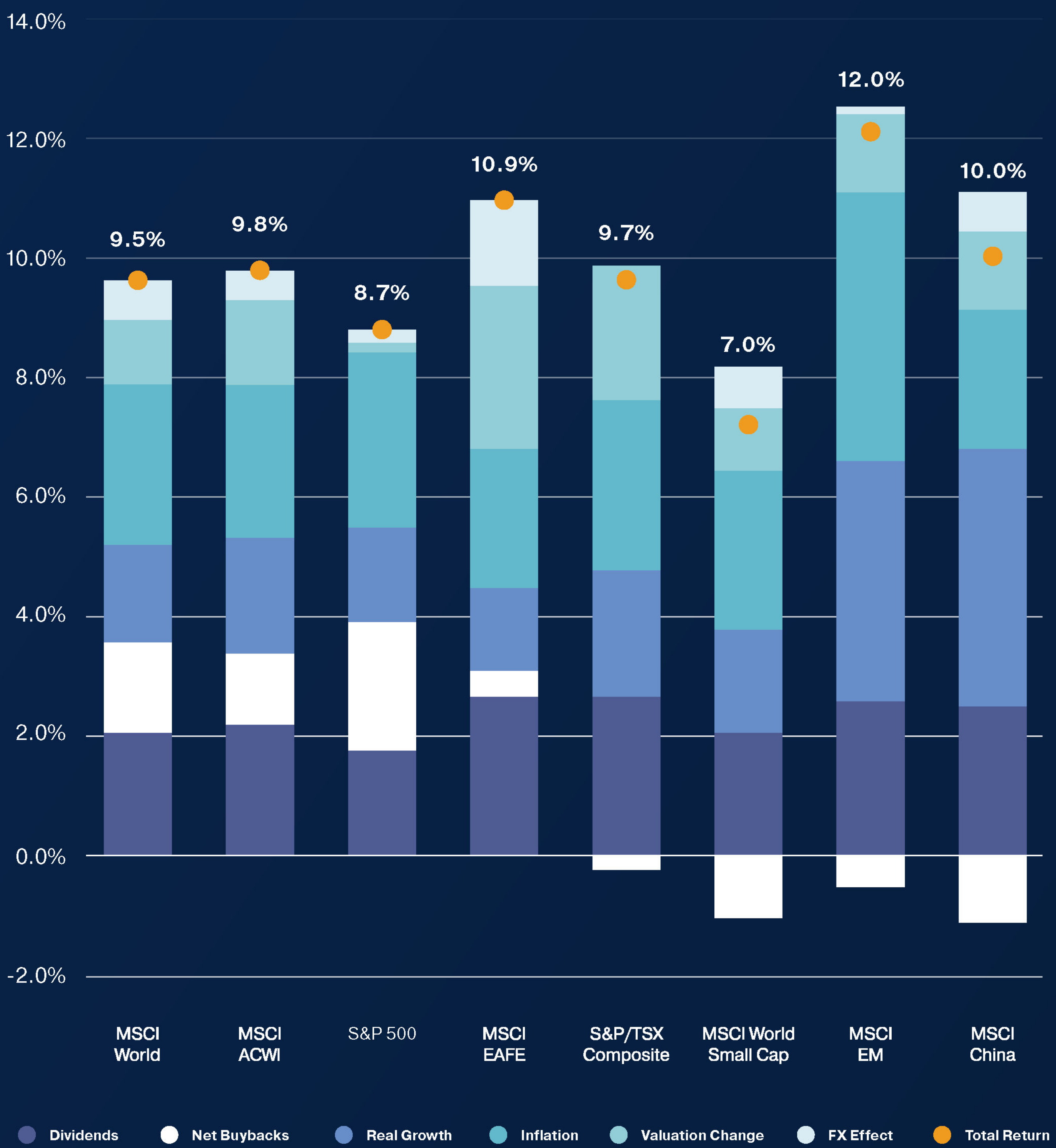
Building Blocks

AIMCo's public equity capital market assumptions are based on forecasts in inflation, real earning growth, dividend yield, buyback yield, net dilution and valuations for the respective index. AIMCo incorporates a currency view through conversion to the Canadian dollar.

For public equity benchmarks, we define the expected return as being the combination of total yield (dividend yield and net buyback yield), expected trend growth (g) in earnings per share EPS, and expected change in valuations (Δv). That is: $E(r) \approx DY+g+\Delta v$

Expected trend growth in EPS is proxied by real GDP growth rates.

Chart 10: Building Blocks for Public Equity Assets



Global Equities

In the past year, Global Equity investors were forced to manage the simultaneous challenges of rising and persistent inflation, central bank hawkishness, and significant geopolitical headwinds. With few exceptions, developed markets all closed out 2022 in the red, with the US technology-heavy NASDAQ among the worst performers. With widespread interest rate hikes globally in 2022, long-duration equities were particularly punished after more than a decade of market leadership. Elsewhere, as global energy security came to the fore, investors in global developed markets are now confronted with the most perplexing set of economic headwinds seen in decades.

Our view is that delayed action by global central banks in response to inflationary pressures has ushered in a new and potentially extended period of monetary tightness. Broadly speaking, 2022 was a year of valuation multiple compression. Indeed, the cost of capital for corporates has seen its biggest 12-month move in four decades. The possibility of a more moderate-to-low economic growth environment in the years to come suggests earnings revisions ahead. As valuations could continue to reset in the short term, we expect corporations to take advantage of this via accelerated shareholder returns in the form of dividends and share repurchases. Nonetheless, the economic backdrop for global equities is now quite distinct from that of the past 40 years and will require rigour, agility, and patience.

Canadian Equities

The strength in Canadian earnings continued in 2022, carrying on the incredible growth and recovery of the prior year. It was inevitable that earnings growth would slow given the higher base, but the broad strength across the economy translated into high profitability for the Canadian market. Return on equity for Canada approached the mid-teens, which are levels not seen since before the great financial crisis. Corporations developed a greater appreciation for capital efficiency and increasingly returned capital to shareholders via increased dividends and share buybacks. This behaviour change was particularly notable in the energy sector where high commodity prices were met with production discipline, which translated into tremendous free cash flow for the industry.

Similar to many parts of the developed world, much of the economic growth of the past two years can be attributed to the lingering effects of the various forms of stimulus introduced during the pandemic. The stimulative effects of these policies highlighted some of the constraints and underinvestment in the world as rising demand for many products and services remains unmet, resulting in significantly higher prices. The Bank of Canada is attempting to cool demand and inflationary pressures through higher interest rates, resulting in 400 bps of tightening in 2022. This has pushed bond yields and discount rates higher. The lower valuation combined with an outlook for a slowing economy have been the primary reasons for the decline in the Canadian equity market this past year.

As has been the case for many years, the Canadian equity market continues to trade at a discount relative to the U.S. due to the country's greater cyclicality and persistently lower profitability. As markets begin to anticipate a pause in central bank tightening, and perhaps some easing in the latter half of 2023, Canadian equities may benefit from Canada's economy pro-cyclical nature alongside less rich valuations than the U.S.

Emerging Market Equities

Last year was another challenging year in emerging markets, driven by rising global interest rates, higher inflation, a stronger U.S. dollar, economic headwinds in China and the largest military conflict in Europe since World War II. These events contributed to lower Emerging Markets' valuation measures, which has made them a more attractive investment opportunity over the longer term. Attractive valuations are supported by expectations for stronger corporate earnings growth as Emerging Market companies benefit from being in the faster-growing parts of the global economy. In 2023, we expect Emerging Markets to deliver stronger growth as China reopens its economy, the outlook for the U.S. dollar weakens, and commodity suppliers continue to perform well. Longer-term geopolitical concerns temper somewhat our enthusiasm for the asset class.

Global Small-Cap Equities

Global small caps performed very much in line with their larger cap peers this past year as the entire market declined. Small-cap earnings growth slowed materially and faced lower valuations as higher interest rates and the prospects for slower, or perhaps even contracting growth presented headwinds to the overall market. With the underperformance of small-cap equities back in 2021, valuations between the two segments of the market have returned to levels more in line with longer-term averages. As the prospect of a shift in central bank policy becomes more apparent and markets begin to anticipate a pause in the tightening cycle, opportunities will present themselves for small-cap markets.

Chinese Equities

China's equity markets have endured a difficult three years of rolling pandemic-related lockdowns, a precipitous property market slump, and most recently, rising geopolitical tensions resulting from Russia's invasion of Ukraine. Yet following the People's Party Congress in the fall of 2022, signals point to a substantial easing in China's COVID restrictions, which could soon unlock China's economic potential. With a backdrop of lower commodity prices, an economic reopening, and an effective put from the Chinese government to property developers, we expect earnings and investor sentiment to significantly improve. If the North American experience is any guide, the relaxation of mobility restrictions should positively impact consumption in China, with goods consumption expected to see an earlier resurgence than that of services. On valuations, although they have been lower relative to history, many domestically oriented sectors in China are near cyclical troughs. Our forecast also acknowledges that foreign investors have been substantially underweighting China equities during the pandemic (and compounded by U.S.-China tensions), and we expect a portfolio normalization over time.

Risk Assumptions Methodology

The AIMCo 2023 long-term risk forecasts are volatility estimates, which are useful for building portfolios based on a mean-variance optimization analysis. However, investors should also consider the broader concept of risk, including tail risks, which can be measured by Value at Risk (VaR) and Expected Tail Loss (ETL). In addition, some assets (e.g. real return bonds) may exhibit high volatility, but they help diminish inflation risk, which is important to investors sensitive to inflation. Although we will not cover the details of alternative risk measures, we would like to highlight the importance of understanding these dynamics, which can become particularly relevant in investment decisions.

Similar to last year, we have incorporated the results of a collaboration with our partner AlphaLayer, to enhance the robustness of our risk forecast. AlphaLayer, a collaboration between AIMCo and AltaML, applies deep understanding and experience in machine learning techniques to deliver solutions specific to the investment management industry. Illiquid asset classes are valued infrequently and may not be marketable securities. As a result, illiquid assets can suffer from various biases and are difficult to compare to higher frequency data available for publicly-traded assets. Adjusting the data for these shortcomings can improve the statistical nature of the dataset and produce more robust and realistic estimates of risk. A few of the specific adjustments made were:

1. Seasonal effects of time series data were removed; for example, accounting effects that are noticeable at year-end.
2. Monthly data were imputed based on the adjusted quarterly data using machine learning techniques. The imputation considered not only an individual asset’s return time series but also the appropriate statistical relationship with other assets.

AIMCo has implemented a VAR-GARCH-DCC[9] statistical model for risk forecasting. This model was proposed by Nobel laureate Robert Engle and Kevin Sheppard to estimate time-varying covariance matrices through the concept of Dynamic Conditional Correlation (DCC) estimators in 2001. The DCC estimators are combined with a multivariate VAR-GARCH in a parsimonious manner to estimate correlation matrices. The following considerations are given during the modelling process:

1. We employed a multivariate time series model, which is a suitable choice when both volatility and correlation vary over time. The use of time series with varying volatilities across time to model asset return data is also supported by extensive academic literature.
2. We used an asymmetric Student's t-distribution to incorporate skewness and kurtosis exhibited by most asset class historical statistical distributions.
3. We ensured the benchmarks would be provided at a high frequency with the requirement of trying to capture the true, underlying volatility properties of the respective asset classes.
4. We made use of long enough benchmark historical data for modelling purposes.
5. We selected the risk benchmarks either following the AIMCo official asset class benchmarks or researched representative benchmarks for the underlying asset class.

The long-term expected risks and correlations have also been reviewed by AIMCo's Chief Investment Officer and the Risk Management group.

[9] Vector Autoregressive - Generalized Autoregressive Conditional Heteroscedasticity - Dynamic Conditional Correlation.

Currency Assumptions Methodology

To convert non-Canadian market returns to Canadian dollar terms, we adjust the expected return using the exchange rate such that purchasing power is maintained between other economies and Canada. Absolute PPP holds that exchange rates are in equilibrium when the value of a national basket of goods and services are the same between two countries.

Disclaimer

Return assumptions and risk estimates were developed by Alberta Investment Management Corporation and are for general information only. This information is not intended to provide specific advice to any individual. We make no representations or warranties with respect o the timeliness, accuracy, or completeness, and bear no liability for any loss arising from its use. All forward-looking information and forecasts contained herein, unless otherwise noted, are the opinion of Alberta Investment Management Corporation, and future market movements may differ significantly from our expectations. Index performance or any index related data is given for illustrative purposes only and is not indicative of the performance of any particular portfolio. Any performance data shown herein represents returns and is no guarantee of future results. Investment returns will fluctuate such that the value of holdings may be worth more or less than their original cost.

AIMCo

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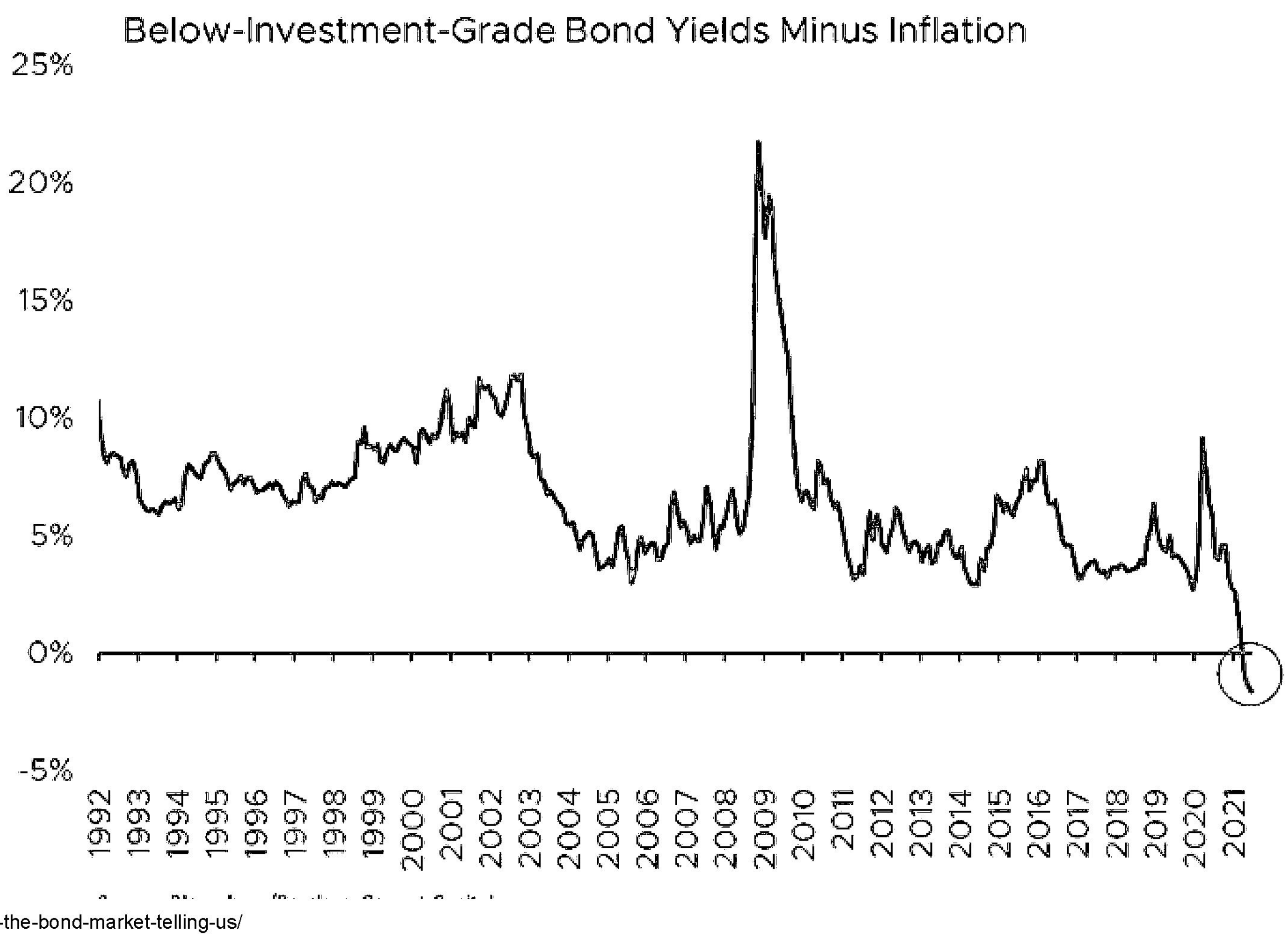
https://cressetcapital.com/post/what-is-the-bond-market-telling-us/

WHAT IS THE BOND MARKET TELLING US?

MARKET COMMENTARY | JACK ABLIN | 7/22/21

The benchmark 10-year yield slid nearly .60 per cent since the end of March against a backdrop of robust economic growth and the highest inflation readings in years. At 1.2 per cent, the 10-year is back to levels we haven't seen since February. With year-over-year inflation running at 5.4 per cent, the "real" rate – the 10-year yield adjusted for inflation – is -4.2 per cent, more than one percentage point more negative than any of our developed market counterparts. The 10-year "junk" bond yield is negative also, for the first time in history. Same for the real earnings yield of the S&P 500, which has never before been negative. Bond holders, unable to earn a yield higher than the inflation rate, are effectively locking in a lower standard of living as the potential growth of their investment trail the cost of living. While negative real rates have occurred historically, they are generally associated with extraordinary events, like an inflation shock or emergency monetary policy, not a recovering economy.



Source: Bloomberg/Barclays; Cresset Capital.
Chart #1254

Bond investors tend to see the glass half empty, since bad news is usually good news for fixed-income investors. But several issues are keeping today's bond holders awake at night. The Delta variant is gaining strength, particularly among the unvaccinated. While the US leads much of the world in vaccinations, we're falling behind projections made earlier this year. President Biden had to walk back his pledge that 70 per cent of US adults would be fully vaccinated by July 4. Credit market investors worry that inadequate vaccination rates could stall reopening efforts in parts of the US where vaccinations trail and in many parts of the developed world. Vaccination rates in Japan, Mexico and South Korea remain at or below 20 per cent of their populations.

Fixed-income investors also worry about waning fiscal support. President Biden's $2 trillion COVID-19 relief package went a long way to support households and unemployed Americans beset with the ravages of the pandemic. Relief in the form of stimulus checks and unemployment benefits are set to expire this fall, leaving a hole in disposable income. Bond investors worry that demand will fall without government support. The economy is expected to have expanded 9 per cent last quarter, fueled in part by government stimulus checks. While economists anticipate growth to continue, it would be at a slower pace. Longer term, growth is expected to trend back to a 2.5 per cent annualized rate.

Federal Reserve governors are tasked with balancing full employment with price stability. Prices are rising even though millions of Americans remain without work. Bond investors worry that the Federal Open Market Committee will tighten monetary policy too soon, quashing the incipient recovery. Bond yields reversed in June, when the Fed updated its “dot plot,” signaling an acceleration of its rate-tightening program. Bond investors now expect the Fed to commence raising its overnight rate next year. That stands in contrast to Chairman Powell’s pledge earlier this year of no rate hikes through 2022.

From a growth perspective, bond investors are concerned today’s growth is situated on the North Pole, where any step taken must be south. They believe we are experiencing peak growth in economic activity, corporate profits and inflation, as the reopening and recovery converge. Inflation is currently running at over 5 per cent year over year, but it’s largely a reflection of pandemic pricing last year, particularly for used car and hotel rooms. Investors expect pricing pressure to retreat, with 5-year inflation expected to average 2.5 per cent and 10-year inflation to average 2.1 per cent. That implies the 5-year rate five years from now is expected to be 1.8 per cent, below the Fed’s 2 per cent inflation target. It remains to be seen if today’s pricing pressure is temporary or longer lasting. A lot depends on what happens to wages and housing costs. Higher wages are needed to fuel demand at higher prices while housing costs, which typically don’t retreat, comprise nearly half of the government’s CPI calculation.

Bond investors worry that profit growth is also peaking. S&P 500 investors are expected to harvest 60+ per cent profit growth this quarter, representing the strongest year-over-year gain since Q4/09, when companies rebounded from the financial crisis. Wary investors understand current profit trends won't last indefinitely. Next quarter, for example, S&P profits are expected to grow 24 per cent from a year earlier, leveling to 14 per cent toward the middle of next year. While mid-teens profit growth is respectable by historical standards, glass-half-empty investors look at growth trends, and see them slowing.

Investment implications

The bond market is sending strong but inconsistent signals. Harried investors should be justifiably worried about the escalation of the Delta variant, especially given the low vaccination rates in certain parts of the world. However, the yield differential between short-term and intermediate-term maturities, albeit narrower, still implies growth. The yield differential touched zero in H2/19 but remains over one per cent today. Financial conditions – an index comprising a combination of credit spreads (the premium lenders require to extend credit to lower-quality borrowers) and market volatility – remain favorable for risk takers as well. Notwithstanding the pullback in growth expectations, lenders have not pulled in their horns.

Putting the pieces together suggests the evolving view of the bond market should not be interpreted as a threat to long-term equity investors. It remains to be seen whether equity investors were justified in driving the S&P 500 to more than 30 all-time highs this year, but in our view a downgrade in growth estimates shouldn't prompt anything worse than a correction. An extended, flatter recovery still leaves bond yields that carry negative real interest rates too low for projected, stable-state conditions. That means that a physical asset, like gold, would do a better job than financial assets in helping short-term bond investors maintain their purchasing power. Persistently negative real rates also mean that equity investors can no longer rely on valuation expansion to help fuel future market returns, a benefit that drove most of the market gains over the last 10 years. Instead, equity investors must rely on organic earnings growth and dividends for returns. From that perspective, cheaper, value-oriented sectors with higher earnings yields, like health care, industrials and materials, will likely be a more important source of investment returns over the coming years.

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CHAPTER 18

The Valuation of Common Stocks

In Chapter 17 it was noted that one purpose of financial analysis is to identify mispriced securities. Fundamental analysis was mentioned as one approach for conducting a search for such securities. With this approach the security analyst makes estimates of such things as the firm's future earnings and dividends. If these estimates are substantially different from the average estimates of other analysts but are felt to be more accurate, then from the viewpoint of the security analyst, a mispriced security will have been identified. If it is also felt that the market price of the security will adjust to reflect these more accurate estimates, then the security will be expected to have an abnormal rate of return. Accordingly, the analyst will issue either a buy or sell recommendation, depending on the direction of the anticipated price adjustment. Based on the capitalization of income method of valuation, dividend discount models have been frequently used by fundamental analysts as a means of identifying mispriced stocks. This chapter will discuss dividend discount models and how they can be related to models based on price-earnings ratios.

18.1 CAPITALIZATION OF INCOME METHOD OF VALUATION

There are many ways to implement the fundamental analysis approach to identifying mispriced securities. A number of them are either directly or indirectly related to what is sometimes referred to as the **capitalization of income method of valuation.**[1] This method states that the "true" or "intrinsic" value of any asset is based on the cash flows that the investor expects to receive in the future from owning the asset. Because these cash flows are expected in the future, they are adjusted by a **discount rate** to reflect not only the time value of money but also the riskiness of the cash flows.

Algebraically, the intrinsic value of the asset V is equal to the sum of the present values of the expected cash flows:

$$V = \frac{C_1}{(1+k)^1} + \frac{C_2}{(1+k)^2} + \frac{C_3}{(1+k)^3} + \cdots$$

$$= \sum_{t=1}^{\infty} \frac{C_t}{(1+k)^t} \tag{18.1}$$

where C_t denotes the expected cash flow associated with the asset at time t and k is the appropriate discount rate for cash flows of this degree of risk. In this equation the discount rate is assumed to be the same for all periods. Because the symbol ∞ above the summation sign in the equation denotes infinity, all expected cash flows, from immediately after making the investment until infinity, will be discounted at the same rate in determining V.[2]

18.1.1 Net Present Value

For the sake of convenience, let the current moment in time be denoted as zero, or $t = 0$. If the cost of purchasing an asset at $t = 0$ is P, then its **net present value** (NPV) is equal to the difference between its intrinsic value and cost, or:

$$\text{NPV} = V - P$$

$$= \left[\sum_{t=1}^{\infty} \frac{C_t}{(1+k)^t}\right] - P. \tag{18.2}$$

The NPV calculation shown here is conceptually the same as the NPV calculation made for capital budgeting decisions that has long been advocated in introductory finance textbooks. Capital budgeting decisions involve deciding whether or not a given investment project should be undertaken. (For example, should a new machine be purchased?) In making this decision, the focal point is the NPV of the project. Specifically, an investment project is viewed favorably if its NPV is positive, and unfavorably if its NPV is negative. For a simple project involving a cash outflow now (at $t = 0$) and expected cash inflows in the future, a positive NPV means that the present value of all the expected cash inflows is greater than the cost of making the investment. Conversely, a negative NPV means that the present value of all the expected cash inflows is less than the cost of making the investment.

The same views about NPV apply when financial assets (such as a share of common stock), instead of real assets (such as a new machine), are being considered for purchase. That is, a financial asset is viewed favorably and said to be underpriced (or undervalued) if NPV > 0. Conversely, a financial asset is viewed unfavorably and said to be overpriced or (overvalued) if NPV < 0. From Equation (18.2), this is equivalent to stating that a financial asset is underpriced if $V > P$:

$$\sum_{t=1}^{\infty} \frac{C_t}{(1+k)^t} > P. \tag{18.3}$$

Conversely, the asset is overvalued if $V < P$:

$$\sum_{t=1}^{\infty} \frac{C_t}{(1+k)^t} < P.$$

18.1.2 Internal Rate of Return

Another way of making capital budgeting decisions in a manner that is similar to the NPV method involves calculating the **internal rate of return** (IRR) associated with the investment project. With IRR, NPV in Equation (18.2) is set equal to zero and the discount rate becomes the unknown that must be calculated. That is, the IRR for a given investment is the discount rate that makes the NPV of the investment equal to zero. Algebraically, the procedure involves solving the following equation for the internal rate of return k^*:

$$0 = \sum_{t=1}^{\infty} \frac{C_t}{(1+k^*)^t} - P. \tag{18.5}$$

Equivalently, Equation (18.5) can be rewritten as:

$$P = \sum_{t=1}^{\infty} \frac{C_t}{(1+k^*)^t}. \tag{18.6}$$

The decision rule for IRR involves comparing the project's IRR (denoted by k^*) with the required rate of return for an investment of similar risk (denoted by k). Specifically, the investment is viewed favorably if $k^* > k$, and unfavorably if $k^* < k$. As with NPV, the same decision rule applies if either a real asset or a financial asset is being considered for possible investment.[3]

18.1.3 Application to Common Stocks

This chapter is concerned with using the capitalization of income method to determine the intrinsic value of common stocks. Because the cash flows associated with an investment in any particular common stock are the dividends that are expected to be paid throughout the future on the shares purchased, the models suggested by this method of valuation are often known as **dividend discount models** (DDMs).[4] Accordingly, D_t will be used instead of C_t to denote the expected cash flow in period t associated with a particular common stock, resulting in the following restatement of Equation (18.1):

$$\begin{aligned} V &= \frac{D_1}{(1+k)^1} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \cdots \\ &= \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t} \end{aligned} \tag{18.7}$$

Usually the focus of DDMs is on determining the "true" or "intrinsic" value of one share of a particular company's common stock, even if larger size purchases are being contemplated. This is because it is usually assumed that larger size purchases can be made at a cost that is a simple multiple of the cost of one share. (For example, the cost of 1,000 shares is usually assumed to be 1,000 times the cost of one share.) Thus the numerator in DDMs is the cash dividends per share that are expected in the future.

However, there is a complication in using Equation (18.7) to determine the intrinsic value of a share of common stock. In particular, in order to use this equation the investor must forecast *all* future dividends. Because a common stock does not have a fixed lifetime, this suggests that an infinitely long stream of dividends must be forecast. Although this may seem to be an impossible task, with the addition of certain assumptions, the equation can be made tractable (that is, usable).

These assumptions center on dividend growth rates. That is, the dividend per share at any time t can be viewed as being equal to the dividend per share at time $t - 1$ times a dividend growth rate of g_t,

$$D_t = D_{t-1}(1 + g_t) \tag{18.8}$$

or, equivalently:

$$\frac{D_t - D_{t-1}}{D_{t-1}} = g_t. \tag{18.9}$$

For example, if the dividend per share expected at $t = 2$ is \$4 and the dividend per share expected at $t = 3$ is \$4.20, then $g_3 = (\$4.20 - \$4)/\$4 = 5\%$.

The different types of tractable DDMs reflect different sets of assumptions about dividend growth rates, and are presented next. The discussion begins with the simplest case, the zero-growth model.

18.2 THE ZERO-GROWTH MODEL

One assumption that could be made about future dividends is that they will remain at a fixed dollar amount. That is, the dollar amount of dividends per share that were paid over the past year D_0 will also be paid over the next year D_1, and the year after that D_2, and the year after that D_3, and so on—that is,

$$D_0 = D_1 = D_2 = D_3 = \cdots = D_\infty.$$

This is equivalent to assuming that all the dividend growth rates are zero, because if $g_t = 0$, then $D_t = D_{t-1}$ in Equation (18.8). Accordingly, this model is often referred to as the **zero-growth** (or no-growth) **model.**

18.2.1 Net Present Value

The impact of this assumption on Equation (18.7) can be analyzed by noting what happens when D_t is replaced by D_0 in the numerator:

$$V = \sum_{t=1}^{\infty} \frac{D_0}{(1 + k)^t}. \tag{18.10}$$

Fortunately, Equation (18.10) can be simplified by noting that D_0 is a f[illegible] lar amount, which means that it can be written outside the summation s[illegible]

$$V = D_0\left[\sum_{t=1}^{\infty} \frac{1}{(1+k)^t}\right]. \qquad (18.11)$$

The next step involves using a property of infinite series from mathematics. If $k > 0$, then it can be shown that:

$$\sum_{t=1}^{\infty} \frac{1}{(1+k)^t} = \frac{1}{k}. \qquad (18.12)$$

Applying this property to Equation (18.11) results in the following formula for the zero-growth model:

$$V = \frac{D_0}{k_0}. \qquad (18.13)$$

Because $D_0 = D_1$, Equation (18.13) is written sometimes as:

$$V = \frac{D_1}{k}. \qquad (18.14)$$

Example

As an example of how this DDM can be used, assume that the Zinc Company is expected to pay cash dividends amounting to \$8 per share into the indefinite future and has a required rate of return of 10%. Using either Equation (18.13) or Equation (18.14), it can be seen that the value of a share of Zinc stock is equal to \$80 (= \$8/.10). With a current stock price of \$65 per share, Equation (18.2) would suggest that the NPV per share is \$15 (= \$80 − \$65). Equivalently, as $V = \$80 > P = \$65$, the stock is underpriced by \$15 per share and would be a candidate for purchase.

18.2.2 Internal Rate of Return

Equation (18.13) can be reformulated to solve for the IRR on an investment in a zero-growth security. First, the security's current price P is substituted for V, and second, k^* is substituted for k. These changes result in:

$$P = \frac{D_0}{k^*}$$

which can be rewritten as:

$$k^* = \frac{D_0}{P} \qquad (18.15a)$$

$$= \frac{D_1}{P}. \qquad (18.15b)$$

Example

Applying this formula to the stock of Zinc indicates that $k^* = 12.3\%$ (= \$8/\$65). Because the IRR from an investment in Zinc exceeds the required rate of return on Zinc (12.3% > 10%), this method also indicates that Zinc is underpriced.[5]

18.2.3 Application

The zero-growth model may seem quite restrictive. After all, it seems unreasonable to assume that a given stock will pay a fixed dollar-size dividend forever. Although such a criticism has validity for common stock valuation, there is one particular situation where this model is quite useful.

Specifically, whenever the intrinsic value of a share of high-grade preferred stock is to be determined, the zero-growth DDM will often be appropriate. This is because most preferred stock is nonparticipating, meaning that it pays a fixed dollar-size dividend that will not change as earnings per share change. Furthermore, for high-grade preferred stock these dividends are expected to be paid regularly into the foreseeable future. Why? Because preferred stock does not have a fixed lifetime, and, by restricting the application of the zero growth model to high-grade preferred stocks, the chance of a suspension of dividends is remote.[6]

18.3 THE CONSTANT-GROWTH MODEL

The next type of DDM to be considered is one that assumes that dividends will grow from period to period at the same rate forever, and is therefore known as the **constant growth model.**[7] Specifically, the dividends per share that were paid over the previous year D_0 are expected to grow at a given rate g, so that the dividends expected over the next year D_1 are expected to be equal to $D_0(1 + g)$. Dividends the year after that are again expected to grow by the same rate g, meaning that $D_2 = D_1(1 + g)$. Because $D_1 = D_0(1 + g)$, this is equivalent to assuming that $D_2 = D_0(1 + g)^2$ and, in general:

$$D_t = D_{t-1}(1 + g) \qquad (18.16a)$$

$$= D_0(1 + g)^t. \qquad (18.16b)$$

18.3.1 Net Present Value

The impact of this assumption on Equation (18.7) can be analyzed by noting what happens when D_t is replaced by $D_0(1 + g)^t$ in the numerator:

$$V = \sum_{t=1}^{\infty} \frac{D_0(1 + g)^t}{(1 + k)^t}. \qquad (18.17)$$

Fortunately, Equation (18.17) can be simplified by noting that D_0 is a dollar amount, which means that it can be written outside the summation sign:

$$V = D_0\left[\sum_{t=1}^{\infty} \frac{(1+g)^t}{(1+k)^t}\right]. \quad (18.18)$$

The next step involves using a property of infinite series from mathematics. If $k > g$, then it can be shown that:

$$\sum_{t=1}^{\infty} \frac{(1+g)^t}{(1+k)^t} = \frac{1+g}{k-g}. \quad (18.19)$$

Substituting Equation (18.19) into Equation (18.18) results in the valuation formula for the constant-growth model:

$$V = D_0\left(\frac{1+g}{k-g}\right). \quad (18.20)$$

Sometimes Equation (18.20) is rewritten as:

$$V = \frac{D_1}{k-g} \quad (18.21)$$

because $D_1 = D_0(1 + g)$.

Example

As an example of how this DDM can be used, assume that during the past year the Copper Company paid dividends amounting to $1.80 per share. The forecast is that dividends on Copper stock will increase by 5% per year into the indefinite future. Thus dividends over the next year are expected to equal $1.89 [= $1.80 × (1 + .05)]. Using Equation (18.20) and assuming a required rate of return k of 11%, it can be seen that the value of a share of Copper stock is equal to $31.50 [= $1.80 × (1 + .05)/(.11 − .05) = $1.89/(.11 − .05)]. With a current stock price of $40 per share, Equation (18.2) would suggest that the NPV per share is −$8.50 (= $31.50 − $40). Equivalently, as $V = \$31.50 < P = \$40$, the stock is overpriced by $8.50 per share and would be a candidate for sale if currently owned.

18.3.2 Internal Rate of Return

Equation (18.20) can be reformulated to solve for the IRR on an investment in a constant-growth security. First, the current price of the security P is substituted for V and then k^* is substituted for k. These changes result in:

$$P = D_0\left(\frac{1+g}{k^*-g}\right). \quad (18.22)$$

which can be rewritten as:

$$k^* = \frac{D_0(1 + g)}{P} + g \tag{18.23a}$$

$$= \frac{D_1}{P} + g. \tag{18.23b}$$

Example

Applying this formula to the stock of Copper indicates that $k^* = 9.72\%$ {= [\$1.80 × (1 + .05)/\$40] + .05 = (\$1.89/\$40) + .05}. Because the required rate of return on Copper exceeds the IRR from an investment in Copper (11% > 9.72%), this method also indicates that Copper is overpriced.

18.3.3 Relationship to the Zero-Growth Model

The zero-growth model of the previous section can be shown to be a special case of the constant-growth model. In particular, if the growth rate g is assumed to be equal to zero, then dividends will be a fixed dollar amount forever, which is the same as saying that there will be zero growth. Letting $g = 0$ in Equations (18.20) and (18.23a) results in two equations that are identical to Equations (18.13) and (18.15a), respectively.

Even though the assumption of constant dividend growth may seem less restrictive than the assumption of zero dividend growth, it may still be viewed as unrealistic in many cases. However, as will be shown next, the constant-growth model is important because it is embedded in the multiple-growth model.

18.4 THE MULTIPLE-GROWTH MODEL

A more general DDM for valuing common stocks is the **multiple-growth model**. With this model, the focus is on a time in the future (denoted by T) after which dividends are expected to grow at a constant rate g. Although the investor is still concerned with forecasting dividends, these dividends do not need to have any specific pattern until this time, after which they will be assumed to have the specific pattern of constant growth. The dividends up until T ($D_1, D_2, D_3, \ldots, D_T$) will be forecast individually by the investor. (The investor also forecasts when this time T will occur.) Thereafter dividends are assumed to grow by a constant rate g that the investor must also forecast, meaning that:

$$D_{T+1} = D_T(1 + g)$$
$$D_{T+2} = D_{T+1}(1 + g) = D_T(1 + g)^2$$
$$D_{T+3} = D_{T+2}(1 + g) = D_T(1 + g)^3$$

…on. Figure 18.1 presents a time line of dividends and growth rates associated with the multiple-growth model.

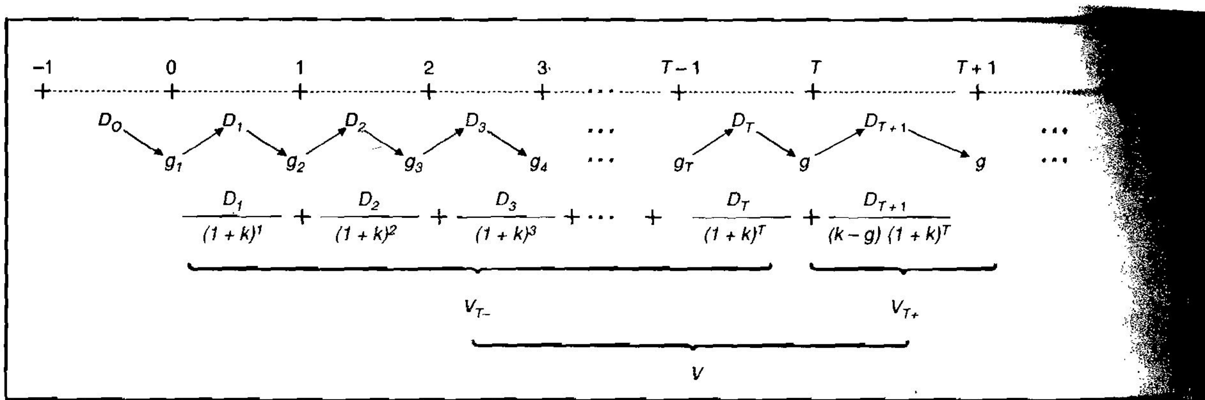

Figure 18.1
Time Line for Multiple-Growth Model

18.4.1 Net Present Value

In determining the value of a share of common stock with the multiple-growth model, the present value of the forecast stream of dividends must be determined. This can be done by dividing the stream into two parts, finding the present value of each part, and then adding these two present values together.

The first part consists of finding the present value of all the forecast dividends that will be paid up to and including time T. Denoting this present value by V_{T-}, it is equal to:

$$V_{T-} = \sum_{t=1}^{T} \frac{D_t}{(1+k)^t}. \tag{18.24}$$

The second part consists of finding the present value of all the forecast dividends that will be paid after time T, and involves the application of the constant-growth model. The application begins by imagining that the investor is not at time zero but is at time T, and has not changed his or her forecast of dividends for the stock. This means that the next period's dividend D_{T+1} and all those thereafter are expected to grow at the rate g. Thus the investor would be viewing the stock as having a constant growth rate, and its value at time T, V_T, could be determined with the constant-growth model of Equation (18.21):

$$V_T = D_{T+1}\left(\frac{1}{k-g}\right). \tag{18.25}$$

One way to view V_T is that it represents a lump sum that is just as desirable as the stream of dividends after T. That is, an investor would find a lump sum of cash equal to V_T, to be received at time T, to be equally desirable as the stream of dividends D_{T+1}, D_{T+2}, D_{T+3}, and so on. Now given that the investor is at time zero, not at time T, the present value at $t = 0$ of the lump sum V_T must be determined. This is done simply by discounting it for T periods at the rate k, resulting in the following formula for finding the present value at time zero for all dividends after T, denoted V_{T+}:

$$V_{T+} = V_T\left[\frac{1}{(1+k)^T}\right] \tag{18.26}$$

$$= \frac{D_{T+1}}{(k-g)(1+k)^T}.$$

Having found the present value of all dividends up to and including time T with Equation (18.24), and the present value of all dividends after time T with Equation (18.26), the value of the stock can be determined by summing up these two amounts:

$$V = V_{T-} + V_{T+}$$

$$= \sum_{t=1}^{T} \frac{D_t}{(1+k)^t} + \frac{D_{T+1}}{(k-g)(1+k)^T}. \tag{18.27}$$

Figure 18.1 illustrates the valuation procedure for the multiple-growth DDM that is given in Equation (18.27).

Example

As an example of how this DDM can be used, assume that during the past year the Magnesium Company paid dividends amounting to \$.75 per share. Over the next year, Magnesium is expected to pay [illegible] $_0)/D_0$ = (\$2 − \$.75)/\$.75 = 167%. The year after that, dividends are expected to amount to \$3 per share, indicating that $g_2 = (D_2 - D_1)/D_1$ = (\$3 − \$2)/\$2 = 50%. At this time, the forecast is that dividends will grow by 10% per year indefinitely, indicating that $T = 2$ and $g = 10\%$. Consequently, $D_{T+1} = D_3$ = \$3(1 + .10) = \$3.30. Given a required rate of return on Magnesium shares of 15%, the values of V_{T-} and V_{T+} can be calculated as follows:

$$V_{T-} = \frac{\$2}{(1+.15)^1} + \frac{\$3}{(1+.15)^2}$$

$$= \$4.01$$

$$V_{T+} = \frac{\$3.30}{(.15-.10)(1+.15)^2}$$

$$= \$49.91.$$

[illegible]ing V_{T-} and V_{T+} results in a value for V of \$4.01 + \$49.91 = \$53.92. With [illegible]ent stock price of \$55 per share, Magnesium appears to be fairly priced. [illegible] Magnesium is not significantly mispriced because V and P are nearly of [illegible]ize.

18.4.2 Internal Rate of Return

The zero-growth and constant-growth models have equations for V that [illegible] reformulated in order to solve for the IRR on an investment in a stock. U[illegible]nately, a convenient expression similar to Equations (18.15a), (18.15b), ([illegible] and (18.23b) is not available for the multiple-growth model. This can be [illegible] noting that the expression for IRR is derived by substituting P for V, and k^* [illegible] in Equation (18.27):

$$P = \sum_{t=1}^{T} \frac{D_t}{(1 + k^*)^t} + \frac{D_{T+1}}{(k^* - g)(1 + k^*)^T}. \qquad (18.28)$$

This equation cannot be rewritten with k^* isolated on the left-hand side, meaning that a closed-form expression for IRR does not exist for the multiple-growth model.

However, all is not lost. It is still possible to calculate the IRR for an investment in a stock conforming to the multiple-growth model by using an "educated" trial-and-error method. The basis for this method is in the observation that the right-hand side of Equation (18.28) is simply equal to the present value of the dividend stream, where k^* is used as the discount rate. Hence the larger the value of k^*, the smaller the value of the right-hand side of Equation (18.28). The trial-and-error method proceeds by initially using an estimate for k^*. If the resulting value on the right-hand side of Equation (18.28) is larger than P, then a larger estimate of k^* is tried. Conversely, if the resulting value is smaller than P, then a smaller estimate of k^* is tried. Continuing this search process, the investor can hone in on the value of k^* that makes the right-hand side equal P on the left-hand side. Fortunately, it is a relatively simple matter to program a computer to conduct the search for k^* in Equation (18.28). Most spreadsheets include a function that does so automatically.

Example

Applying Equation (18.28) to the Magnesium Company results in:

$$\$55 = \frac{\$2}{(1 + k^*)^1} + \frac{\$3}{(1 + k^*)^2} + \frac{\$3.30}{(k^* - .10)(1 + k^*)^2}. \qquad (18.29)$$

Initially a rate of 14% is used in attempting to solve this equation for k^*. Inserting 14% for k^* in the right-hand side of Equation (18.29) results in a value of \$67.54. Earlier 15% was used in determining V and resulted in a value of \$53.92. This means that k^* must have a value between 14% and 15%, since \$55 is between \$67.54 and \$53.92. If 14.5% is tried next, the resulting value is \$59.97, suggesting that a higher rate should be tried. If 14.8% and 14.9% are subsequently tried, the respective resulting values are \$56.18 and \$55.03. As \$55.03 is the closest to P, the IRR associated with an investment in Magnesium is 14.9%. Given a required return of 15% and an IRR of approximately that amount, the stock of Magnesium appears to be fairly priced.

18.4.3 Relationship to the Constant-Growth Model

The constant-growth model can be shown to be a special case of the multiple-growth model. In particular, if the time when constant growth is assumed to begin is set equal to zero, then:

$$V_{T-} = \sum_{t=1}^{T} \frac{D_t}{(1 + k)^t} = 0$$

and

$$V_{T+} = \frac{D_{T+1}}{(k - g)(1 + k)^T} = \frac{D_1}{k - g}$$

because $T = 0$ and $(1 + k)^0 = 1$. Given that the multiple-growth model states that $V = V_{T-} + V_{T+}$, it can be seen that setting $T = 0$ results in $V = D_1/(k - g)$, a formula that is equivalent to the formula for the constant-growth model.

18.4.4 Two-Stage and Three-Stage Models

Two dividend discount models that investors sometimes use are the two-stage model and the three-stage model.[8] The two-stage model assumes that a constant growth rate g_1 exists only until some time T, when a different growth rate g_2 is assumed to begin and continue thereafter. The three-stage model assumes that a constant growth rate g_1 exists only until some time T_1, when a second growth rate is assumed to begin and last until a later time T_2, when a third growth rate is assumed to begin and last thereafter. By letting V_{T+} denote the present value of all dividends after the last growth rate has begun and V_{T-} the present value of all the preceding dividends, it can be seen that these models are just special cases of the multiple-growth model.

In applying the capitalization of income method of valuation to common stocks, it might seem appropriate to assume that the stock will be sold at some point in the future. In this case the expected cash flows would consist of the dividends up to that point as well as the expected selling price. Because dividends after the selling date would be ignored, the use of a dividend discount model may seem to be improper. However, as will be shown next, this is not so.

18.5 VALUATION BASED ON A FINITE HOLDING PERIOD

capitalization of income method of valuation involves discounting all divi- that are expected throughout the future. Because the simplified models growth, constant growth, and multiple growth are based on this method, involve a future stream of dividends. Upon reflection it may seem that odels are relevant only for an investor who plans to hold a stock forever, uch an investor would expect to receive this stream of future dividends.

But what about an investor who plans to sell the stock in a year? In this situation, the cash flows that the investor expects to receive from purchasing a share of the stock are equal to the dividend expected to be paid one year from now (for ease of exposition, it is assumed that common stocks pay dividends annually) and the expected selling price of the stock. Thus it would seem appropriate to determine the intrinsic value of the stock to the investor by discounting these two cash flows at the required rate of return as follows:

$$V = \frac{D_1 + P_1}{1 + k}$$

$$= \frac{D_1}{1 + k} + \frac{P_1}{1 + k} \tag{18.30}$$

where D_1 and P_1 are the expected dividend and selling price at $t = 1$, respectively.

In order to use Equation (18.30), the expected price of the stock at $t = 1$ must be estimated. The simplest approach assumes that the selling price will be based on the dividends that are expected to be paid after the selling date. Thus the expected selling price at $t = 1$ is:

$$P_1 = \frac{D_2}{(1 + k)^1} + \frac{D_3}{(1 + k)^2} + \frac{D_4}{(1 + k)^3} + \cdots$$

$$= \sum_{t=2}^{\infty} \frac{D_t}{(1 + k)^{t-1}}. \tag{18.31}$$

Substituting Equation (18.31) for P_1 in the right-hand side of Equation (18.30) results in:

$$V = \frac{D_1}{1 + k} + \left[\frac{D_2}{(1 + k)^1} + \frac{D_3}{(1 + k)^2} + \frac{D_4}{(1 + k)^3} + \cdots\right]\left(\frac{1}{1 + k}\right)$$

$$= \frac{D_1}{(1 + k)^1} + \frac{D_2}{(1 + k)^2} + \frac{D_3}{(1 + k)^3} + \frac{D_4}{(1 + k)^4} + \cdots$$

$$= \sum_{t=1}^{\infty} \frac{D_t}{(1 + k)^t}$$

which is exactly the same as Equation (18.7). Thus valuing a share of common stock by discounting its dividends up to some point in the future and its expected selling price at that time is equivalent to valuing stock by discounting all future dividends. Simply stated, the two are equivalent because the expected selling price is itself based on dividends to be paid after the selling date. Thus Equation (18.7), as well as the zero-growth, constant-growth, and multiple-growth models that are based on it, is appropriate for determining the intrinsic value of a share of common stock regardless of the length of the investor's planned holding period.

Example

As an example, reconsider the common stock of the Copper Company. Over the past year it was noted that Copper paid dividends of $1.80 per share, with the forecast that the dividends would grow by 5% per year forever. This means that dividends over the next two years (D_1 and D_2) are forecast to be \$1.89 [= \$1.80 × (1 + .05)] and \$1.985 [= \$1.89 × (1 + .05)], respectively. If the investor plans to sell the stock after one year, the selling price could be estimated by noting that at $t = 1$, the forecast of dividends for the forthcoming year would be D_2, or \$1.985. Thus the anticipated selling price at $t = 1$, denoted P_1, would be equal to \$33.08 [= \$1.985/(.11 − .05)]. Accordingly, the intrinsic value of Copper to such an investor would equal the present value of the expected cash flows, which are D_1 = \$1.89 and P_1 = \$33.08. Using Equation (18.30) and assuming a required rate of 11%, this value is equal to \$31.50 [= (\$1.89 + \$33.08)/(1 + .11)]. Note that this is the same amount that was calculated earlier when all the dividends from now to infinity were discounted using the constant-growth model: $V = D_1/(k - g)$ = \$1.89/(.11 − .05) = \$31.50.

18.6 MODELS BASED ON PRICE-EARNINGS RATIOS

Despite the inherent sensibility of DDMs, many security analysts use a much simpler procedure to value common stocks. First, a stock's earnings per share over the forthcoming year E_1 are estimated, and then the analyst (or someone else) specifies a "normal" **price-earnings ratio** for the stock. The product of these two numbers gives the estimated future price P_1. Together with estimated dividends D_1 to be paid during the period and the current price P, the estimated return on the stock over the period can be determined:

$$\text{Expected return} = \frac{(P_1 - P) + D_1}{P} \tag{18.32}$$

where $P_1 = (P_1/E_1) \times E_1$.

Some security analysts expand this procedure, estimating earnings per share and price-earnings ratios for optimistic, most likely, and pessimistic scenarios to produce a rudimentary probability distribution of a security's return. Other analysts determine whether a stock is underpriced or overpriced by comparing the stock's actual price-earnings ratio with its "normal" price-earnings ratio, as will be shown next.[10]

In order to make this comparison, Equation (18.7) must be rearranged and some new variables introduced. To begin, it should be noted that earnings per share E_t are related to dividends per share D_t by the firm's **payout ratio** p_t,

$$D_t = p_t E_t. \tag{18.33}$$

Furthermore, if an analyst has forecast earnings-per-share and payout ratios, he or she has implicitly forecast dividends.

Equation (18.33) can be used to restate the various DDMs where the focus is on estimating what the stock's price-earnings ratio should be instead of on estimating the intrinsic value of the stock. In order to do so, $p_t E_t$ is substituted for D_t

in the right-hand side of Equation (18.7), resulting in a general formula for determining a stock's intrinsic value that involves discounting earnings:

$$V = \frac{D_1}{(1+k)^1} + \frac{D_2}{(1+k)^2} + \frac{D_3}{(1+k)^3} + \cdots$$

$$= \frac{p_1 E_1}{(1+k)^1} + \frac{p_2 E_2}{(1+k)^2} + \frac{p_3 E_3}{(1+k)^3} + \cdots$$

$$= \sum_{t=1}^{\infty} \frac{p_t E_t}{(1+k)^t}. \quad (18.34)$$

Earlier it was noted that dividends in adjacent time periods could be viewed as being "linked" to each other by a dividend growth rate g_t. Similarly, earnings per share in any year t can be "linked" to earnings per share in the previous year $t-1$ by a growth rate in earnings per share, g_{et},

$$E_t = E_{t-1}(1 + g_{et}). \quad (18.35)$$

This implies that

$$E_1 = E_0(1 + g_{e1})$$
$$E_2 = E_1(1 + g_{e2}) = E_0(1 + g_{e1})(1 + g_{e2})$$
$$E_3 = E_2(1 + g_{e3}) = E_0(1 + g_{e1})(1 + g_{e2})(1 + g_{e3})$$

and so on, where E_0 is the actual level of earnings per share over the past year, E_1 is the expected level of earnings per share over the forthcoming year, E_2 is the expected level of earnings per share for the year after E_1, and E_3 is the expected level of earnings per share for the year after E_2.

These equations relating expected future earnings per share to E_0 can be substituted into Equation (18.34), resulting in:

$$V = \frac{p_1[E_0(1 + g_{e1})]}{(1+k)^1} + \frac{p_2[E_0(1 + g_{e1})(1 + g_{e2})]}{(1+k)^2} + \frac{p_3[E_0(1 + g_{e1})(1 + g_{e2})(1 + g_{e3})]}{(1+k)^3} + \cdots. \quad (18.36)$$

As V is the intrinsic value of a share of stock, it represents what the stock would be selling for if it were fairly priced. It follows that V/E_0 represents what the price-earnings ratio would be if the stock were fairly priced, and is sometimes referred to as the stock's "normal" price-earnings ratio. Dividing both sides of Equation (18.36) by E_0 and simplifying results in the formula for determining the "normal" price-earnings ratio:

$$\frac{V}{E_0} = \frac{p_1(1 + g_{e1})}{(1+k)^1} + \frac{p_2(1 + g_{e1})(1 + g_{e2})}{(1+k)^2} + \frac{p_3(1 + g_{e1})(1 + g_{e2})(1 + g_{e3})}{(1+k)^3} + \cdots. \quad (18.37)$$

This shows that, other things being equal, a stock's "normal" price-earnings ratio will be higher:

The *greater* the expected payout ratios ($p_1, p_2, p_3, \ldots$),
The *greater* the expected growth rates in earnings per share ($g_{e1}, g_{e2}, g_{e3}, \ldots$),
The *smaller* the required rate of return (k).

The qualifying phrase "other things being equal" should not be overlooked. For example, a firm cannot increase the value of its shares by simply making greater payouts. This will increase $p_1, p_2, p_3, \ldots$, but will decrease the expected growth rates in earnings per share $g_{e1}, g_{e2}, g_{e3}, \ldots$. Assuming that the firm's investment policy is not altered, the effects of the reduced growth in its earnings per share will just offset the effects of the increased payouts, leaving its share value unchanged.

Earlier it was noted that a stock was viewed as underpriced if $V > P$ and overpriced if $V < P$. Because dividing both sides of an inequality by a positive constant will not change the direction of the inequality, such a division can be done here to the two inequalities involving V and P, where the positive constant is E_0. The result is that a stock can be viewed as being underpriced if $V/E_0 > P/E_0$ and overpriced if $V/E_0 < P/E_0$. Thus a stock will be underpriced if its "normal" price-earnings ratio is greater than its actual price-earnings ratio, and overpriced if its "normal" price-earnings ratio is less than its actual price-earnings ratio.

Unfortunately, Equation (18.37) is intractable, meaning that it cannot be used to estimate the "normal" price-earnings ratio for any stock. However, simplifying assumptions can be made that result in tractable formulas for estimating "normal" price-earnings ratios. These assumptions, along with the formulas, parallel those made previously regarding dividends and are discussed next.

18.6.1 The Zero-Growth Model

The zero-growth model assumed that dividends per share remained at a fixed dollar amount forever. This is most likely if earnings per share remain at a fixed dollar amount forever, with the firm maintaining a 100% payout ratio. Why 100%? Because if a lesser amount were assumed to be paid out, it would mean that the firm was retaining part of its earnings. These retained earnings would be put to some use, and would thus be expected to increase future earnings and hence dividends per share.

Accordingly, the zero-growth model can be interpreted as assuming $p_t = 1$ for all time periods and $E_0 = E_1 = E_2 = E_3$ and so on. This means that $D_0 = E_0$ = $D_1 = E_1 = D_2 = E_2$ and so on, allowing valuation Equation (18.13) to be restated as:

$$V = \frac{E_0}{k}. \tag{18.38}$$

Dividing Equation (18.38) by E_0 results in the formula for the "normal" price-earnings ratio for a stock having zero growth:

$$\frac{V}{E_0} = \frac{1}{k}. \tag{18.39}$$

Example

Earlier it was assumed that the Zinc Company was a zero-growth firm idends of $8 per share, selling for $65 a share, and having a required turn of 10%. Because Zinc is a zero-growth company, it will be assum has a 100% payout ratio which, in turn, means that E_0 = $8. At this po tion (18.38) can be used to note that a "normal" price-earnings ratio for $1/.10 = 10$. As Zinc has an actual price-earnings ratio of $65/$8 = 8.1, cause $V/E_0 = 10 > P/E_0 = 8.1$, it can be seen that Zinc stock is underpric

18.6.2 The Constant-Growth Model

Earlier it was noted that dividends in adjacent time periods could be viewed being connected to each other by a dividend growth rate g_t. Similarly, it was noted that earnings per share can be connected by an earnings growth rate g_e. The constant-growth model assumes that the growth rate in dividends per share will be the same throughout the future. An equivalent assumption is that earnings per share will grow at a constant rate g_e throughout the future, with the payout ratio remaining at a constant level p. This means that:

$$E_1 = E_0(1 + g_e) = E_0(1 + g_e)^1$$
$$E_2 = E_1(1 + g_e) = E_0(1 + g_e)(1 + g_e) = E_0(1 + g_e)^2$$
$$E_3 = E_2(1 + g_e) = E_0(1 + g_e)(1 + g_e)(1 + g_e) = E_0(1 + g_e)^3$$

and so on. In general, earnings in year t can be connected to E_0 as follows:

$$E_t = E_0(1 + g_e)^t. \tag{18.40}$$

Substituting Equation (18.40) into the numerator of Equation (18.34) and recognizing that $p_t = p$ results in:

$$V = \sum_{t=1}^{\infty} \frac{pE_0(1 + g_e)^t}{(1 + k)^t}$$

$$= pE_0\left[\sum_{t=1}^{\infty} \frac{(1 + g_e)^t}{(1 + k)^t}\right]. \tag{18.41}$$

The same mathematical property of infinite series given in Equation (18.19) can be applied to Equation (18.41), resulting in:

$$V = pE_0\left(\frac{1 + g_e}{k - g_e}\right). \tag{18.42}$$

It can be noted that the earnings-based constant-growth model has a numerator that is identical to the numerator of the dividend-based constant-growth model, because $pE_0 = D_0$. Furthermore, the denominators of the two models are identical. Both assertions require that the growth rates in earnings and dividends be the same (that is, $g_e = g$). Examination of the assumptions of the models reveals that these growth rates must be equal. This can be seen by recalling that constant earnings growth means:

$$E_t = E_{t-1}(1 + g_e).$$

Now when both sides of this equation are multiplied by the constant payout ratio, the result is:

$$pE_t = pE_{t-1}(1 + g_e).$$

Because $pE_t = D_t$ and $pE_{t-1} = D_{t-1}$, this equation reduces to:

$$D_t = D_{t-1}(1 + g_e)$$

which indicates that dividends in any period $t - 1$ will grow by the earnings growth rate, g_e. Because the dividend-based constant-growth model assumed that dividends in any period $t - 1$ would grow by the dividend growth rate g, it can be seen that the two growth rates must be equal for the two models to be equivalent.

Equation (18.42) can be restated by dividing each side by E_0, resulting in the following formula for determining the "normal" price-earnings ratio for a stock with constant growth:

$$\frac{V}{E_0} = p\left(\frac{1 + g_e}{k + g_e}\right). \tag{18.43}$$

Example

Earlier it was assumed that the Copper Company had paid dividends of \$1.80 per share over the past year, with a forecast that dividends would grow by 5% per year forever. Furthermore, it was assumed that the required rate of return on Copper was 11%, and the current stock price was \$40 per share. Now assuming that E_0 was \$2.70, it can be seen that the payout ratio was equal to 66⅔% (= \$1.80/\$2.70). This means that the "normal" price-earnings ratio for Copper, according to Equation (18.43), is equal to 11.7 [= .6667 × (1 + .05) /(.11 − .05)]. Because this is less than Copper's actual price-earnings ratio of 14.8 (= \$40/\$2.70), it follows that the stock of Copper Company is overpriced.

18.6.3 The Multiple-Growth Model

Earlier it was noted that the most general DDM is the multiple-growth model, where dividends are allowed to grow at varying rates until some point in time T, after which they are assumed to grow at a constant rate. In this situation the present value of all the dividends is found by adding the present value of all dividends up to and including T, denoted by V_{T-}, and the present value of all dividends after T, denoted by V_{T+}:

$$V = V_{T-} + V_{T+}$$
$$= \sum_{t=1}^{T} \frac{D_t}{(1 + k)^t} + \frac{D_{T+1}}{(k - g)(1 + k)^T}. \tag{18.27}$$

In general, earnings per share in any period t can be expressed as equal to E_0 times the product of all the earnings growth rates from time zero to time t:

$$E_t = E_0(1 + g_{e1})(1 + g_{e2}) \cdots (1 + g_{et}). \qquad (18.44)$$

Because dividends per share in any period t are equal to the payout ratio for that period times the earnings per share, it follows from Equation (18.44) that:

$$\begin{aligned} D_t &= p_t E_t \\ &= p_t E_0(1 + g_{e1})(1 + g_{e2}) \cdots (1 + g_{et}). \end{aligned} \qquad (18.45)$$

Replacing the numerator in Equation (18.37) with the right-hand side of Equation (18.45) and then dividing both sides by E_0 gives the following formula for determining a stock's "normal" price-earnings ratio with the multiple-growth model:

$$\begin{aligned} \frac{V}{E_0} &= \frac{p_1(1 + g_{e1})}{(1 + k)^1} + \frac{p_2(1 + g_{e1})(1 + g_{e2})}{(1 + k)^2} + \cdots \\ &+ \frac{p_T(1 + g_{e1})(1 + g_{e2}) \cdots (1 + g_{eT})}{(1 + k)^T} \\ &+ \frac{p(1 + g_{e1})(1 + g_{e2}) \cdots (1 + g_{eT})(1 + g)}{(k - g)(1 + k)^T}. \end{aligned} \qquad (18.46)$$

Example

Consider the Magnesium Company again. Its share price is currently \$55, and per share earnings and dividends over the past year were \$3 and \$.75, respectively. For the next two years, forecast earnings and dividends, along with the earnings growth rates and payout ratios, are:

$$\begin{array}{llll} D_1 = \$2.00 & E_1 = \$5.00 & g_{e1} = 67\% & p_1 = 40\% \\ D_2 = \$3.00 & E_2 = \$6.00 & g_{e2} = 20\% & p_2 = 50\%. \end{array}$$

Constant growth in dividends and earnings of 10% per year is forecast to begin at $T = 2$, which means that $D_3 = \$3.30$, $E_3 = \$6.60$, $g = 10\%$, and $p = 50\%$.

Given a required return of 15%, Equation (18.46) can be used as follows to estimate a "normal" price-earnings ratio for Magnesium:

$$\begin{aligned} \frac{V}{E_0} &= \frac{.40(1 + .67)}{(1 + .15)^1} + \frac{.50(1 + .67)(1 + .20)}{(1 + .15)^2} + \frac{.50(1 + .67)(1 + .20)(1 + .10)}{(.15 - .10)(1 + .15)^2} \\ &= .58 + .76 + 16.67 \\ &= 18.01. \end{aligned}$$

Because the actual price-earnings ratio of 18.33 (= \$55/\$3) is close to the "normal" ratio of 18.01, the stock of the Magnesium Company can be viewed as fairly priced.

So far no explanation has been given as to why earnings or dividends will be expected to grow in the future. One way of providing such an explanation uses the constant-growth model. Assuming that no new capital is obtained externally and no shares are repurchased (meaning that the number of shares outstanding does not increase or decrease), the portion of earnings not paid to stockholders as dividends will be used to pay for the firm's new investments. Given that p_t denotes the payout ratio in year t, then $(1 - p_t)$ will be equal to the portion of earnings not paid out, known as the **retention ratio.** Furthermore, the firm's new investments, stated on a per-share basis and denoted by I_t, will be:

$$I_t = (1 - p_t)E_t. \tag{18.47}$$

If these new investments have an average return on equity of r_t in period t and every year thereafter, they will add $r_t I_t$ to earnings per share in year $t + 1$ and every year thereafter. If all previous investments also produce perpetual earnings at a constant rate of return, next year's earnings will equal this year's earnings plus the new earnings resulting from this year's new investments:

$$\begin{aligned} E_{t+1} &= E_t + r_t I_t \\ &= E_t + r_t(1 - p_t)E_t \\ &= E_t[1 + r_t(1 - p_t)]. \end{aligned} \tag{18.48}$$

Because it was shown earlier that the growth rate in earnings per share is:

$$E_t = E_{t-1}(1 + g_{et}) \tag{18.35}$$

it follows that:

$$E_{t+1} = E_t(1 + g_{et+1}). \tag{18.49}$$

A comparison of Equations (18.48) and (18.49) indicates that:

$$g_{et+1} = r_t(1 - p_t). \tag{18.50}$$

If the growth rate in earnings per share g_{et+1} is to be constant over time, then the average return on equity for new investments r_t and the payout ratio p_t must also be constant over time. In this situation Equation (18.50) can be simplified by removing the time subscripts:

$$g_e = r(1 - p). \tag{18.51a}$$

Because the growth rate in dividends per share g is equal to the growth rate in earnings per share g_e, this equation can be rewritten as:

$$g = r(1 - p). \tag{18.51b}$$

From this equation it can be seen that the growth rate g depends on (1) the proportion of earnings that is retained $1 - p$, and (2) the average return on equity on the earnings that are retained r.

The constant-growth valuation formula given in Equation (18.20) can be modified by replacing g with the expression on the right-hand side of Equation (18.51b), resulting in:

$$V = D_0\left(\frac{1 + g}{k - g}\right)$$

$$= D_0\left[\frac{1 + r(1 - p)}{k - r(1 - p)}\right].$$

$$= D_1\left[\frac{1}{k - r(1 - p)}\right].$$

Under these assumptions, a stock's value (and hence its price) should be greater, the greater its average return on equity for new investments, other things being equal.

Example

Continuing with the Copper Company, recall that $E_0 = \$2.70$ and $p = 66⅔\%$. This means that 33⅓% of earnings per share over the past year were retained and reinvested, an amount equal to \$.90 (= .3333 × \$2.70). The earnings per share in the forthcoming year E_1 are expected to be \$2.835 [= \$2.70 × (1 + .05)] because the growth rate g for Copper is 5%.

The source of the increase in earnings per share of \$.135 (= \$2.835 − \$2.70) is the \$.90 per share that was reinvested at $t = 0$. The average return on equity for new investments r is 15%, because \$.135/\$.90 = 15%. That is, the reinvested earnings of \$.90 per share can be viewed as having generated an annual increase in earnings per share of \$.135. This increase will occur not only at $t = 1$, but also at $t = 2$, $t = 3$, and so on. Equivalently, a \$.90 investment at $t = 0$ will generate a perpetual annual cash inflow of \$.135 beginning at $t = 1$.

Expected dividends at $t = 1$ can be calculated by multiplying the expected payout ratio p of 66⅔% times the expected earnings per share E_1 of \$2.835, or .6667 × \$2.835 = \$1.89. It can also be calculated by multiplying 1 plus the growth rate g of 5% times the past amount of dividends per share D_0 of \$1.80, or 1.05 × \$1.80 = \$1.89.

It can be seen that the growth rate in dividends per share of 5% is equal to the product of the retention rate (33⅓%) and the average return on equity for new investments (15%), an amount equal to 5% (= .3333 × .15).

Two years from now ($t = 2$), earnings per share are anticipated to be \$2.977 [= \$2.835 × (1 + .05)], a further increase of \$.142 (= \$2.977 − \$2.835) that is due to the retention and reinvestment of \$.945 (= .3333 × \$2.835) per share at $t = 1$. This expected increase in earnings per share of \$.142 is the result of earning (15%) on the reinvestment (\$.945), because .15 × \$.945 = \$.142.

The expected earnings per share at $t = 2$ can be viewed as having three components. The first is the earnings attributable to the assets held at $t = 0$, an amount equal to \$2.70. The second is the earnings attributable to the reinvestment of \$.90 at $t = 0$, earning \$.135. The third is the earnings attributable to the reinvestment of \$.945 at $t = 1$, earning \$.142. These three components, when summed, can be seen to equal $E_2 = \$2.977$ (= \$2.70 + \$.135 + \$.142).

Dividends at $t = 2$ are expected to be 5% larger than at $t = 1$, or \$1.985 (= 1.05 × \$1.89) per share. This amount corresponds to the amount calculated by multiplying the payout ratio times the expected earnings per share at $t = 2$, or \$1.985 (= .6667 × \$2.977). Figure 18.2 summarizes the example.

−1		0		1		2	→ ∞
	E_0 = \$2.70			\$2.700		\$2.700	...
			\$.90 x .15 =	.135		.135	...
			E_1 =	\$2.835	\$.945 x .15 =	.142	...
					E_2 =	\$2.977	...
	I_0 = \$ .90		I_1 =	\$ .945	I_2 =	\$ .992	...
	D_0 = 1.80		D_1 =	1.890	D_2 =	1.985	...
	E_0 = \$2.70		E_1 =	\$2.835	E_2 =	\$2.977	...

Figure 18.2
Growth in Earnings for Copper Company

18.8 A THREE-STAGE DDM

As this chapter's Institutional Issues discusses, the three-stage DDM is the most widely applied form of the general multiple-growth DDM. Consider analyzing the *ABC* Company.

18.8.1 Making Forecasts

Over the past year, *ABC* has had earnings per share of \$1.67 and dividends per share of \$.40. After carefully studying ABC, the security analyst has made the following forecasts of earnings per share and dividends per share for the next five years:

$E_1 = \$2.67$ $\quad E_2 = \$4.00$ $\quad E_3 = \$6.00$ $\quad E_4 = \$8.00$ $\quad E_5 = \$10.00$
$D_1 = \$\ .60$ $\quad D_2 = \$1.60$ $\quad D_3 = \$2.40$ $\quad D_4 = \$3.20$ $\quad D_5 = \$\ 5.00.$

These forecasts imply the following payout ratios and earnings-per-share growth rates:

$p_1 = 22\%$ $\quad p_2 = 40\%$ $\quad p_3 = 40\%$ $\quad p_4 = 40\%$ $\quad p_5 = 50\%$
$g_{e1} = 60\%$ $\quad g_{e2} = 50\%$ $\quad g_{e3} = 50\%$ $\quad g_{e4} = 33\%$ $\quad g_{e5} = 25\%.$

Furthermore, the analyst believes that *ABC* will enter a transition stage at the end of the fifth year (that is, the sixth year will be the first year of the transition stage), and that the transition stage will last three years. Earnings per share and payout ratio for year 6 are forecast to be $E_6 = \$11.90$ and $p_6 = 55\%$. {Thus g_{e6} = 19% [= (\$11.90 − \$10.00)/\$10.00] and $D_6 = \$6.55$ (= .55 × \$11.90)}. The last stage, known as the maturity stage, is forecast to have an earnings-per-share growth rate of 4% and a payout ratio of 70%. Now it was shown in Equation (18.51b) that with the constant-growth model, $g = r(1 - p)$, where r is the average return on equity for new investment and p is the payout ratio. Given

Applying Dividend Discount Models

Over the last 30 years, dividend discount models (DDMs) have achieved broad acceptance among professional common stock investors. Although few investment managers rely solely on DDMs to select stocks, many have integrated DDMs into their security valuation procedures.

The reasons for the popularity of DDMs are twofold. First, DDMs are based on a simple, widely understood concept: The fair value of any security should equal the discounted value of the cash flows expected to be produced by that security. Second, the basic inputs for DDMs are standard outputs for many large investment management firms—that is, these firms employ security analysts who are responsible for projecting corporate earnings.

Valuing common stocks with a DDM technically requires an estimate of future dividends over an infinite time horizon. Given that accurately forecasting dividends three years from today, let alone 20 years in the future, is a difficult proposition, how do investment firms actually go about implementing DDMs?

One approach is to use constant or two-stage dividend growth models, as described in the text. However, although such models are relatively easy to apply, institutional investors typically view the assumed dividend growth assumptions as overly simplistic. Instead, these investors generally prefer three-stage models, believing that they provide the best combination of realism and ease of application.

Whereas many variations of the three-stage DDM exist, in general, the model is based on the assumption that companies evolve through three stages during their lifetimes. (Figure 18.3 portrays these stages.)

1. **Growth stage:** Characterized by rapidly expanding sales, high profit margins, and abnormally high growth in earnings per share. Because of highly profitable expected investment opportunities, the payout ratio is low. Competitors are attracted by the unusually high earnings, leading to a decline in the growth rate.
2. **Transition stage:** In later years, increased competition reduces profit margins and earnings growth slows. With fewer new investment opportunities, the company begins to pay out a larger percentage of earnings.

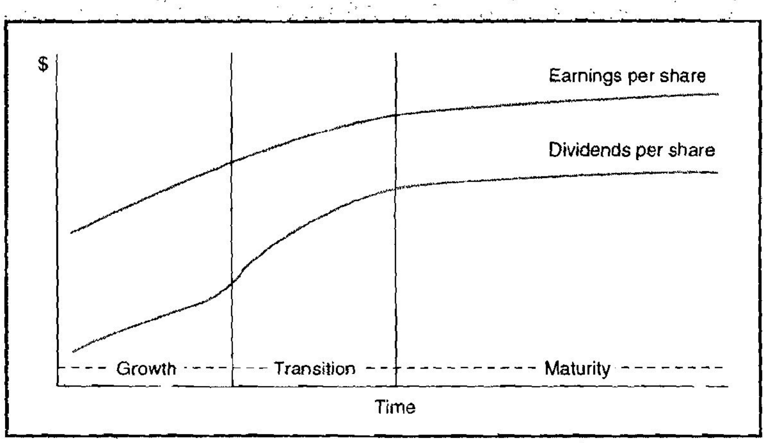

Figure 18.3
The Three Stages of the Multiple-Growth Model
Source: Adapted from Carmine J. Grigoli, "Demystifying Dividend Discount Models," *Merrill Lynch Quantitative Research*, April 1982.

3. **Maturity (steady-state) stage:** Eventually the company reaches a position where its new investment opportunities offer, on average, only slightly attractive returns on equity. At that time its earnings growth rate, payout ratio, and return on equity stabilize for the remainder of its life.

The forecasting process of the three-stage DDM involves specifying earnings and dividend growth rates in each of the three stages. Although one cannot expect a security analyst to be omniscient in his or her growth forecast for a particular company, one can hope that the forecast pattern of growth—in terms of magnitude and duration—resembles that actually realized by the company, particularly in the short run.

Investment firms attempt to structure their DDMs to make maximum use of their analysts' forecasting capabilities. Thus the models emphasize specific forecasts in the near term, when it is realistic to expect security analysts to project earnings and dividends more accurately. Conversely, the models emphasize more general forecasts over the longer term, when distinctions between companies' growth rates become less discernible. Typically, analysts are required to supply the following for their assigned companies:

1. expected annual earnings and dividends for the next several years;
2. after these specific annual forecasts end, earnings growth and the payout ratio forecasts until the end of the growth stage;
3. the number of years until the transition stage is reached;
4. the duration (in years) of the transition stage—that is, once abnormally high growth ends, the number of years until the maturity stage is reached.

Most three-stage DDMs assume that during the transition stage, earnings growth declines and payout ratios rise linearly to the maturity-stage steady-state levels. (For example, if the transition stage is ten years long, earnings growth at the maturity stage is 5% per year, and earnings growth at the end of the growth stage is 25%, then earnings growth will decline 2% in each year of the transition stage.) Finally, most three-stage DDMs make standard assumptions that all companies in the maturity stage have the same growth rates, payout ratios, and return on equity.

With analysts' inputs, plus an appropriate required rate of return for each security, all the necessary information for the three-stage DDM is available. The last step involves merely calculating the discounted value of the estimated dividends to determine the stock's "fair" value.

The seeming simplicity of the three-stage DDM should not lead one to believe that it is without its implementation problems. Investment firms must strive to achieve consistency across their analysts' forecasts. The long-term nature of the estimates involved, the substantial training required to make even short-term earnings forecasts accurately, and the coordination of a number of analysts covering many companies severely complicate the problem. Considerable discipline is required if the DDM valuations generated by a firm's analysts are to be sufficiently comparable and reliable to guide investment decisions. Despite these complexities, if successfully implemented, DDMs can combine the creative insights of security analysts with the rigor and discipline of quantitative investment techniques.

that the maturity stage has constant growth, this equation can be reformulated and used to determine r:

$$r = g/(1 - p).$$

Thus r for *ABC* has an implied value of 13.33% [= 4%/(100% − 70%)], which is assumed to be consistent with the long-run growth forecasts for similar companies.

At this point there are only two missing pieces of information that are needed to determine the value of *ABC*—the earnings-per-share growth rates and the payout ratios for the transition stage. Taking earnings per share first, forecast that $g_{e6} = 19\%$ and $g_{e9} = 4\%$. One method of determining "decay" to 4% is to note that there are three years between the sixth years, and 15% between 19% and 4%. A "linear decay" rate would be d by noting that 15%/3 years = 5% per year. This rate of 5% would be from 19% to get g_{e7}, resulting in 14% (= 19% − 5%). Then it would be ed from 14% to get g_{e8}, resulting in 9% (= 14% − 5%). Finally, as a check be noted that 4% (= 9% − 5%) is the value that was forecast for g_{e9}.

A similar procedure can be used to determine how the payout ratio o in year 6 will grow to 70% in year 9. The "linear growth" rate will be (70 55%)/3 years = 15%/3 years = 5% per year, indicating that p_7 = 60% (= 5 + 5%) and p_8 = 65% (= 60% + 5%). Again a check indicates that 70% (65% + 5%) is the value that was forecast for p_9.

With these forecasts of earnings-per-share growth rates and payout ratios in hand, forecasts of dividends per share can now be made:

$$
\begin{aligned}
D_7 &= p_7E_7 \\
&= p_7E_6(1 + g_{e7}) \\
&= .60 \times \$11.90 \times (1 + .14) \\
&= .60 \times \$13.57 \\
&= \$8.14 \\
D_8 &= p_8E_8 \\
&= p_8E_6(1 + g_{e7})(1 + g_{e8}) \\
&= .65 \times \$11.90 \times (1 + .14) \times (1 + .09) \\
&= .65 \times \$14.79 \\
&= \$9.61 \\
D_9 &= p_9E_9 \\
&= p_9E_6(1 + g_{e7})(1 + g_{e8})(1 + g_{e9}) \\
&= .70 \times \$11.90 \times (1 + .14) \times (1 + .09) \times (1 + .04) \\
&= .70 \times \$15.38 \\
&= \$10.76.
\end{aligned}
$$

18.8.2 Estimating the Intrinsic Value

Given a required rate of return on *ABC* of 12.4%, all the necessary inputs for the multiple-growth model have been determined. Hence it is now possible to estimate *ABC*'s intrinsic (or fair) value. To begin, it can be seen that $T = 8$, indicating that V_{T-} involves determining the present value of D_1 through D_8,

$$
\begin{aligned}
V_{T-} &= \left[\frac{\$.60}{(1 + .124)^1}\right] + \left[\frac{\$1.60}{(1 + .124)^2}\right] + \left[\frac{\$2.40}{(1 + .124)^3}\right] \\
&+ \left[\frac{\$3.20}{(1 + .124)^4}\right] + \left[\frac{\$5.00}{(1 + .124)^5}\right] + \left[\frac{\$6.55}{(1 + .124)^6}\right] \\
&+ \left[\frac{\$8.14}{(1 + .124)^7}\right] + \left[\frac{\$9.61}{(1 + .124)^8}\right] \\
&= \$18.89.
\end{aligned}
$$

Then V_{T+} can be determined using D_9:

$$V_{T+} = \frac{\$10.76}{(.124 - .04)(1 + .124)^8}$$
$$= \$50.28.$$

Combining V_{T-} and V_{T+} results in the intrinsic value of *ABC:*

$$V = V_{T-} + V_{T+}$$
$$= \$18.89 + \$50.28$$
$$= \$69.17.$$

Given a current market price for *ABC* of \$50, it can be seen that its stock is underpriced by \$19.17 (= \$69.17 − \$50) per share. Equivalently, it can be noted that the actual price-earnings ratio for *ABC* is 29.9 (= \$50/\$1.67) but that a "normal" price-earnings ratio would be higher, equal to 41.4 (= \$69.17/\$1.67), again indicating that *ABC* is underpriced.

18.8.3 Implied Returns

As shown with the previous example, once the analyst has made certain forecasts, it is relatively straightforward to determine a company's expected dividends for each year up through the first year of the maturity stage. Then the present value of these predicted dividends can be calculated for a given required rate of return. However, many investment firms use a computerized trial-and-error procedure to determine the discount rate that equates the present value of the stock's expected dividends with its current price. Sometimes this long-run internal rate of return is referred to as the security's **implied return.** In the case of *ABC*, its implied return is 14.8%.

18.8.4 The Security Market Line

After implied returns have been estimated for a number of stocks, the associated beta for each stock can be estimated. Then for all the stocks analyzed, this information can be plotted on a graph that has implied returns on the vertical axis and estimated betas on the horizontal axis.

At this point there are alternative methods for estimating the security market line (SML).[11] One method involves determining a line of best fit for this graph by using a statistical procedure known as simple regression (as discussed in Chapter 17). That is, the values of an intercept term and a slope term are determined from the data, thereby indicating the location of the straight line that best describes the relationship between implied returns and betas.[12]

Figure 18.4 provides an example of the estimated SML. In this case the SML has been determined to have an intercept of 8% and a slope of 4%, indicating that, in general, securities with higher betas are expected to have higher implied returns in the forthcoming period. Depending on the sizes of the implied returns, such lines can have steeper or flatter slopes, or even negative slopes.

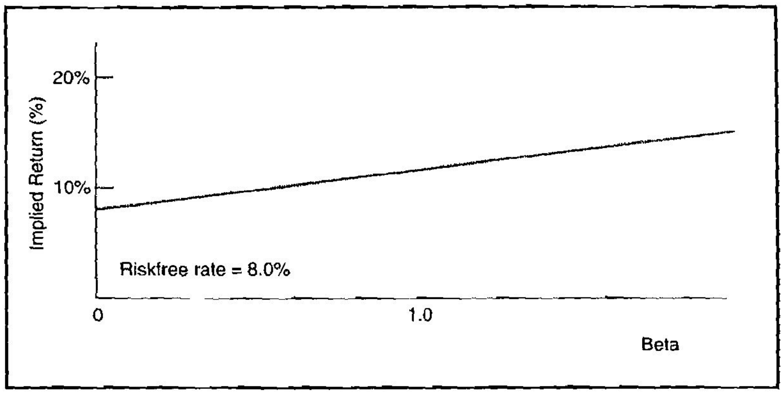

Figure 18.4
A Security Market Line Estimated from Implied Returns

The second method of estimating the SML involves calculating the implied return for a portfolio of common stocks. This is done by taking a value-weighted average of the implied returns of the stocks in the portfolio, with the resulting return being an estimate of the implied return on the market portfolio. Given this return and a beta of 1, the "market" portfolio can be plotted on a graph having implied returns on the vertical axis and betas on the horizontal axis. Next the riskfree rate, having a beta of 0, can be plotted on the same graph. Finally, the SML is determined by simply connecting these two points with a straight line.

Either of these SMLs can be used to determine the required return on a stock. However, they will most likely result in different numbers, as the two lines will most likely have different intercepts and slopes. For example, note that in the first method the SML may not go through the riskfree rate, whereas the second method forces the SML to go through this rate.

18.8.5 Required Returns and Alphas

Once a security's beta has been estimated, its required return can be determined from the estimated SML. For example, the equation for the SML shown in Figure 18.4 is:

$$k_i = 8 + 4\beta_i.$$

Thus if *ABC* has an estimated beta of 1.1, then it would have a required return equal to 12.4% [= 8 + (4 × 1.1)].

Once the required return on a stock has been determined, the difference between the stock's implied return (from the DDM) and this required return can be calculated. This difference is then viewed as an estimate of the stock's *alpha* and represents "... the degree to which a stock is mispriced. Positive alphas indicate undervalued securities and negative alphas indicate overvalued securities."[13] In the case of *ABC*, its implied and required returns were 14.8% and 12.4%, respectively. Thus its estimated alpha would be 2.4% (= 14.8% − 12.4%). Because this is a positive number, *ABC* can be viewed as being underpriced.

18.8.6 The Implied Return on the Stock Market

Another product of this analysis is that the implied return for a portfolio of stocks can be compared with the expected return on bonds. (The latter is typically represented by the current yield-to-maturity on long-term Treasury bonds.) Specifically, the difference between stock and bond returns can be used as an input for recommendations concerning asset allocation between stocks and bonds. That is, it can be used to form recommendations regarding what percent of an investor's money should go into stocks and what percent should go into bonds. For example, the greater the implied return on stocks relative to bonds, the larger the percentage of the investor's money that should be placed in common stocks.

18.9 DIVIDEND DISCOUNT MODELS AND EXPECTED RETURNS

The procedures described here are similar to those employed by a number of brokerage firms and portfolio managers.[14] A security's implied return, obtained from a DDM, is often treated as an expected return, which in turn can be divided into two components—the security's required return and alpha.

However, the expected return on a stock over a given holding period may differ from its DDM-based implied rate k^*. A simple set of examples will indicate why this difference can exist.

Assume that a security analyst predicts that a stock will pay a dividend of $1.10 per year forever. On the other hand, the consensus opinion of "the market" (most other investors) is that the dividend will equal $1.00 per year forever. This suggests that the analyst's prediction is a deviant or nonconsensus one.

Assume that both the analyst and other investors agree that the required rate of return for a stock of this type is 10%. Using the formula for the zero-growth model, the value of the stock is $D_1/.10 = 10D_1$, meaning that the stock should sell for ten times its expected dividend. Because other investors expect to receive $1.00 per year, the stock has a current price P of $10 per share. The analyst feels that the stock has a value of $1.10/.10 = $11 and thus feels that it is underpriced by $11 − $10 = $1 per share.

18.9.1 Rate of Convergence of Investors' Predictions

[illegible] this situation the implied return according to the analyst is $1.10/$10 = 11%. [illegible] the analyst buys a share now with a plan to sell it a year later, what rate of re[illegible] might the analyst expect to earn? The answer depends on what assumption [illegible] made regarding the *rate of convergence of investors' predictions*—that is, the an[illegible] depends on the expected market reaction to the mispricing that the analyst [illegible] currently exists.

[illegible] cases shown in Table 18.1 are based on an assumption that the analyst is [illegible] that his or her forecast of future dividends is correct. That is, in all of [illegible] the analyst expects that at the end of the year, the stock will pay the [illegible] dividend of $1.10.

TABLE 18.1

ALPHA AND THE CONVERGENCE OF PREDICTIONS

	Expected Amount of Convergence		
	0% (A)	100% (B)	50% (C)
Dividend predictions D_2			
Consensus of other investors	1.00	1.10	1.05
Analyst	1.10	1.10	1.10
Expected stock price P_1	10.00	11.00	10.50
Expected return:			
Dividence yield D_1/P	11%	11%	11%
Capital gain $(P_1 - P)/P$	0	10	5
Total expected return	11%	21%	16%
Less required return	10	10	10
Alpha	1%	11%	6%

Note: P_1 is equal to the consensus dividend prediction at $t = 1$ divided by the required return of 10%. The example assumes that the current stock price P is \$10, and dividends are forecast by the consensus at $t = 0$ to remain constant at \$1.00 per share, whereas the analyst forecasts the dividends at $t = 0$ to remain constant at \$1.10 per share.

No Convergence

In column (A), it is assumed that other investors will regard the higher dividend as a fluke and steadfastly refuse to alter their projections of subsequent dividends from their initial estimate of \$1.00. As a result, the security's price at $t = 1$ can be expected to remain at \$10 (= \$1.00/.10). In this case the analyst's total return is expected to be 11% (= \$1.10/\$10), which will be attributed entirely to dividends as no capital gains are expected.

The 11% expected return can also be viewed as consisting of the required return of 10% plus an alpha of 1% that is equal to the portion of the dividend unanticipated by other investors, \$.10/\$10. Accordingly, if it is assumed that there will be no convergence of predictions, the expected return would be set at the implied rate of 11% and the alpha would be set at 1%.

Complete Convergence

Column (B) shows a very different situation. Here it is assumed that the other investors will recognize their error and completely revise their predictions. At the end of the year, it is expected that they too will predict future dividends of \$1.10 per year thereafter; thus the stock is expected to be selling for \$11 (= \$1.10/.10) at $t = 1$. Under these conditions, the analyst can expect to achieve a total return of 21% by selling the stock at the end of the year for \$11, obtaining 11% (= \$1.10/\$10) in dividend yield and 10% (= \$1/\$10) in capital gains.

The 10% expected capital gains result directly from the expected repricing of the security because of the complete convergence of predictions. In this case the fruits of the analyst's superior prediction are expected to be obtained all in one year. Instead of 1% "extra" per year forever, as in column (A), the analyst expects to obtain 1% (= \$.10/\$10) in extra dividend yield plus 10% (= \$1/\$10) in capital gains this year. By continuing to hold the stock in subsequent years, the analyst would expect to earn only the required return of 10% over those years. Accordingly, the expected return is 21% and the alpha is 11% when it is assumed that there is complete convergence of predictions.

Partial Convergence

Column (C) shows an intermediate case. Here the predictions of the other investors are expected to converge only halfway toward those of the analyst (that is, from \$1.00 to \$1.05 instead of to \$1.10). Total return in the first year is expected to be 16%, consisting of 11% (= \$1.10/\$10) in dividend yield plus 5% (= \$.50/\$10) in capital gains.

Since the stock is expected to be selling for \$10.50 (= \$1.05/.10) at $t = 1$, the analyst will still feel that it is underpriced at $t = 1$ because it will have an intrinsic value of \$11 (= \$1.10/.10) at that time. To obtain the remainder of the "extra return" owing to this underpricing, the stock would have to be held past $t = 1$. Accordingly, the expected return would be set at 16% and the alpha would be set at 6% when it is assumed that there is halfway convergence of predictions.

In general, a security's expected return and alpha will be larger, the faster the assumed rate of convergence of predictions.[15] Many investors use the implied rate (that is, the internal rate of return k^*) as a surrogate for a relatively short-term (for example, one year) expected return, as in column (A). In doing so, they are assuming that the dividend forecast is completely accurate, but that there is no convergence. Alternatively, investors could assume that there is some degree of convergence, thereby raising their estimate of the security's expected return. Indeed, investors could further alter their estimate of the security's expected return by assuming that the security analyst's deviant prediction is less than perfectly accurate, as will be seen next.[16]

18.9.2 Predicted versus Actual Returns

An alternative approach does not simply use outputs from a model "as is," but *adjusts* them, based on relationships between previous predictions and actual outcomes. Panels (a) and (b) of Figure 18.5 provide examples.

Each point in Figure 18.5(a) plots a *predicted return* on the stock market as a whole (on the horizontal axis) and the subsequent *actual return* for that period (on the vertical axis). The line of best fit (determined by simple regression) through the points indicates the general relationship between prediction and outcome. If the current prediction is 14%, history suggests that an estimate of 16% would be superior.

Each point in Figure 18.5(b) plots a predicted alpha value for a security (on the horizontal axis) and the subsequent "abnormal return" for that period (on the vertical axis). Such a diagram can be made for a given security, or for all the securities that a particular analyst makes predictions about, or for all the securities that the investment firm makes predictions about. Again a line of best fit can be drawn through the points. In this case, if the current prediction of a security's

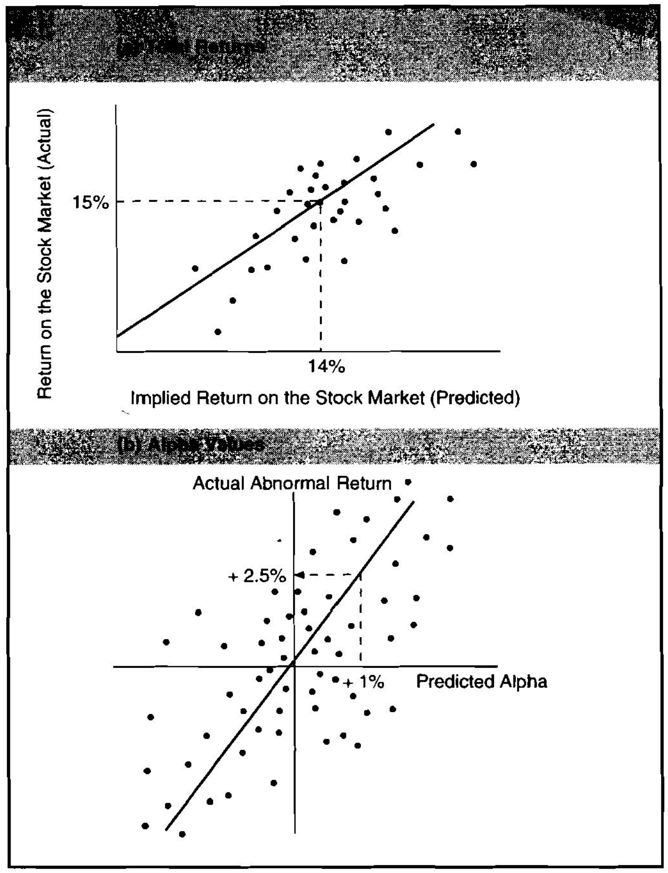

Figure 18.5
Adjusting Predictions

alpha is +1%, this relationship suggests that an "adjusted" estimate of +2.5% would be superior.

An important by-product of this type of analysis is the measure of correlation between predicted and actual outcomes, indicating the nearness of the points to the line. This **information coefficient** (IC) can serve as a measure of predictive accuracy. If it is too small to be significantly different from zero in a statistical sense, the value of the predictions is subject to considerable question.[17]

18.10 SUMMARY

1. The capitalization of income method of valuation states that the intrinsic value of any asset is equal to the sum of the discounted cash flows investors expect to receive from that asset.

2. Dividend discount models (DDMs) are a specific application of the capitalization of income method of valuation to common stocks.
3. To use a DDM, the investor must implicitly or explicitly supply a forecast of all future dividends expected to be generated by a security.
4. Investors typically make certain simplifying assumptions about the growth of common stock dividends. For example, a common stock's dividends may be assumed to exhibit zero growth or growth at a constant rate. More complex assumptions may allow for multiple growth rates over time.
5. Instead of applying DDMs, many security analysts use a simpler method of security valuation that involves estimating a stock's "normal" price-earnings ratio and comparing it with the stock's actual price-earnings ratio.
6. The growth rate in a firm's earnings and dividends depends on its earnings retention rate and its average return on equity for new investments.
7. Determining whether a security is mispriced using a DDM can be done in one of two ways. First, the discounted value of expected dividends can be compared with the stock's current price. Second, the discount rate that equates the stock's current price to the present value of forecast dividends can be compared with the required return for stocks of similar risk.
8. The rate of return that an analyst with accurate non-consensus dividend forecasts can expect to earn depends on the rate of convergence of other investors' predictions to the predictions of the analyst.

QUESTIONS AND PROBLEMS

1. Consider five annual cash flows (the first occurring one year from today):

Year	Cash Flow
1	\$5
2	\$6
3	\$7
4	\$8
5	\$9

Given a discount rate of 10%, what is the present value of this stream of cash flows?

2. Alta Cohen is considering buying a machine to produce baseballs. The machine costs \$10,000. With the machine, Alta expects to produce and sell 1,000 baseballs per year for \$3 per baseball, net of all costs. The machine's life is five years (with no salvage value). Based on these assumptions and an 8% discount rate, what is the net present value of Alta's investment?
3. Hub Collins has invested in a project that promised to pay \$100, \$200, and \$300, respectively, at the end of the next three years. If Hub paid \$513.04 for this investment, what is the project's internal rate of return?
4. [illegible]ton Products currently pays a dividend of \$4 per share on its common stock.

Explaining Market-to-Book

The relative impact of firm performance, growth, and risk

By Anurag Sharma, Ben Branch, Chetan Chgawla, and Liping Qiu

Peer Reviewed

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ABSTRACT

The Market-to-Book ratio, as a rough proxy for Tobin's q, has been a common measure of firm value for over two decades. The ratio has, however, had two distinct interpretations. One emphasizes it as reflecting efficiency and growth, and the other as proxy for risk. Herein we explore these interpretations in light of the constant growth discount model. We argue that both perspectives are theoretically sound. Upon testing these interpretations, we find that efficiency and growth variables explain the bulk of the variance in the MB ratio, and the contribution of risk is both mixed and limited. Our results suggest that the MB ratio largely reflects the success of managers in delivering strong operating performance and growth in the net assets of the firm.