In calculating an expected market risk premium by averaging historical data, projecting historical data using growth models, or even conducting a survey, one must determine a proxy for the "market". Common proxies for the US market include the S&P 500, the NYSE index, and the NYSE, AMEX, and NASDAQ index.[22] For the purpose of this paper, we use the S&P 500 and its antecedents as the market. However, in the various research surveyed, many different market proxies are assumed. We have already discussed using international versus domestic data when describing different MRP types. With international data, different proxies for other country, region, or world markets are used.[23] For domestic data, different proxies have been used over time as stock market exchanges have expanded.[24] Fortunately, as shown in the Ibbotson Valuation yearbook, the issue of a US market proxy does not have a large effect on the MRP estimate because the various indices are highly correlated. For example, the S&P 500 and the NYSE have a correlation of 0.95, the S&P 500 and NYSE/AMEX/NASDAQ 0.97, and the NYSE and NYSE/AMEX/NASDAQ 0.90.[25] Therefore, the market proxy selected is one reason for slight differences in the estimates of the market risk premium.

As a final note, stock returns and risk-free rates can be stated in nominal or real terms. Nominal includes inflation; real removes inflation. The equity risk premium should not be affected by inflation because either the stock return and risk-free rate both include the effects of inflation (both stated in nominal terms) or neither have inflation (both stated in real terms). If both returns are nominal, the difference in the returns is generally assumed to remove inflation. Otherwise, both terms are real, so inflation is removed prior to finding the equity risk premium. While numerical differences in the real and nominal approaches may exist, their magnitudes are expected to be small.

Equity Risk Premia 1926-2002

As an example of the importance of knowing the types of equity risk premium estimates under consideration, Table 5 displays ERP returns that each use the same historical data, but are based on arithmetic or geometric returns and the type of horizon. The ERP estimates are quite different.[26]

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[22] 2003 Ibbotson Valuation Yearbook, p92.
[23] For example, Dimson (2002) and Claus and Thomas (2001) use international market data.
[24] For a data series that is a mixture of the NYSE exchange, NYSE, AMEX, and NASDAQ stock exchange, and the Wilshire 5000, see Dimson (2002), p306.
[25] 2003 Ibbotson Valuation Yearbook, p93; using data from October 1997 to September 2002.
[26] The nominal and real ERPs are identical in Table 5 because the ERPs are calculated as arithmetic differences, and the same value of inflation will reduce the market return and the risk-free return equally. Geometric differences would produce minimally different estimates for the same types.

ERP using same historical data (1926-2002)		
RFR Description	**ERP Description**	**ERP Historical Return**
Short nominal	Arithmetic Short-horizon	8.4%
Short nominal	Geometric Short-horizon	6.4%
Short real	Arithmetic Short-horizon	8.4%
Short real	Geometric Short-horizon	6.4%
Intermediate nominal	Arithmetic Inter-horizon	7.4%
Intermediate nominal	Geometric Inter-horizon	5.4%
Intermediate real	Arithmetic Inter-horizon	7.4%
Intermediate real	Geometric Inter-horizon	5.4%
Long nominal	Arithmetic Long-horizon	7.0%
Long nominal	Geometric Long-horizon	5.0%
Long real	Arithmetic Long-horizon	7.0%
Long real	Geometric Long-horizon	5.0%

Table 5

Historical Methods

The historical methodology uses averages of past returns to forecast future returns. Different time periods may be selected, but the two most common periods arise from data provided by either Ibbotson or Siegel. The Ibbotson series begins in 1926 and is updated each year. The Siegel series begins in 1802 with the most recent compilation using returns through 2001. Appendix A provides equity risk premium estimates using Ibbotson data for the 1926-2002 period that we use in this paper for most illustrations. We begin with a look at the ERP history through a time series analysis of the Ibbotson data.

Time Series Analysis

Much of the analysis addressing the equity risk premium puzzle relies on the annual time series of market, risk-free and risk premium returns. Two opposite views can be taken of these data. One view would have the 1926-2002 Ibbotson data, or the 1802-2001 Siegel data, represent one data point; i.e., we have observed one path for the ERP through time from the many possible 77 or 200 year paths. This view rests upon the existence or assumption of a stochastic process with (possibly) inter-temporal correlations. While mathematically sophisticated, this model is particularly unhelpful without some testable hint at the details of the generating stochastic process. The practical view is that the observed returns are random samples from annual distributions that are iid, independent and identically distributed about the mean. The obvious advantage is that we have at hand 77 or 200 observations on the iid process to analyze. We adopt the latter view.

Some analyses adopt the assumption of stationarity of ERP, i.e., the true mean does not change with time. Figure 1 displays the Ibbotson ERP data and highlights two subperiods, 1926-1959 and 1960-2002.[27] While the mean ERP for the two subperiods appear quite different (11.82% vs. 5.27%), the large variance of the process (std dev 20.24%) should make them indistinguishable statistically speaking.

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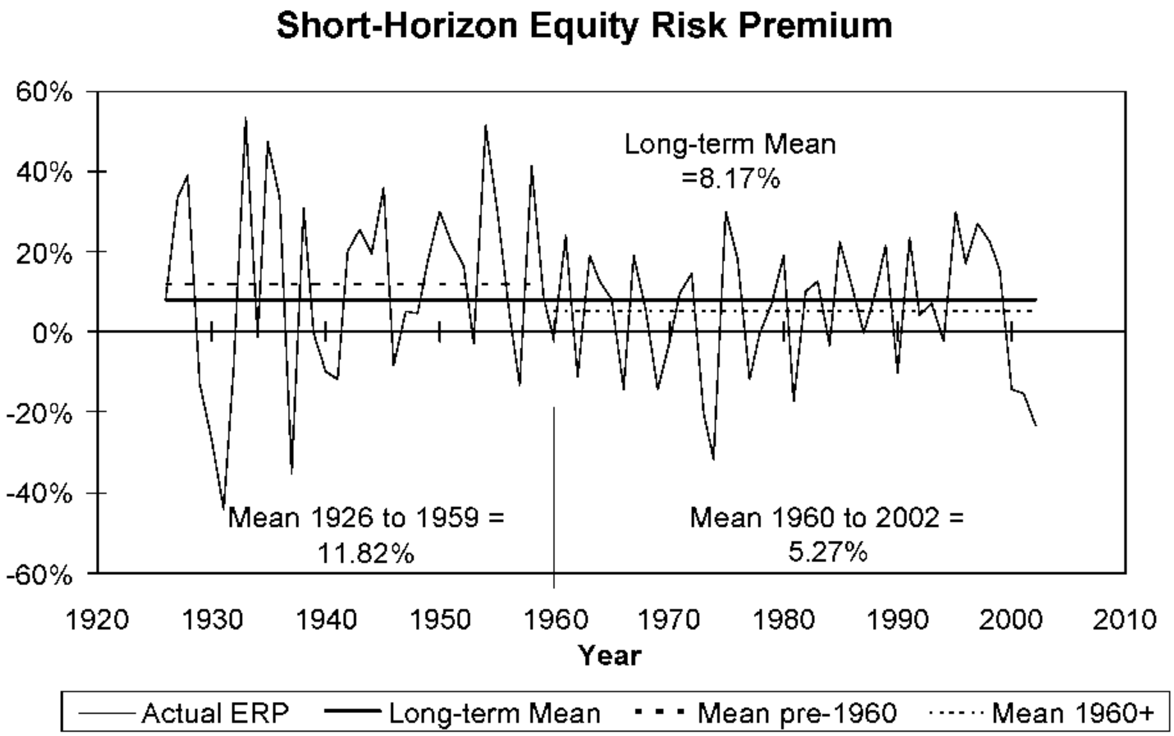
[27] The ERP shown here are the geometric differences (calculated) rather than the simple arithmetic differences in Table 1; i.e. ERP = $[(1+r_m)/(1+r_f)] - 1$. The test results are qualitatively the same for the arithmetic differences.



Source: Ibbotson Yearbook (2003), Geometric Differences

Figure 1

T-Tests

The standard T-test can be used for the null hypothesis H_o : mean 1960-2002 = 8.17%, the 77 year mean.[28] The outcome of the test is shown in Table 6; the null hypothesis cannot be rejected.

T-Test Under the Null Hypothesis that ERP (1960-2002) = ERP (1926-2002) = 8.17%	
Sample mean 1960-2002	5.27%
Sample s.d. 1960-2002	15.83%
T value (DF=42)	-1.20
PR > |T|	0.2374
Confidence Interval 95%	(0.0040, 0.1014)
Confidence Interval 90%	(0.0121, 0.0933)

Table 6

Another T-Test can be used to test whether the subperiod means are different in the presence of unequal variances.[29] The result is similar to Table 6 and the difference of subperiod means equal to zero cannot be rejected.[30]

[28] Standard statistical procedures in SAS 8.1 have been used for all tests.
[29] Equality of variances is rejected at the one percent level by an F test (F=2.39, DF=33,42)
[30] t-value 1.35, PR> |T| = 0.1850 with the Cochran method.

Time Trends
The supposition of stationarity of the ERP series can be supported by ANOVA regressions. The results of regressing the ERP series on time is shown in Table 7.

ERP ANOVA Regressions on Time		
Period	**Time Coefficient**	**P-Value**
1926-1959	0.004	0.355
1960-2002	0.001	0.749
1926-2002	-0.001	0.443

Table 7

There are no significant time trends in the Ibbotson ERP data.[31]

ARIMA Model
Time series analysis using the well established Box-Jenkins approach can be used to predict future series values through the lag correlation structure.[32] The SAS ARIMA procedure applied to the full 77 time series data shows:

(1) No significant autocorrelation lags.
(2) An identification of the series as white noise.
(3) ARIMA projection of year 78+ ERP is 8.17%, the 77 year average.

All of the above single time series tests point to the reasonability of the stationarity assumption for (at least) the Ibbotson ERP 77 year series.[33]

Social Security Administration
In the current debate on whether to allow private accounts that may invest in equities, the Office of the Chief Actuary of the Social Security Administration has selected certain assumptions to assess various proposals (Goss, 2001). The relevant selection is to use 7 percent as the real (geometric) annual rate of return for equities.[34] This assumption is based on the historical return of the 20^{th} century. SSA received further support that showed the historical return for the last 200 years is consistent with this estimate, along with the Ibbotson series beginning in 1926. For SSA, the calculation of the equity risk premium uses a long-run real yield on Treasury bonds as the risk-free rate. From the assumptions in the 1995 Trustees Report, the long-run real yield on Treasury bonds that the Advisory Council proposals use is 2.3%. Using a future Treasury securities real yield of 2.3% produces a geometric equity risk premium of 4.7% over long-term Treasury securities. More recently, the Treasury securities assumption has increased to 3%[35], yielding a 4% geometric ERP over long-term Treasury securities.

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[31] The result is confirmed by a separate Chow test on the two subperiods.
[32] See Harvey (1990), p30.
[33] The same tests applied to the Wilson and Jones 1871-2002 data series show similar results: Neither the 1871-1925 period nor the 1926-2002 period is different from the overall 1871-2002 period. The overall period and subperiods also show no trends over time.
[34] Compare Table 3, subperiod III.
[35] 1999 Social Security Trustees Report.

At the request of the Office of the Chief Actuary of the Social Security Administration (OCACT), John Campbell, Peter Diamond, and John Shoven were engaged to give their expert opinions on the assumptions Social Security mode. Each economist begins with the Social Security assumptions and then explains any difference he feels would be more appropriate.

In John Campbell's response, he considers valuation ratios as a comparison to the returns from the historical approach (Campbell 2001). The current valuation ratios are at unusual levels, with a low dividend-price ratio and high price-earnings ratio. He reasons that the prices are what have dramatically changed these ratios. Campbell presents two views as to the effect of valuation ratios in their current state. One view is that valuations will remain at the current level, suggesting much lower expected returns. The second view is a correction to the ratios, resulting in less favorable returns until the ratios readjust. He decides to give some weight to both possibilities, so he lowers the geometric equity return estimate to 5-5.5% from 7%. For the risk-free rate, he uses the yield on the long-term inflation-indexed bonds[36] of 3.5% or the OCACT assumption of 3%. Therefore, his geometric equity premium estimate is around 1.5 to 2.5%.

Peter Diamond uses the Gordon growth formula to calculate an estimate of the equity return (Diamond 2001). The classic Gordon Dividend Growth model is[37]:

$$K = (D_1 / P_0) + g$$

K = Expected Return or Discount Rate P_0 = Price this period
D_1 = Expected Dividend next period g = Expected growth in dividends in perpetuity

Based on his analysis, he feels that the equity return assumption of 7% for the next 75 years is not consistent with a reasonable level of stock value compared to GDP. Even when increasing the GDP growth assumption, he still does not feel that the equity return is plausible. By reasoning that the next decade of returns will be lower than normal, only then is the equity return beyond that time frame consistent with the historical return. By considering the next 75 years together, he would lower the overall projected equity return to 6-6.5%. He argues that the stock market is overvalued, and a correction is required before the long-run historical return is a reasonable projection for the future. By using the OCACT assumption of 3.0% for the long-term real yield on Treasury bonds, Diamond estimates a geometric equity risk premium of about 3-3.5%.

John Shoven begins by explaining why the traditional Gordon growth model is not appropriate, and he suggests a modernized Gordon model that allows share repurchases to be included instead of only using the dividend yield and growth rate (Shoven 2001). By assuming a long-term price-earnings ratio between its current and historical value, he comes up with an estimate for the long-term real equity return of 6.125%. Using his general estimate of 6-6.5% for the equity return and the OCACT assumptions for the long-term bond yield, he projects a long-term equity risk premium of approximately 3-3.5%. All the SSA experts begin by accepting the long-run historical

[36] See discussion of current yields on TIPS below.
[37] Brealey and Myers (2000), p67.

ERP analyses and then modifying that by changes in the risk-free rate or by decreases in the long-term ERP based on their own personal assessments. We now turn to the major strains in ERP puzzle research.

ERP Puzzle Research

Campbell and Shiller (2001) begin with the assumption of mean reversion of dividend/price and price/earnings ratios. Next, they explain the result of prior research which finds that the dividend-price ratio predicts future prices, and historically, the price corrects the ratio when it diverts from the mean.[38] Based on this result, they then use regressions of the dividend-price ratio and the price-smoothed-earnings[39] ratio to predict future stock prices out ten years. Both regressions predict large losses in stock prices for the ten year horizon. Although Campbell and Shiller do not rerun the regression on the dividend-price ratio to incorporate share repurchases, they point out that the dividend-price ratio should be upwardly adjusted, but the adjustment only moves the ratio to the lower range of the historical fluctuations (as opposed to the mean). They conclude that the valuation ratios indicate a bear market in the near future[40]. They predict for the next ten year period negative real stock returns. They caution that because valuation ratios have changed so much from their normal level, they may not completely revert to the historical mean, but this does not change their pessimism about the next decade of stock market returns.

Arnott and Ryan (2001) take the perspective of fiduciaries, such as pension fund managers, with an investment portfolio. They begin by breaking down the historical stock returns (past 74 years since December 1925) by analyzing dividend yields and real dividend growth. They point out that the historical dividend yield is much higher than the current dividend yield of about 1.2%. They argue that the changes from stock repurchases, reinvestment, and mergers and acquisitions, which affect the lower dividend yield, can be represented by a higher dividend growth rate. However, they cap real dividend or earnings growth at the level of real economic growth. They add the dividend yield and the growth in real dividends to come up with an estimate for the future equity return; the current dividend yield of 1.2% and the economic growth rate of 2.0% add to the 3.2% estimated real stock return. This method corresponds to the dividend growth model or earnings growth model and does not take into account changing valuation levels. They cite a TIPS yield of 4.1% for the real risk-free rate return.[41] These two estimates yield a negative geometric long-horizon conditional equity risk premium.

Arnott and Bernstein (2002) begin by arguing that in 1926 investors were not expecting the realized, historical compensation that they later received from stocks. They cite bonds' reaction to inflation, increasing valuations, survivorship bias[42], and changes in

[38] Campbell and Shiller (1989).
[39] Earnings are "smoothed" by using ten year averages.
[40] The stock market correction from year-end 1999 to year-end 2002 is a decrease of 37.6% or 14.6% per year. Presumably, the "next ten years" refers to 2000 to 2010.
[41] See the current TIPS yield discussion near end of paper.
[42] See Brown et al. (1992, 1995) for details on potential survivorship bias.

regulation as positive events that helped investors during this period. They only use the dividend growth model to predict a future expected return for investors. They do not agree that the earnings growth model is better than the dividend growth model both because earnings are reported using accounting methods and earnings data before 1870 are inaccurate. Even if the earnings growth model is chosen instead, they find that the earnings growth rate from 1870 only grows 0.3% faster than dividends, so their results would not change much. Because of the Modigliani-Miller theorem[43], a change in dividend policy should not change the value of the firm. They conclude that managers benefited in the "era of 'robber baron' capitalism" instead of the conclusion reached by others that the dividend growth model under-represents the value of the firm.

By holding valuations constant and using the dividend yield and real growth of dividends, Arnott and Bernstein calculate the equity return that an investor might have expected during the historical time period starting in 1802. They use an expected dividend yield of 5.0%, close to the historical average of 1810 to 2001. For the real growth of dividends, they choose the real per capita GDP growth less a reduction for entrepreneurial activity in the economy plus stock repurchases. They conclude that the net adjustment is negative, so the real GDP growth is reduced from 2.5-3% to only 1%. A fair expectation of the stock return for the historical period is close to 6.1% by adding 5.0% for the dividend yield and a net real GDP per capita growth of 1.1%. They use a TIPS yield of 3.7% for the real risk-free rate, which yields a geometric intermediate-horizon equity risk premium of 2.4% as a fair expectation for investors in the past. They consider this a "normal" equity risk premium estimate. They also opine that the current ERP is zero; i.e. they expect stocks and (risk-free) bonds to return the same amounts.

Fama and French (2002) use both the dividend growth model and the earnings growth model to investigate three periods of historical returns: 1872 to 2000, 1872 to 1950, and 1951 to 2000. Their ultimate aim is to find an unconditional equity risk premium. They cite that by assuming the dividend-price ratio and the earnings-price ratio follow a mean reversion process, the result follows that the dividend growth model or earnings growth model produce approximations of the unconditional equity return. Fama and French's analysis of the earlier period of 1872 to 1950 shows that the historical average equity return and the estimate from the dividend growth model are about the same. In contrast, they find that the 1951 to 2000 period has different estimates for returns when comparing the historical average and the growth models' estimates. The difference in the historical average and the model estimates for 1951 to 2000 is interpreted to be "unexpected capital gains" over this period. They find that the unadjusted growth model estimates of the ERP, 2.55% from the dividend model and 4.32% from the earnings model, fall short of the realized average excess return for 1951-2000. Fama and French prefer estimates from growth models instead of the historical method because of the lower standard error using the dividend growth model. Fama and French provide 3.83% as the unconditional expected equity risk premium return (referred to as the annual bias-adjusted ERP estimate) using the dividend growth model with underlying data from 1951 to 2000. They give 4.78% as the unconditional expected equity risk

[43] Brealey and Myers (2000), p447. See also discussion in Ibbotson and Chen (2003).

premium return using the earnings growth model with data from 1951 to 2000. Note that using a one-month Treasury bill instead of commercial paper for the risk-free rate would increase the ERP by about 1% to nearly 6% for the 1951-2000 period.

Ibbotson and Chen (2003) examine the historical real geometric long-run market and long risk-free returns using their "building block" methodology.[44] They use the full 1926-2000 Ibbotson Associates data and consider as building blocks all of the fundamental variables of the prior researchers. Those blocks include (not all simultaneously):

- Inflation
- Real risk-free rates (long)
- Real capital gains
- Growth of real earnings per share
- Growth of real dividends
- Growth in payout ratio (dividend/earnings)
- Growth in book value
- Growth in ROE
- Growth in price/earnings ratio
- Growth in real GDP/population
- Growth in equities excess of GDP/POP
- Reinvestment

Their calculations show that a forecast real geometric long run return of 9.4% is a reasonable extrapolation of the historical data underlying a realized 1926-2000 return of 10.7%, yielding a long horizon arithmetic ERP of 6%, or a short horizon arithmetic ERP of about 7.5%.

The authors construct six building block methods; i.e., they use combinations of historic estimates to produce an expected geometric equity return. They highlight the importance of using both dividends and capital gains by invoking the Modigliani-Miller theorem. The methods, and their component building blocks are:

- Method 1: Inflation, real risk free rate, realized ERP
- Method 2: Inflation, income, capital gains and reinvestment
- Method 3: Inflation, income, growth in price/earnings, growth in real earnings per share and reinvestment.
- Method 4: Inflation, growth rate of price/earnings, growth rate of real dividends, growth rate of payout ratio dividend yield and reinvestment
- Method 5: Inflation, income growth rate of price/earnings, growth of real book value, ROE growth and reinvestment
- Method 6: Inflation, income, growth in real GDP/POP, growth in equities excess GDP/POP and reinvestment.

[44] See Appendix D for a summary of their building block estimates. See also Pratt (1998) for a discussion of the Building Block, or Build-Up Model, cost of capital estimation method.

All six methods reproduce the historical long horizon geometric mean of 10.70% as shown in Appendix D. Since the source of most other researchers' lower ERP is the dividend yield, the authors recast the historical results in terms of ex ante forecasts for the next 75 years. Their estimate of 9.37% using supply side methods 3 and 4 is approximately 130 basis points lower than the historical result. Within their methods, they also show how the substantially lower expectation of 5.44% for the long mean geometric return is calculated by omitting one or more relevant variables. Underlying these ex ante methods are the assumptions of stationarity of the mean ERP return and market efficiency, the absence of the assumption that the market has mispriced equities. All of their methods are aimed at producing an unconditioned estimate of the ex ante ERP.

As opposed to short-run, conditional estimates from Campbell and Shiller and others, Constantinides (2002) seeks to estimate the unconditional equity risk premium, more in line with the goal of Fama and French (2002) and Ibbotson and Chen (2003). He begins with the premise that the unconditional ERP can be estimated from the historical average using the assumption that the ERP follows a stationary path. He suggests most of the other research produces conditional estimates, conditioned upon beliefs about the future paths of fundamentals such as dividend growth, price-earnings ratio and the like. While interesting in themselves, they add little to the estimation of the unconditional mean ERP.

Constantinides uses the historical return and adjusts downward by the growth in the price-earnings ratio to calculate the unconditional equity risk premium. He removes the growth in the price-earnings ratio because he is assuming no change in valuations in the unconditional state. He gives estimates using three periods. For 1872-2000, he uses the historical equity risk premium which is 6.9%, and after amortizing the growth in the price-dividend ratio or price-earnings ratio over a period as long as 129 years, the effect of the potential reduction is no change. Therefore, he finds an unconditional arithmetic, short-horizon equity risk premium of 6.9% using the 1872-2000 underlying data. For 1951-2000, he again starts with the historical equity risk premium which is 8.7% and lowers this estimate by the growth in the price-earnings ratio of 2.7% to find an unconditional arithmetic, short-horizon equity risk premium of 6.0%. For 1926-2000, he uses the historical equity risk premium which is 9.3% and reduces this estimate by the growth in the price-earnings ratio of 1.3% to find an unconditional arithmetic, short-horizon equity risk premium of 8.0%. He appeals to behavioral finance to offer explanations for such high unconditional equity risk premium estimates.

From the perspective of giving practical investor advice, Malkiel (1999) discusses "the age of the millennium" to give some indication of what investors might expect for the future. He specifically estimates a reasonable expectation for the first few decades of the twenty-first century. He estimates the future bond returns by giving estimates if bonds are held to maturity with corporate bonds of 6.5-7%, long-term zero-coupon Treasury bonds of about 5.25%, and TIPS with a 3.75% return. Depending on the desired level of risk, Malkiel indicates bondholders should be more favorably compensated in the future compared to the historical returns from 1926 to 1998. Malkiel uses the earnings growth model to predict future equity returns. He uses the current dividend yield of 1.5% and an earnings growth estimate of 6.5%, yielding an 8% equity return estimate compared with an 11% historical return. Malkiel's estimated range of the equity risk premium is from 1% to 4.25%, depending on the risk-free instrument selected. Although his equity risk premium is lower than the historical return, his selection of a relatively high earnings growth rate is similar to Ibbotson and Chen's forecasted models. In contrast with Ibbotson and Chen, Malkiel allows for a changing equity risk premium and advises investors to not rely solely on the past "age of exuberance" as a guide for the future. Malkiel points out the impact of changes in valuation ratios, but he does not attempt to predict future valuation levels.

Finally, Mehra (2002) summarizes the results of the research since the ERP puzzle was posed. The essence of the puzzle is the inconsistency of the ERPs produced by descriptive and prescriptive economic models of asset pricing on the one hand and the historical ERPs realized in the US market on the other. Mehra and Prescott (1985) speculated that the inconsistency could arise from the inadequacy of standard models to incorporate market imperfections and transaction costs. Failure of the models to reflect reality rather than failure of the market to follow the theory seems to be Mehra's conclusion as of 2002. Mehra points to two promising threads of model-modifying research. Campbell and Cochrane (1999) incorporate economic cycles and changing risk aversion while Constantinides et al. (2002) propose a life cycle investing modification, replacing the representative agent by segmenting investors into young, middle aged, and older cohorts. Mehra sums up by offering:

> "Before we dismiss the premium, we not only need to have an understanding of the observed phenomena but also why the future is likely to be different. In the absence of this, we can make the following claim based on what we know. Over the long horizon the equity premium is likely to be similar to what it has been in the past and the returns to investment in equity will continue to substantially dominate those in bonds for investors with a long planning horizon."

<u>Financial Analyst Estimates</u>

Claus and Thomas (2001) and Harris and Marston (2001) both provide equity premium estimates using financial analysts' forecasts. However, their results are rather different. Claus and Thomas use an abnormal earnings model with data from 1985 to 1998 to calculate an equity risk premium as opposed to using the more common dividend growth model. Financial analysts project five year estimates of future earnings growth rates. When using this five year growth rate for the dividend growth rate in perpetuity in the Gordon growth model, Claus and Thomas explain that there is a potential upward bias in estimates for the equity risk premium. Therefore, they choose to use the abnormal earnings model instead and only let earnings grow at the level of inflation after five years. The abnormal earnings model replaces dividends with "abnormal earnings" and discounts each flow separately instead of using a perpetuity. The average estimate that they find is 3.39% for the equity risk premium. Although it is generally recognized that financial analysts' estimates have an upward bias, Claus and Thomas propose that in the current literature, financial analysts' forecasts have underestimated short-term earnings in order for management to achieve earnings estimates in the slower economy. Claus and Thomas conclude that their findings of the ERP using data from the past fifteen years are not in line with historical values.

Harris and Marston use the dividend growth model with data from 1982 to 1998. They assume that the dividend growth rate should correspond to investor expectations. By using financial analysts' longest estimates (five years) of earnings growth in the model, they attempt to estimate these expectations. They argue that if investors are in accord with the optimism shown in analysts' estimates, even biased estimates do not pose a drawback because these market sentiments will be reflected in actual returns. Harris and Marston find an equity risk premium estimate of 7.14%. They find fluctuations in the equity risk premium over time. Because their estimates are close to historical returns, they contend that investors continue to require a high equity risk premium.

Survey Methods

One method to estimate the ex ante equity risk premium is to find the consensus view of experts. John Graham and Campbell Harvey perform a survey of Chief Financial Officers to determine the average cost of capital used by firms. Ivo Welch surveys financial economists to determine the equity risk premium that academic experts in this area would estimate.

Graham and Harvey administer surveys from the second quarter of 2000 to the third quarter of 2002 (Graham and Harvey, 2002). For their survey format, they show the current ten year bond yield and then ask CFOs to provide their estimate of the S&P 500 return for the next year and over the next ten years. CFOs are actively involved in setting a company's individual hurdle[45] rate and are therefore considered knowledgeable about investors' expectations.[46] When comparing the survey responses of the one and ten year returns, the one year returns have so much volatility that they conclude that the ten-year equity risk premium is the more important and appropriate return of the two when making financial decisions such as hurdle rates and estimating cost of capital. The average ten-year equity risk premium estimate varies from 3% to 4.7%.

The most current Welch survey compiles the consensus view of about five hundred financial economists (Welch 2001). The average arithmetic estimate for the 30-year equity risk premium relative to Treasury bills is 5.5%; the one-year arithmetic equity risk premium consensus is 3.4%. Welch deduces from the average 30-year geometric

[45] A "hurdle" rate is a benchmark cost of capital used to evaluate projects to accept (expected returns greater than hurdle rate) or reject (expected returns less than hurdle rate).
[46] Graham and Harvey claim three-fourths of the CFOs use CAPM to estimate hurdle rates.

equity return estimate of 9.1% that the arithmetic equity return forecast is approximately 10%.[47]

Welch's survey question allows the participants to self select into different categories based upon their knowledge of ERP. The results indicate that the responses of the less ERP knowledgeable participants showed more pessimism than those of the self reported experts. The experts gave 30-year estimates that are 30 to 150 basis points above the estimates of the non-expert group.

Differences in Forecasts across Expertise Level				
Relative Expertise	**Statistic**	**Stock Market**	**Equity Premium**	
		30-Year Geometric	30-Year Arithmetic	30-Year Geometric
188 Less Involved	Mean	8.5%	4.9%	4.4%
	Median	8%	5%	4%
	IQ Range	6%-10%	3%-6%	2%-5.5%
235 Average	Mean	9.2%	5.8%	4.8%
	Median	9%	5%	4%
	IQ Range	7.5%-10%	3.5%-7%	3%-6%
72 Experts	Mean	10.1%	6.2%	5.4%
	Median	9%	5.4%	5%
	IQ Range	8%-11%	4%-7.5%	3.4%-6%
Data Source: Welch (2001), Table 5				

Table 8

Table 8 shows that there may be a "lemming" effect, especially among economists who are not directly involved in the ERP question. Stated differently, all the academic and popular press, together with the prior Welch survey[48] could condition the non-expert, the "less involved", that the expected ERP was lower than historic levels.

The Behavioral Approach

Benartzi and Thaler (1995) analyze the equity risk premium puzzle from the point of view of prospect theory (Kahneman and Tversky; 1979). Prospect theory[49] has "loss aversion", the fact that individuals are more sensitive to potential loss than gain, as one of its central tenets. Once an asymmetry in risk aversion is introduced into the model of the rational representative investor or agent, the unusual risk aversion problem raised initially by Mehra and Prescott (1985) can be "explained" within this behavioral model of decision-making under uncertainty. Stated differently, given the historical ERP series, there exists a model of investor behavior that can produce those or similar results. Benartzi and Thaler combine loss aversion with "mental accounting", the behavioral process people use to evaluate their status relative to gains and losses compared to expectations, utility and wealth, to get "myopic loss aversion". In particular, mental

[47] For the Ibbotson 1926-2002 data, the arithmetic return is about 190 basis points higher than the geometric return rather than the inferred 90 basis points. This suggests the participant's beliefs may not be internally consistent.
[48] The prior Welch survey in 1998 had a consensus ERP of about 7%.
[49] A current survey of the applications of prospect theory to finance can be found in Benartzi et al. (2001).

accounting for a portfolio needs to take place infrequently because of loss aversion, in order to reduce the chances of observing loss versus gain. The authors concede that there is a puzzle with the standard expected utility-maximizing paradigm but that the myopic loss aversion view may resolve the puzzle. The authors' views are not free of controversy; any progress along those lines is sure to match the advance of behavioral economics in the large.

The adoption of other behavioral aspects of investing may also provide support for the historical patterns of ERPs we see from 1802-2002. For example, as the true nature of risk and rewards has been uncovered by the virtual army of 20th century researchers, and as institutional investors held sway in the latter fifty years of the century, the demand for higher rewards seen in the later historical data may be a natural and rational response to the new and expanded information set. Dimson et al. (2002, Figure 4-6) displays increasing real US equity returns of 6.7, 7.4, 8.2 and 10.2 for periods of 101, 75, 50 and 25 years ending in 2001 consistent with this "risk-learning" view.

Next Ten Years

The "next ten years" is an issue that experts reviewing Social Security assumptions and Campbell and Shiller address either explicitly or implicitly. Experts evaluating Social Security's proposals predicted that the "next ten years", indicating a period beginning around 2000, of returns were likely to be below the historical return. However, a historical return was recommended as appropriate for the remaining 65 of the 75 years to be projected. For Campbell and Shiller (2001), the period they discuss is approximately 2000-2010. Based on the current state of valuation ratios, they predict lower stock market returns over "the next ten years". These expert predictions, and other pessimistic low estimates, have already come to fruition as market results 2000 through 2002.[50] The US equities market has decreased 37.6% since 1999, or an annual decrease of 14.6%. Although these forecasts have proved to be accurate in the short term, for future long-run projections, the market is not at the same valuation today as it was when these conditional estimates were originally given. Therefore, actuaries should be wary of using the low long-run estimates made prior to the large market correction of 2000-2002.

Treasury Inflation Protection Securities (TIPS)

Several of the ERP researchers refer to TIPS when considering the real risk free rates. Historically, they adjust Treasury yields downward to a real rate by an estimate of inflation, presumably for the term of the Treasury security. As Table 3 shows using the Siegel data, the modern era data show a low real long-term risk-free rate of return (2.2%). This contrasts with the initial[51] TIPS issue yields of 3.375%. Some researchers use those TIPS yields as (market) forecasts of real risk-free returns for intermediate and long-horizon, together with reduced (real) equity returns to produce low estimates of ex ante ERPs. None consider the volatility of TIPS as indicative of the accuracy of their ERP estimate.

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[50] The Social Security Advisory Board will revisit the seventy five year rate of return assumption during 2003, Social Security Advisory Board (2002).

[51] TIPS were introduced by the Treasury in 1996 with the first issue in January, 1997.

Table 9 shows a recent market valuation of ten and thirty year TIPS issued in 1998-2002.

Inflation-Indexed Treasury Securities		
Maturity	Coupon Issue Rate	Yield to Maturity
1/11	3.500	1.763
1/12	3.375	1.831
7/12	3.000	1.878
4/28	3.625	2.498
4/29	3.875	2.490
4/32	3.375	2.408
Source: WSJ 1 2/24/2003		

Table 9

Note the large 90-180 basis point decrease in the current "real" yields from the issue yields as recent as ten months ago. While there can be several explanations for the change (revaluation of the inflation option, flight to Treasury quality, paucity of 30 year Treasuries), the use of these current "real" risk free yields, with fixed expected returns, would raise ERPs by at least one percent.

Conclusion

This paper has sought to bring the essence of recent research on the equity risk premium to practicing actuaries. The researchers covered here face the same ubiquitous problems that actuaries face daily: Do I rely on past data to forecast the future (costs, premiums, investments) or do I analyze the past and apply informed judgment as to future differences, if any, to arrive at actuarially fair forecasts? Most of the ERP estimates lower than the unconditional historical estimate have an undue reliance on recent lower dividend yields (without a recognition of capital gains[52]) and/or on data prior to 1926.

Despite a spate of research suggesting ex ante ERPs lower than recent realized ERPs, actuaries should be aware of the range of estimates covered here (Appendix B); be aware of the underlying assumptions, data and terminology; and be aware that their independent analysis is required before adopting an estimate other than the historical average. We believe that the Ibbotson-Chen (2003) layout, reproduced here as Appendix D, offers the actuary both an understanding of the fundamental components of the historical ERP and the opportunity to change the estimates based upon good judgment and supportable beliefs. We believe that reliance solely on "expert" survey averages, whether of financial analysts, academic economists, or CFOs, is fraught with risks of statistical bias to fair estimates of the forward ERP.

[52] Under the current US tax code, capital gains are tax-advantaged relative to dividend income for the vast majority of equity holders (households and mutual funds are 55% of the total equity holders, Federal Flow of Funds, 2002 Q3, Table L-213). Curiously, the reverse is true for property-liability insurers because of the 70% stock dividend exclusion afforded insurers.

It is dangerous for actuaries to engage in simplistic analyses of historical ERPs to generate ex ante forecasts that differ from the realized mean.[53] The research we have catalogued in Appendix B, the common level ERPs estimated in Appendix C, and the building block (historical) approach of Ibboston and Chen in Appendix D all discuss important concepts related to both ex post and ex ante ERPs and cannot be ignored in reaching an informed estimate. For example, Richard Wendt, writing in a 2002 issue of Risks and Rewards, a newsletter of the Society of Actuaries, concludes that a linear relationship is a better predictor of future returns than a "constant" ERP based on the average historical return. He arrives at this conclusion by estimating a regression equation[54] relating long bond yields with 15-year geometric mean market returns starting monthly in 1960. First, there is no significant relationship between short, intermediate or long-term income returns over 1926-2002 (or 1960-2002) and ERPs, as evidenced by simple regressions using Ibbotson data.[55] Second, if the linear structural equation indeed held, there would be no need for an ERP since the (15-year) return could be predicted within small error bars. Third, there is always a negative bias introduced when geometric averages are used as dependent variables (Brennan and Schwartz, 1985). Finally, the results are likely to be spurious due to the high autocorrelations of the target and independent variables; an autocorrelation correction would eliminate any significant relationship of long-yields to the ERP.

Actuaries should also be aware of the variability of both the ERP and risk-free rate estimates discussed in this paper (see Tables 4 and 9). All too often, return estimates are made without noting the error bars and that can lead to unexpected "surprises". As one example, recent research by Francis Longstaff (2002), proposes that a 1991-2001 "flight to quality" has created a valuation premium (and lowered yields) in the entire yield curve of Treasuries. He finds a 10 to 16 basis point liquidity premium throughout the zero coupon Treasury yield curve. He translates that into a 10% to 15% pricing difference at the long end. This would imply a simple CAPM market estimate for the long horizon might be biased low.

Finally, actuaries should know that the research catalogued in Appendix B is not definitive. No simple model of ERP estimation has been universally accepted. Undoubtedly, there will be still more empirical and theoretical research into this data rich financial topic. We await the potential advances in understanding the return process that the behavioral view may uncover.

Post Script: Appendices A-D

We provide four appendices that catalogue the ERP approaches and estimates discussed in the paper. Actuaries, in particular, should find the numerical values, and descriptions of assumptions underlying those values, helpful for valuation work that

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[53] ERPs are derived from historical or expected after corporate tax returns. Pre-tax returns depend uniquely on the tax schedule for the differing sources of income.

[54] 15-year mean returns = 2.032 (Long Government Bond Yield) – 0.0242, R^2 = 0.882.

[55] The p-values on the yield-variables in an ERP/Yield regression using 1926-2002 annual data are 0.1324, 0.2246, and 0.3604 for short, intermediate and long term yields respectively with adjusted r square virtually zero.

adjusts for risk. Appendix A provides the annual Ibbotson data from 1926 through 2002 from Ibbotson Associates referred to throughout this paper. The equity risk-premium shown is a simple difference of the arithmetic stock returns and the arithmetic U.S. Treasury Bills total returns. Appendix B is a compilation of articles and books related to the equity risk premium. The puzzle research section contains the articles and books that were most related to addressing the equity risk premium puzzle. Page 1 of Appendix B gives each source, along with risk-free rate and equity risk premium estimates. Then, each source's estimate is classified by type (indicated with an X for the appropriate type). Page 2 of Appendix B shows further details collected from each source. This page adds the data period used, if applicable, and the projection period. We also list the general methodology used in the reference. The final three pages of Appendix B provide the footnotes which give additional details on the sources' intent.

Appendix C adjusts all the equity risk premium estimates to a short-horizon, arithmetic, unconditional ERP estimate. We begin with the authors' estimates for a stock return (the risk-free rate plus the ERP estimate). Next, we make adjustments if the ERP "type" given by the author(s) is not given in this format. For example, to adjust from a geometric to an arithmetic ERP estimate, we adjust upwards by the 1926-2002 historical difference in the arithmetic large company stocks' total return and the geometric large company stocks' total return of 2%. Next, if the estimate is given in real instead of nominal terms, we adjust the stock return estimate upwards by 3.1%, the 1926-2002 historical return for inflation.

We make an approximate adjustment to move the estimate from a conditional to unconditional estimate based on Fama and French (2002). Using the results for the 1951-2000 period shown in Table 4 of their paper and the standard deviations provided in Table 1, we have four adjustments based on their data. For the 1951-2000 period, Fama and French use an adjustment of 1.28% for the dividend growth model and 0.46% for the earnings growth model. Following a similar calculation, the 1872-2000 period would require a 0.82% adjustment using a dividend growth model; the 1872-1950 period would require a 0.54% adjustment using a dividend growth model. Earnings growth models were used by Fama and French only for the 1951-2000 data period. Therefore, we selected the lowest adjustment (0.46%) as a minimum adjustment from a conditional estimate to an unconditional estimate. Finally, we subtract the 1926-2002 historical U.S. Treasury Bills' total return to arrive at an adjusted equity risk premium.

These adjustments are only approximations because the various sources rely on different underlying data, but the changes in the ERP estimate should reflect the underlying concept that different "types" of ERPs cannot be directly compared and require some attempt to normalize the various estimates.

Page 1 of Appendix D is a table from Ibbotson and Chen which breaks down historical returns using various methods that correspond to their 2003 paper (reprinted with permission of Ibbotson Associates). The bottom portion provides forward-looking estimates. Page 2 of Appendix D is provided to show the formulas that Ibbotson and Chen develop within their paper.

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Appendix A Ibbotson Market Data 1926-2002*			
	Common Stocks	U. S. Treasury Bills	Arithmetic Short-Horizon
Year	Total Annual Returns	Total Annual Returns	Equity Risk Premia
1926	11.62%	3.27%	8.35%
1927	37.49%	3.12%	34.37%
1928	43.61%	3.56%	40.05%
1929	- 8.42%	4.75%	-13.17%
1930	-24.90%	2.41%	-27.31%
1931	-43.34%	1.07%	-44.41%
1932	- 8.19%	0.96%	- 9.15%
1933	53.99%	0.30%	53.69%
1934	- 1.44%	0.16%	- 1.60%
1935	47.67%	0.17%	47.50%
1936	33.92%	0.18%	33.74%
1937	-35.03%	0.31%	-35.34%
1938	31.12%	- 0.02%	31.14%
1939	- 0.41%	0.02%	- 0.43%
1940	- 9.78%	0.00%	- 9.78%
1941	-11.59%	0.06%	-11.65%
1942	20.34%	0.27%	20.07%
1943	25.90%	0.35%	25.55%
1944	19.75%	0.33%	19.42%
1945	36.44%	0.33%	36.11%
1946	- 8.07%	0.35%	- 8.42%
1947	5.71%	0.50%	5.21%
1948	5.50%	0.81%	4.69%
1949	18.79%	1.10%	17.69%
1950	31.71%	1.20%	30.51%
1951	24.02%	1.49%	22.53%
1952	18.37%	1.66%	16.71%
1953	- 0.99%	1.82%	- 2.81%
1954	52.62%	0.86%	51.76%
1955	31.56%	1.57%	29.99%
1956	6.56%	2.46%	4.10%

Appendix A Ibbotson Market Data 1926-2002*			
	Common Stocks	U. S. Treasury Bills	Arithmetic Short-Horizon
Year	Total Annual Returns	Total Annual Returns	Equity Risk Premia
1957	-10.78%	3.14%	-13.92%
1958	43.36%	1.54%	41.82%
1959	11.96%	2.95%	9.01%
1960	0.47%	2.66%	- 2.19%
1961	26.89%	2.13%	24.76%
1962	- 8.73%	2.73%	-11.46%
1963	22.80%	3.12%	19.68%
1964	16.48%	3.54%	12.94%
1965	12.45%	3.93%	8.52%
1966	-10.06%	4.76%	-14.82%
1967	23.98%	4.21%	19.77%
1968	11.06%	5.21%	5.85%
1969	- 8.50%	6.58%	-15.08%
1970	4.01%	6.52%	- 2.51%
1971	14.31%	4.39%	9.92%
1972	18.98%	3.84%	15.14%
1973	-14.66%	6.93%	-21.59%
1974	-26.47%	8.00%	-34.47%
1975	37.20%	5.80%	31.40%
1976	23.84%	5.08%	18.76%
1977	- 7.18%	5.12%	-12.30%
1978	6.56%	7.18%	- 0.62%
1979	18.44%	10.38%	8.06%
1980	32.42%	11.24%	21.18%
1981	- 4.91%	14.71%	-19.62%
1982	21.41%	10.54%	10.87%
1983	22.51%	8.80%	13.71%
1984	6.27%	9.85%	- 3.58%
1985	32.16%	7.72%	24.44%
1986	18.47%	6.16%	12.31%
1987	5.23%	5.47%	- 0.24%
1988	16.81%	6.35%	10.46%
1989	31.49%	8.37%	23.12%

Appendix A
Ibbotson Market Data 1926-2002*

	Common Stocks	**U. S. Treasury Bills**	**Arithmetic Short-Horizon**
Year	**Total Annual Returns**	**Total Annual Returns**	**Equity Risk Premia**
1990	- 3.17%	7.81%	-10.98%
1991	30.55%	5.60%	24.95%
1992	7.67%	3.51%	4.16%
1993	9.99%	2.90%	7.09%
1994	1.31%	3.90%	- 2.59%
1995	37.43%	5.60%	31.83%
1996	23.07%	5.21%	17.86%
1997	33.36%	5.26%	28.10%
1998	28.58%	4.86%	23.72%
1999	21.04%	4.68%	16.36%
2000	- 9.11%	5.89%	-15.00%
2001	-11.88%	3.83%	-15.71%
2002	-22.10%	1.65%	-23.75%
mean=	**12.20%**	**3.83%**	**8.37%**
Standard Dev=	**20.49%**	**3.15%**	**20.78%**

* *2003 SBBI Yearbook pages 38 and 39*

Appendix B

Source	Risk-free-Rate	ERP Estimate	Real risk-free rate	Nominal risk-free rate	Geometric	Arithmetic	Long-horizon	Short-horizon	Short-run expectation	Long-run expectation	Conditional	Unconditional
Historical												
Ibbotson Associates	3.8% [7]	8.4% [31]		X		X		X		X		X
Social Security												
Office of the Chief Actuary [1]	2.3%,3.0% [8]	4.7%,4.0% [32]	X		X		X			X		X
John Campbell [2]	3% to 3.5% [9]	1.5-2.5%, 3-4% [33]	X		X	X	X	X		X	X	
Peter Diamond	2.2% [10]	<4.8% [34]	X		X		X			X	X	
Peter Diamond [3]	3.0% [11]	3.0% to 3.5% [35]	X		X		X			X	X	
John Shoven [4]	3.0%,3.5% [12]	3.0% to 3.5% [36]	X		X		X			X	X	
Puzzle Research												
Robert Arnott and Peter Bernstein	3.7% [13]	2.4% [37]	X		X		X			X	X	
Robert Arnott and Ronald Ryan	4.1% [14]	-0.9% [38]	X		X		X			X	X	
John Campbell and Robert Shiller	N/A	Negative [39]	X		?		?		X		X	
James Claus and Jacob Thomas	7.64% [15]	3.39% or less [40]		X		X	X			X	X	
George Constantinides	2.0% [16]	6.9% [41]	X			X		X		X		X
Bradford Cornell	5.6%, 3.8% [17]	3.5-5.5%, 5-7% [42]		X		X	X	X		X	X	
Dimson, Marsh, & Staunton	1.0% [18]	5.4% [43]	X			X		X		X	X	
Eugene Fama and Kenneth French	3.24% [19]	3.83% & 4.78% [44]	X			X		X		X		X
Robert Harris and Felicia Marston	8.53% [20]	7.14% [45]		X		X	X		X		X	
Roger Ibbotson and Peng Chen	2.05% [21]	4% and 6% [46]	X		X	X	X			X		X
Jeremy Siegel	4.0% [22]	-0.9% to -0.3% [47]	X		X		X			X	X	
Jeremy Siegel	3.5% [23]	2-3% [48]	X		X		X			?	X	
Surveys												
John Graham and Campbell Harvey	? by survey [24]	3-4.7% [49]		X		?	X		X		X	
Ivo Welch	N/A [25]	7% [50]		X		X		X		X	X	
Ivo Welch [5]	5% [26]	5.0% to 5.5% [51]		X		X		X		X	X	
Misc.												
Barclays Global Investors	5% [27]	2.5%, 3.25% [52]		X	X		X		X		X	
Richard Brealey and Stewart Myers	N/A [28]	6 to 8.5% [53]		X		X		X		X		X
Burton Malkiel	5.25% [29]	2.75% [54]		X	X		X			X	X	
Richard Wendt [6]	5.5% [30]	3.3% [55]		X		X	X			X	X	

Long-run expectation considered to be a forecast of more than 10 years.
Short-run expectation considered to be a forecast of 10 years or less.

Source	Risk-free Rate	ERP Estimate	Data Period	Methodology
Historical				
Ibbotson Associates	3.8% [7]	8.4% [31]	1926-2002	Historical
Social Security				
Office of the Chief Actuary [1]	2.3%, 3.0% [8]	4.7%, 4.0% [32]	1900-1995, Projecting out 75 years	Historical
John Campbell [2]	3% to 3.5% [9]	1.5-2.5%, 3-4% [33]	Projecting out 75 years	Historical & Ratios (Div/Price & Earn Gr)
Peter Diamond	2.2% [10]	<4.8% [34]	Last 200 yrs for eq/ 75 for bonds, Proj 75 yrs	Fundamentals: Div Yld, GDP Gr
Peter Diamond [3]	3.0% [11]	3.0% to 3.5% [35]	Projecting out 75 years	Fundamentals: Div/Price
John Shoven [4]	3.0%, 3.5% [12]	3.0% to 3.5% [36]	Projecting out 75 years	Fundamentals: P/E, GDP Gr
Puzzle Research				
Robert Arnott and Peter Bernstein	3.7% [13]	2.4% [37]	1802 to 2001, normal	Fundamentals: Div Yld & Gr
Robert Arnott and Ronald Ryan	4.1% [14]	-0.9% [38]	Past 74 years, 74 year projection [56]	Fundamentals: Div Yld & Gr
John Campbell and Robert Shiller	N/A	Negative [39]	1871 to 2000, ten-year projection	Ratios: P/E and Div/Price
James Claus and Jacob Thomas	7.64% [15]	3.39% or less [40]	1985-1998, long-term	Abnormal Earnings model
George Constantinides	2.0% [16]	6.9% [41]	1872 to 2000, long-term	Hist. and Fund.: Price/Div & P/E
Bradford Cornell	5.6%, 3.8% [17]	3.5-5.5%, 5-7% [42]	1926-1997, long run forward-looking	Weighing theoretical and empirical evid
Dimson, Marsh, & Staunton	1.0% [18]	5.4% [43]	1900-2000, prospective	Adj hist ret, Var of Gordon gr model
Eugene Fama and Kenneth French	3.24% [19]	3.83% & 4.78% [44]	Estimate for 1951-2000, long-term	Fundamentals: Dividends and Earnings
Robert Harris and Felicia Marston	8.53% [20]	7.14% [45]	1982-1998, expectational	Fin analysts' est, div gr model
Roger Ibbotson and Peng Chen	2.05% [21]	4% and 6% [46]	1926-2000, long-term	Historical and supply side approaches
Jeremy Siegel	4.0% [22]	-0.9% to -0.3% [47]	1871 to 1998, forward-looking	Fundamentals: P/E, Div Yld, Div Gr
Jeremy Siegel	3.5% [23]	2-3% [48]	1802-2001, forward-looking	Earnings yield
Surveys				
John Graham and Campbell Harvey	? by survey [24]	3-4.7% [49]	2Q 2000 thru 3Q 2002, 1 & 10 year proj	Survey of CFO's
Ivo Welch	N/A [25]	7% [50]	30-Year forecast, surveys in 97/98 & 99	Survey of financial economists
Ivo Welch [5]	5% [26]	5.0% to 5.5% [51]	30-Year forecast, survey around August 2001	Survey of financial economists
Misc.				
Barclays Global Investors	5% [27]	2.5%, 3.25% [52]	Long-run (10-year) expected return	Fundamentals: Inc, Earn Gr, & Repricing
Richard Brealey and Stewart Myers	N/A [28]	6 to 8.5% [53]	1926-1997	Predominantly Historical
Burton Malkiel	5.25% [29]	2.75% [54]	1926 to 1997, estimate millennium [57]	Fundamentals: Div Yld, Earn Gr
Richard Wendt [6]	5.5% [30]	3.3% [55]	1960-2000, estimate for 2001-2015 period	Linear regression model

Footnotes:

[1] Social Security Administration.

[2] Presented to the Social Security Advisory Board.

[3] Presented to the Social Security Advisory Board. Update of 1999 article.

[4] Presented to the Social Security Advisory Board.

[5] Update to Welch 2000.

[6] Newsletter of the Investment Section of the Society of Actuaries.

[7] Arithmetic mean of U.S. Treasury bills annual total returns from 1926-2002.

[8] 2.3% Long-run real yield on Treasury bonds; used for Advisory Council proposals. 3.0% Long-term real yield on Treasury bonds; used in 1999 Social Security Trustees Report.

[9] Estimate for safe real interest rates in the future based on yield of long-term inflation-indexed Treasury securities of 3.5% and short-term real interest rates recently averaging about 3%.

[10] Real long-term bond yield using 75 year historical average.

[11] Real yield on long-term Treasuries (assumption by OCACT).

[12] 3.0% is the OCACT assumption. 3.5% is the real return on long-run (30-year) inflation-indexed Treasury securities.

[13] Long-term expected real geometric bond return (10 year-horizon).

[14] The yield on US government inflation-indexed bonds (starting bond real yield in Jan 2000).

[15] Average 10-year Government T-bond yield between 1985 and 1998 (yield of 11.43% in 1985 to 5.64% in 1998. The mean 30-year risk-free rate for each year of the U.S. sample period is 31 basis points higher than the mean 10-year risk-free rate.

[16] Rolled-over real arithmetic return of three-month Treasury bills and certificates.

[17] Historical 20-year Treasury bond return of 5.6%. Yield on 20-year Treasury bonds in 1998 was approximately 6%. Historical 1-month Treasury bill return of 3.8%. Yield on 1-month Treasury bills in 1998 was approximately 4%.

[18] United States historical arithmetic real Treasury bill return over 1900-2000 period. 0.9% geometric Treasury bill return.

[19] Average real return on six-month commercial paper (proxy for risk-free interest rate). Substituting the one-month Treasury bill rate for the six-month commercial paper rate causes estimates of the annual equity premium for 1951-2000 to rise by about 1.00%.

[20] Average yield to maturity on long-term U.S. government bonds, 1982-1998.

[21] Real, geometric risk-free rate. Geometric risk-free rate with inflation (nominal) 5.13%.
Nominal yield equivalent to historical geometric long-term government bond income return for 1926-2000.

[22] The ten- and thirty-year TIPS bond yielded 4.0% in August 1999.

[23] Return on inflation-indexed securities.

[24] Current 10-year Treasury bond yield. Survey administered from June 6, 2000 to June 4, 2002. The rate on the 10-year Treasury bond changes in each survey. For example, in the Dec. 1, 2000 survey, the current annual yield on the 10-year Treasury bond was 5.5%. For the June 6, 2001 survey, the current annual yield on the 10-year Treasury bond was 5.3%.

[25] Arithmetic per-annum average return on rolled-over 30-day T-bills.

[26] Average forecast of arithmetic risk-free rate of about 5% by deducting ERP from market return.

[27] Current nominal 10-year bond yield.

[28] Return on Treasury bills. Treasury bills yield of about 5 percent in mid-1998. Average historical return on Treasury bills 3.8 percent.

[29] Good quality corporate bonds will earn approximately 6.5% to 7%. Long-term zero-coupon Treasury bonds will earn about 5.25%. Long-term TIPS will earn a real return of 3.75%.

[30] 1/1/01 Long T-Bond yield; uses initial bond yields in predictive model.

[31] Arithmetic short-horizon expected equity risk premium. Arithmetic intermediate-horizon expected equity risk premium 7.4%. Arithmetic long-horizon expected equity risk premium 7.0%. Geometric short-horizon expected equity risk premium 6.4%.

[32] Geometric equity premium over long-term Treasury securities. OCACT assumes a constant geometric real 7.0% stock return.

[33] Long-run average equity premium of 1.5% to 2.5% in geometric terms and 3% to 4% in arithmetic terms.

[34] Lower return over the next decade, followed by a geometric, real 7.0% stock return for remaining 65 years or lower rate of return for entire 75-year period (obscures pattern of returns).

[35] Most likely poor return over the next decade followed by a return to historic yields. Working from OCACT stock return assumption, he gives a single rate of return on equities for projection purposes of 6.0 to 6.5% (geometric, real).

[36] Geometric real stock return over the geometric real return on long-term government bonds.

[37] Expected geometric return over long-term government bonds. Their current risk premium is approximately zero, and their recommended expectation for the future real return for both stocks and bonds is 2-4 percent. The "normal" level of the risk premium is modest (2.4 percent or quite possibly less).

[38] Geometric real returns on stocks are likely to be in the 3%-4% range for the foreseeable future (10-20 years).

[39] Substantial declines in real stock prices, and real stock returns below zero, over the next ten years (2001-2010).

[40] The equity premium for each year between 1985 and 1998 in the United States. Similar results for five other markets.

[41] Unconditional, arithmetic mean aggregate equity premium over the 1872-2000 period. Over the period 1951 to 2000, the adjusted estimate of the unconditional mean premium is 6.0%. The corresponding estimate over the 1926 to 2000 period is 8.0%. Sharp distinction between conditional, short-term forecasts of the mean equity return and premium and estimates of the unconditional mean.

[42] Long run arithmetic future ERP of 3.5% to 5.5% over Treasury bonds and 5% to 7% over Treasury bills. Compares estimates to historical returns of 7.4% for bond premium and 9.2% for bill premium.

[43] 5.4% United States arithmetic expected future ERP relative to bills. 4.0% World (16 countries) arithmetic expected future ERP relative to bills. 4.1% United States geometric expected future ERP relative to bills. 3.0% World (16 countries) geometric expected future ERP relative to bills.

[44] 3.83% unconditional expected annual simple equity premium return (referred to as the annual-bias adjusted estimate of the annual equity premium) using dividend growth model. 4.78% unconditional expected annual simple equity premium return (referred to as the annual-bias adjusted estimate of the annual equity premium) using earnings growth model. Compares these results against historical real equity risk premium of 7.43% for 1951-2000.

[45] Average expectational risk premium. Because of the possible bias of analysts' optimism, the estimates are interpreted as "upper bounds" for the market premium. The average expectational risk premium is approximately equal to the arithmetic (7.5%) long-term differential between returns on stocks and long-term government bonds.

[46] 4% geometric (real) and 6% arithmetic (real). Forward looking long-horizon sustainable equity risk premium.

[47] Using the dividend discount model, the forward-looking real long-term geometric return on equity is 3.3%. Based on the earnings yield, the forward-looking real long-term geometric return on equity is between 3.1% and 3.7%.

[48] Future geometric equity premium. Future real return on equities of about 6%.

[49] The 10-year premium. The one-year risk premium averages between 0.4 and 5.2% depending on the quarter surveyed.

[50] Arithmetic 30-year forecast relative to short-term bills; 10-year same estimate. Second survey 6.8% for 30 and 10-year estimate. 1-year horizon between 0.5% and 1.5% lower. Geometric 30-year forecast around 5.2% (50% responded to this question).

[51] Arithmetic 30-year equity premium (relative to short-term T-bills). Geometric about 50 basis points below arithmetic. Arithmetic 1-year equity premium 3 to 3.5%.

[52] 2.5% current (conditional) geometric equity risk premium. 3.25% long-run, geometric normal or equilibrium equity risk premium.

[53] Extra arithmetic return versus Treasury bills. "Brealey and Myers have no official position on the exact market risk premium, but we believe a range of 6 to 8.5 percent is reasonable for the United States. We are most comfortable with figures towards the upper end of the range."

[54] The projected geometric (nominal) total return for the S&P 500 is 8 percent per year.

[55] Arithmetic mean 15 year horizon.

[56] 74 years since Dec 1925 and 74 years starting Jan 2000.

[57] Estimate the early decades of the twenty-first century.

Appendix C
Estimating a Short-Horizon Arithmetic Unconditional Equity Risk Premium

Source	Risk-free Rate	ERP Estimate	Stock Return Estimate	Geometric to arithmetic	Real to nominal	Conditional to unconditional [60]	Fixed short-horizon RFR	Short-horizon arithmetic unconditional ERP estimate
	I	II	III	IV	V	VI	VII	VIII
Historical								
Ibbotson Associates	3.8% [7]	8.4% [31]	12.2%	0.0%	0.0%	0.00%	3.8%	8.4%
Social Security								
Office of the Chief Actuary [1]	2.3%,3.0% [8]	4.7%,4.0% [32]	7.0%	2.0%	3.1%	0.00%	3.8%	8.3%
John Campbell [2]	3% to 3.5% [9]	1.5-2.5%, 3-4% [33]	6.0%-7.5%	0.0%	3.1%	0.46%	3.8%	5.8%-7.3%
Peter Diamond	2.2% [10]	<4.8% [34]	<7.0%	2.0%	3.1%	0.46%	3.8%	<8.8%
Peter Diamond [3]	3.0% [11]	3.0% to 3.5% [35]	6.0%-6.5%	2.0%	3.1%	0.46%	3.8%	7.8%-8.3%
John Shoven [4]	3.0%,3.5% [12]	3.0% to 3.5% [36]	6.0%-7.0%	2.0%	3.1%	0.46%	3.8%	7.8%-8.8%
Puzzle Research								
Robert Arnott and Peter Bernstein	3.7% [13]	2.4% [37]	6.1%	2.0%	3.1%	0.46%	3.8%	7.9%
Robert Arnott and Ronald Ryan	4.1% [14]	-0.9% [38]	3.2%	2.0%	3.1%	0.46%	3.8%	5.0%
John Campbell and Robert Shiller	N/A	Negative [39]	Negative	N/A	N/A	N/A	N/A	N/A
James Claus and Jacob Thomas	7.64% [15]	3.39% or less [40]	11.03%	0.0%	0.0%	0.46%	3.8%	7.69%
George Constantinides	2.0% [16]	6.9% [41]	8.9%	0.0%	3.1%	0.00%	3.8%	8.2%
Bradford Cornell	5.6%, 3.8% [17]	3.5-5.5%, 5-7% [42]	8.8%-10.8%	0.0%	0.0%	0.46%	3.8%	5.5%-7.5%
Dimson, Marsh, & Staunton	1.0% [18]	5.4% [43]	6.4% [58]	0.0%	3.1%	0.46%	3.8%	6.2% [61]
Eugene Fama and Kenneth French	3.24% [19]	3.83% & 4.78% [44]	7.07%-8.02%	0.0%	3.1%	0.00%	3.8%	6.37%-7.32%
Robert Harris and Felicia Marston	8.53% [20]	7.14% [45]	12.34% [59]	0.0%	0.0%	0.46%	3.8%	9.00%
Roger Ibbotson and Peng Chen	2.05% [21]	4% and 6% [46]	8.05%	0.0%	3.1%	0.00%	3.8%	7.35%
Jeremy Siegel	4.0% [22]	-0.9% to -0.3% [47]	3.1%-3.7%	2.0%	3.1%	0.46%	3.8%	4.9%-5.5%
Jeremy Siegel	3.5% [23]	2-3% [48]	5.5%-6.5%	2.0%	3.1%	0.46%	3.8%	7.3%-8.3%
Surveys								
John Graham and Campbell Harvey	? by survey [24]	3-4.7% [49]	8.3%-10.2%	N/A	0.0%	0.46%	3.8%	5.0%-6.9%
Ivo Welch	N/A [25]	7% [50]	N/A	0.0%	0.0%	0.46%	0.0%	7.5%
Ivo Welch [5]	5% [26]	5.0% to 5.5% [51]	10.0%-10.5%	0.0%	0.0%	0.46%	3.8%	6.7%-7.2%
Misc.								
Barclays Global Investors	5% [27]	2.5%, 3.25% [52]	7.5%,8.25%	2.0%	0.0%	0.46%	3.8%	6.16%-6.91%
Richard Brealey and Stewart Myers	N/A [28]	6 to 8.5% [53]	N/A	0.0%	0.0%	0.00%	0.0%	6.0%-8.5%
Burton Malkiel	5.25% [29]	2.75% [54]	8.0%	2.0%	0.0%	0.46%	3.8%	6.7%
Richard Wendt [6]	5.5% [30]	3.3% [55]	8.8%	0.0%	0.0%	0.46%	3.8%	5.5%

Column formulas:
III = I + II
VIII = III + IV + V + VI – VII

Source for adjustments:
2003 Ibbotson Yearbook Table 2-1 page 33
Fama French 2002 (see footnote 60)

Footnotes (1-57 from Appendix B):
[58] World estimate of 5.0%.
[59] Long risk-free of 5.2% plus 7.14%.
[60] For the 1951-2000 period, Fama and French (2002) adjust the conditional dividend growth model estimate upwards by 1.28% for an unconditional estimate, and they make a 0.46% upwards adjustment to the earnings growth model. We select the smaller of the two as an approximate minimum adjustment. For the longer period of 1872-2000, a comparable adjustment would be 0.82% for the dividend growth model and 0.54% for the 1872-1950 period using a dividend growth model. Earnings growth rates are shown by Fama and French only for the 1951-2000 period.
[61] World estimate of 4.8%.

Appendix D

Historical and Forecasted Equity Returns- All Ibbotson and Chen Models (Percent).

Method/ Model	Sum	Inflation	Real Risk-Free Rate	Equity Risk Premium	Real Capital Gain	g(Real EPS)	g(Real Div)	- g (Pay out Ratio)	g (BV)	g (ROE)	g P/E)	g(Real GDP/ POP)	g(FS-GDP/ POP)	Income Return	Re-Investment + Interaction	Additional Growth	Forecast Earnings Growth
Column #	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII	XIII	XIV	XV	XVI	XVII
Historical																	
Method 1	10.70	3.08	2.05	5.24											0.33		
Method 2	10.70	3.08			3.02									4.28	0.32		
Method 3	10.70	3.08				1.75					1.25			4.28	0.34		
Method 4	10.70	3.08					1.23	0.51			1.25			4.28	0.35		
Method 5	10.70	3.08							1.46	0.31	1.25			4.28	0.31		
Method 6	10.70	3.08										2.04	0.96	4.28	0.32		
Forecast with Historical Dividend Yield																	
Model 3F	9.37	3.08				1.75								4.28	0.26		
Model 3F (ERP)	9.37	3.08	2.05	3.97											0.27		
Forecast with Current Dividend Yield																	
Model 4F	5.44	3.08					1.23							1.10 [a]	0.03		
Model 4F (ERP)	5.44	3.08	2.05	0.24											0.07		
Model $4F_2$	9.37	3.08					1.23	0.51						2.05 [b]	0.21	2.28	
Model $4F_2$ (FG)	9.37	3.08												1.10 [a]	0.21		4.98

Source: The data and format was made available by Ibbotson/Chen and is reprinted with permission by Ibbotson Associates.
Corresponds to Ibbotson/Chen Table 2 Exhibit; column numbers have been added.

[a] 2000 dividend yield

[b] Assuming the historical average dividend-payout ratio, the 2000 dividend yield is adjusted up 0.95 pps.

	Formula*	Description of Method
Historical		
Method 1	I=(1+II)*(1+III)*(1+IV)-1	Building Blocks Method: inflation, real risk-free rate, and equity risk premium.
Method 2	I=[(1+II)*(1+V)-1]+XIV+XV	Capital Gain and Income Method: inflation, real capital gain, and income return.
Method 3	I=[(1+II)*(1+VI)*(1+XI)-1]+XIV+XV	Earnings Model: inflation, growth in earnings per share, growth in price to earnings ratio, and income return.
Method 4	I=[(1+II)*(1+XI)*(1+VII)/(1-VIII)-1]+XIV+XV	Dividends Model: inflation, growth rate of price earnings ratio, growth rate of the dollar amount of dividends after inflation, growth rate of payout ratio, and dividend yield (income return).
Method 5	I=[(1+II)*(1+XI)*(1+IX)*(1+X)-1]+XIV+XV	Return on Book Equity Model: inflation, growth rate of price earnings ratio, growth rate of book value, growth rate of ROE, and income return.
Method 6	I=[(1+II)*(1+XII)*(1+XIII)-1]+XIV+XV	GDP Per Capita Model: inflation, real growth rate of the overall economic productivity (GDP per capita), increase of the equity market relative to the overall economic productivity, and income return.
Forecast with Historical Dividend Yield		
Model 3F	I=[(1+II)*(1+VI)-1]+XIV+XV	Forward-looking Earnings Model: inflation, growth in real earnings per share, and income return.
Model 3F (ERP)	IV=(1+I)/[(1+II)*(1+III)]-1	Using Model 3F result to calculate ERP.
Forecast with Current Dividend Yield		
Model 4F	I=[(1+II)*(1+VII)-1]+XIV+XV	Forward-looking Dividends Model: inflation, growth in real dividend, and dividend yield (income return); also referred to as Gordon model.
Model 4F (ERP)	IV=(1+I)/[(1+II)*(1+III)]-1	Using Model 4F result to calculate ERP.
Model $4F_2$	I=[(1+II)*(1+VII)*(1+VIII)-1]+XIV+XV+XVI	Attempt to reconcile Model 4F and Model 3F.
Model $4F_2$ (FG)	XVII=[(1+I)/(1+II)-1]-XIV-XV	Using Method $4F_2$ result to calculate forecasted earnings.

Explanation of Ibbotson/Chen Table 2 Exhibit; using column numbers to represent formula.

Estimating the Ex Ante Equity Premium

R. Glen Donaldson
Sauder School of Business
University of British Columbia
glen.donaldson AT sauder.ubc.ca

Mark J. Kamstra
Schulich School of Business
York University
mkamstra AT schulich.yorku.ca

Lisa A. Kramer
Rotman School of Management
University of Toronto
Lkramer AT rotman.utoronto.ca

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Estimating the Ex Ante Equity Premium

Abstract

We find that the true ex ante equity premium very likely lies within 50 basis points of 3.5%. This estimate is similar to values obtained in some recent studies but is considerably more precise. In addition to narrowing the range of plausible ex ante equity premia, we also find that equity premium models that allow for time-variation, breaks, and/or trends are the models that best match the experience of US markets and are the only models not rejected by our specification tests. This suggests that time-variation, breaks, and/or trends are critical features of the equity premium process. Our approach involves simulating the distribution from which interest rates, dividend growth rates, and equity premia are drawn and determining the prices and returns consistent with these distributions. We achieve the narrower range of ex ante equity premium values and the narrower set of plausible models by comparing statistics that arise from our simulations with key financial characteristics of the US economy, including the mean dividend yield, return volatility, and mean return. Our findings are achieved in part with the imposition of more structure than is typically exploited in the literature. In order to mitigate the potential for misspecification with this additional structure, we consider a broad collection of models that variously do or do not incorporate features such as an adjustment in dividend growth rates to account for recently increased share repurchase activity, sampling uncertainty in generating model parameters, and cross-correlation between interest rates, dividend growth rates, and equity premia.

Estimating the Ex Ante Equity Premium

Financial economic theory is often concerned with the premium that investors demand ex ante, when they first decide whether to purchase risky stocks instead of risk-free debt. In contrast, empirical tests of the equity premium often focus on the return investors received ex post.[1] It is well known that estimates of the ex ante equity premium based on ex post data can be very imprecise; such estimates have very wide margins of error, as wide as 1000 basis points in typical studies and 320 basis points in some recent studies. This fact makes it challenging to employ the equity premium estimates for common practical purposes, including evaluating the equity premium puzzle, performing valuation, and conducting capital budgeting. The imprecision of traditional equity premium estimates also makes it difficult to determine if the equity premium has changed over time. Our goals, therefore, are to develop a more precise estimate of the ex ante equity premium and to determine what kind of equity premium model can be supported by the experience of US markets. We accomplish these goals by employing simulation techniques that identify a range of models of the equity premium and the values of the ex ante equity premium that are consistent with values of several key financial statistics that are observed in US market data, including dividend growth rates, interest rates, Sharpe ratios, price-dividend ratios, volatilities, and of course the ex post equity premium.

Our results suggest that the mean ex ante equity premium lies within 50 basis points of 3.5%. These results stand even when we allow for investors' uncertainty about the true state of the world. The tightened bounds are achieved in part with the imposition of more structure than has been commonly employed in the equity premium literature. In order to mitigate the potential for misspecification with this additional structure, we consider a broad collection of models that variously do or do not incorporate features such as a conditionally time-varying equity premium, a downward trend in the equity premium, a structural break in the equity premium, an adjustment in dividend growth rates to account for increased share repurchase activity in the last 25 years, sampling uncertainty in generating model parameters, a range of time series models, and cross-correlation between interest rates, dividend growth rates, and equity premia. We also find that

[1]The equity premium literature is large, continuously growing, and much too vast to fully cite here. For recent work, see Bansal and Yaron (2004), Graham and Harvey (2005), and Jain (2005). For excellent surveys see Kocherlakota (1996), Siegel and Thaler (1997), Mehra and Prescott (2003), and Mehra (2003).

equity premium models that allow for time-variation, breaks, and/or trends in the equity premium process are the models that best match the experience of US markets and are the only models not rejected by our specification tests. This suggests that time-variation, breaks, and/or trends are critical features of the equity premium process, itself an important finding.

We draw on two relatively new techniques in order to provide a more precise estimate of the equity premium than is currently available. The first technique builds on the fundamental valuation dividend discounting method of Donaldson and Kamstra (1996). This technique permits the simulation of fundamental prices, returns, and return volatility for a given ex ante equity premium. Donaldson and Kamstra find that if we allow dividend growth rates and discount rates to be time-varying and dependent, as well as cross-correlated, the fundamental prices and returns that come out of dividend discounting match observed prices and returns, even during extreme events like stock market crashes. The second technique is simulated method of moments (SMM).[2] An attractive feature of SMM is that the estimation of parameters requires only that the model, with a given set of parameters, can generate data. SMM forms estimates of model parameters by using a given model with a given set of parameter values to simulate moments of the data (for instance means or volatilities), measuring the distance between the simulated moments and the actual data moments, and repeating with new parameter values until the parameter values that minimize the (weighted) distance are found.[3] The parameter estimates that minimize this distance are consistent for the true values, are asymptotically normally distributed, and display the attractive feature of permitting tests that can reject misspecified models. The SMM technique has been described as "estimating on one group of moments, testing on another." See Cochrane (2001, Section 11.6). We use SMM rather than GMM because, as we show below, the economic model we use is nonlinear in the parameters and cannot be solved without the use of SMM.

We exploit the dividend discounting method of Donaldson and Kamstra to generate simulated fundamental prices, dividends, returns, and derivative moments such as the mean ex post equity

[2]Simulated method of moments was developed by McFadden (1989) and Pakes and Pollard (1989), and a helpful introduction to the technique is provided in Carrasco and Florens (2002). Examples of papers that employ SMM in an asset pricing context are Duffie and Singleton (1993) and Corradi and Swanson (2005).

[3]The typical implementation of SMM is to weight the moments inversely to their estimated precision; that is minimize the product of the moments weighted by the inverse of the covariance matrix of the moments. This is the approach we adopt.

premium, mean dividend yield, and return volatility for a given ex ante equity premium. We minimize (by choice of the ex ante equity premium) the distance between the simulated moments that the model produces and the moments observed in US stock markets over the past half century. That is, given various characteristics of the US economic experience (such as low interest rates and a high ex post equity premium, high Sharpe ratios and low dividend yields, *etc.*), we determine the range of values of the ex ante equity premium and the set of equity premium models that are most likely to have generated the observed collection of sample moments.

To undertake our study, we consider a broad collection of models, including models with and without conditional time-variation in the equity premium process, with and without trends in the equity premium, with and without breaks in the equity premium, with and without breaks in the dividend growth rate, as well as various autoregressive specifications for dividend growth rates, interest rates, and the equity premium. Virtually every model we consider achieves a minimum distance between the simulated moments and the actual data moments by setting the ex ante equity premium between 3% and 4%, typically very close to 3.5%. That is, the equity premium estimate is very close to 3.5% across our models. Further, the range of ex ante equity premium values that can be supported by the US data for a given model is typically within plus or minus 50 basis points of 3.5%. Our models of fundamentals, which capture the dynamics of actual US dividend and interest rate data, imply that the true ex ante equity premium is 3.5% plus or minus 50 basis points. Simpler models of fundamental valuation, such as the Gordon (1962) constant dividend growth model, are overwhelmingly rejected by the data. Models of the equity premium which do not allow time-variation, trends, or breaks are also rejected by the SMM model specification tests. While we restrict our attention to a stock market index in this study, the technique we employ is more broadly applicable to estimating the equity premium of an individual firm.

In the literature to date, empirical work investigating the equity premium has largely consisted of a series of innovations around a common theme: producing a better estimate of the mean ex ante equity premium. Recent work in the area has included insights such as exploiting dividend yields or earnings yields to provide new, more precise estimates of the return to holding stocks (see Fama and French, 2002, and Jagannathan, McGrattan, and Scherbina, 2000), looking across many countries to account for survivorship issues (see Jorion and Goetzmann, 1999), looking across many countries to decompose the equity premium into dividend growth, price-dividend ratio, dividend yield, and real exchange rate components (see Dimson, Marsh, and Staunton, 2007), modeling equity premium structural breaks in a Bayesian econometric framework (see Pástor and Stambaugh, 2001), or computing out-of-sample forecasts of the distribution of excess returns, allowing for structural breaks which are identified in real time (see Maheu and McCurdy, 2007). Most of this work estimates the ex ante equity premium by considering one moment of the data at a time, typically the mean difference between an estimate of the return to holding equity and a risk free rate, though Maheu and McCurdy (2007) consider higher-order moments of the excess return distribution and Pástor and Stambaugh (2001) incorporate return volatility and direction of price movements through their use of priors.

Unfortunately, the equity premium is still estimated without much precision. Pástor and Stambaugh (2001), exploiting extra information from return volatility and prices, narrow a two standard deviation confidence interval around the value of the ex ante equity premium to plus or minus roughly 280 basis points around a mean premium estimate of roughly 4.8% (a range that spans 2% to 7.6%) and determine that the data strongly support at least one break in the equity premium in the last half century. Fama and French (2002), based on data from 1951 to 2000, provide point estimates of the ex post equity premium of 4.32% (based on earnings growth rate fundamentals) plus or minus roughly 400 basis points (again, two standard deviations) and of 2.55% (based on dividend growth rate fundamentals) plus or minus roughly 160 basis points: a range of approximately 0.95% to 4.15%. That is, the plausible range of equity premia that emerge from Fama and French's study occupy a confidence bound with a width of anywhere from 320 to 800 basis points. Claus and Thomas (2001), like Fama and French (2002), make use of fundamental information to form lower estimates of the ex post equity premium, but their study covers a shorter time period relative to the Fama and French study   14 years versus 50 years   yielding point estimates that are subject to at least as much variability as the Fama and French estimates.

Not only are the point estimates from the existing literature imprecisely estimated in terms of their standard error, there is also less of an emerging consensus than one would hope. Fama and French (2002) produce point estimates of 2.55% (using dividend yields) and 4.78% (using earnings yields), Pástor and Stambaugh (2001) estimate the equity premium at the end of the 1990s to be 4.8%, and Claus and Thomas (2001) estimate the equity premium to be *no more* than 3%. Welch (2000), surveying academic financial economists, estimates the consensus equity premium to be between 6% and 7% (depending on the horizon). Based on a survey of US CFOs, Graham and Harvey (2005) estimate the ten-year equity premium to be 3.66%. We believe that the lack of consensus across the literature is intimately tied to the imprecision of techniques typically used to estimate the equity premium, such as the simple average excess return. That is, the various estimates cited above all fall within two standard errors of the sample mean estimate of the equity premium, based on US data. Further, the studies that provide these estimates do not explicitly consider which models of the equity premium process can be rejected by actual data, though Pástor and Stambaugh's analysis strongly supports a model that incorporates breaks in the equity premium process.

The remainder of our paper proceeds as follows. The basic methodology of our simulation approach to estimating equity premia is presented in Section 1, along with important details on estimating the equity premium. (Appendices to the paper provide detailed explanations of the technical aspects of our simulations, including calibration of key model parameters.) In Section 2 we compare univariate financial statistics that arise in our simulations with US market data, including dividend yields, Sharpe ratios, and conditional moments including ARCH coefficients. Our results confirm that the simulations generate data broadly consistent with the US market data and, taken one-at-a-time, these financial statistics imply that the ex ante equity premium lies in a range much narrower than between 2% and 8%. We determine how much narrower in Section 3 by exploiting the full power of the simulation methodology. We compare joint multivariate distributions of our simulated data with observed US data, yielding a very precise estimate of the ex ante equity premium and providing strong rejections of models of the equity premium process that fail to incorporate time variation, breaks, and/or trends. We find the range of ex ante equity premium values is very narrow: 3.5% plus or minus 50 basis points. Our consideration of a broad collection of possible data generating processes and models lends confidence to the findings. Section 4 concludes.

I Methodology

Consider a stock for which the price P_t is set at the beginning of each period t and which pays a dividend D_{t+1} at the end of period t. The return to holding this stock (denoted R_t) is defined as

$$R_t = \frac{D_{t+1} + P_{t+1} - P_t}{P_t}.$$

The risk-free rate, set at the beginning of each period, is denoted $r_{t,f}$. The ex ante equity premium, π, is defined as the difference between the expected return on risky assets, $E\{R_t\}$, and the expected risk-free rate, $E\{r_{t,f}\}$:[4]

$$\pi \equiv E\{R_t\} - E\{r_{t,f}\}. \tag{1}$$

We do not observe this ex ante equity premium. Empirically, we only observe the returns that investors actually receive ex post, after they have purchased the stock and held it over some period of time during which random economic shocks impact prices. Hence, the ex post equity premium is typically estimated using historical equity returns and risk-free rates. Define $\overline{R}$ as the average historical annual return on the S&P 500 and $\overline{r}_f$ as the average historical return on US T-bills. Then we can calculate the estimated ex post equity premium, $\hat{\pi}$, as follows:

$$\hat{\pi} \equiv \overline{R} - \overline{r}_f. \tag{2}$$

Given that the world almost never unfolds exactly as one expects, there is no reason to believe that the stock return we estimate ex post is exactly the same as the return investors anticipated ex ante. It is therefore difficult to argue that just because we observe a 6% ex post equity premium in the US data, the premium that investors demand ex ante is also 6% and thus a puzzling challenge to economic theory. So we ask the following question: If investors' true ex ante premium is π, what is the probability that the US economy could randomly produce an ex post premium of at least 6%? The answer to this question has implications for whether or not the 6% ex post premium

[4]See, for instance, Mehra and Prescott (1985), Equation (14). We will consider time-varying equity premium models below.

observed in the US data is consistent with various ex ante premium values, π, with which standard economic theory may be more compatible. We also ask a deeper question: If investors' true ex ante premium is π, what is the probability that we would observe the various *combinations* of key financial statistics and yields that have been realized in the US, such as high Sharpe ratios and low dividend yields, high return volatility and a high ex post equity premium, and so on? The analysis of multivariate distributions of these statistics allows us to narrow substantially the range of equity premia consistent with the US market data, especially relative to previous studies that have considered univariate distributions.

Because the *empirical* joint distribution of the financial statistics we wish to consider is difficult or impossible to estimate accurately, in particular the joint distribution *conditional* on various ex ante equity premium values, we use simulation techniques to estimate this distribution. The simulated joint distribution allows us to conduct formal statistical tests that a given ex ante equity premium could have produced the US experience. Most of our models employ a time-varying ex ante equity premium, so that a simulation described as having an ex ante equity premium of 2.75% actually has a mean ex ante equity premium of 2.75%, while period-by-period the ex ante equity premium can vary somewhat from this mean value. In what follows we refer to the ex ante equity premium and the mean ex ante equity premium interchangeably.

A Matching Moments

Consider the valuation of a stock. Define $1 + r_t$ as the gross rate investors use to discount payments received during period t. The price of the stock is then given by Equation (3),

$$P_t = E_t \left\{ \frac{D_{t+1} + P_{t+1}}{1 + r_t} \right\}, \tag{3}$$

where E_t is the conditional expectations operator incorporating information available to the market when P_t is formed, up to but not including the beginning of period t (*i.e.*, information from the end of period $t - 1$ and earlier).

Assuming the usual transversality conditions, we can derive Equation (4) by recursively substituting out for future prices in Equation (3):

$$P_t = E_t\left\{\sum_{j=0}^{\infty}\left(\Pi_{i=0}^{j}\frac{1}{1+r_{t+i}}\right)D_{t+j+1}\right\}. \tag{4}$$

Defining the growth rate of dividends over the period t as $g_t \equiv (D_{t+1} - D_t)/D_t$, we can re-write Equation (4) as

$$P_t = D_t E_t\left\{\sum_{j=0}^{\infty}\left(\Pi_{i=0}^{j}\left[\frac{1+g_{t+i}}{1+r_{t+i}}\right]\right)\right\}. \tag{5}$$

Hence we can re-write Equation (1) as

$$\pi \equiv E\left\{\frac{D_{t+1} + D_{t+1}E_{t+1}\left\{\sum_{j=0}^{\infty}\Pi_{i=0}^{j}\frac{1+g_{t+1+i}}{1+r_{t+1+i}}\right\} - D_t E_t\left\{\sum_{j=0}^{\infty}\Pi_{i=0}^{j}\frac{1+g_{t+i}}{1+r_{t+i}}\right\}}{D_t E_t\left\{\sum_{j=0}^{\infty}\Pi_{i=0}^{j}\frac{1+g_{t+i}}{1+r_{t+i}}\right\}} - r_{t,f}\right\} \tag{6}$$

or

$$\pi \equiv E\left\{\frac{(1+g_t)\left(1+E_{t+1}\left\{\sum_{j=0}^{\infty}\Pi_{i=0}^{j}\frac{1+g_{t+1+i}}{1+r_{t+1+i}}\right\}\right) - E_t\left\{\sum_{j=0}^{\infty}\Pi_{i=0}^{j}\frac{1+g_{t+i}}{1+r_{t+i}}\right\}}{E_t\left\{\sum_{j=0}^{\infty}\Pi_{i=0}^{j}\frac{1+g_{t+i}}{1+r_{t+i}}\right\}} - r_{t,f}\right\}. \tag{7}$$

In the case of a constant equity premium π and a possibly time-varying risk-free interest rate we can re-write Equation (7) as

$$\pi \equiv E\left\{\frac{(1+g_t)\left(1+E_{t+1}\left\{\sum_{j=0}^{\infty}\Pi_{i=0}^{j}\frac{1+g_{t+1+i}}{1+\pi+r_{t+1+i,f}}\right\}\right) - E_t\left\{\sum_{j=0}^{\infty}\Pi_{i=0}^{j}\frac{1+g_{t+i}}{1+\pi+r_{t+i,f}}\right\}}{E_t\left\{\sum_{j=0}^{\infty}\Pi_{i=0}^{j}\frac{1+g_{t+i}}{1+\pi+r_{t+i,f}}\right\}} - r_{t,f}\right\}. \tag{8}$$

Under interesting conditions, such as risk-free rates and dividend growth rates that conditionally time-vary and covary (we consider, for instance, ARMA models and correlated errors for dividend growth rates and interest rates), the individual conditional expectations in Equation (8) are analytically intractable. The difference between the sample mean return and the sample mean risk-free interest rate provides a consistent estimate of π, as shown by Mehra and Prescott (1985), but unfortunately the sample mean difference is very imprecisely estimated, even based on more than 100 years of data.

We note that another consistent estimator of π is one that directly exploits the method of Donaldson and Kamstra (1996), hereafter referred to as the DK method. The DK method uses (ARMA) models for dividend growth rates and interest rates to simulate the conditional expectations $E_t\left\{\sum_{j=0}^{\infty}\prod_{i=0}^{j}\frac{1+g_{t-i}}{1+\pi+r_{t-i,f}}\right\}$ and $E_{t+1}\left\{\sum_{j=0}^{\infty}\prod_{i=0}^{j}\frac{1+g_{t-1+i}}{1+\pi+r_{t+1+i,f}}\right\}$. The DK method allows us, for a given *ex ante* equity premium (or time-varying equity premium process), to simulate the conditional expectations in Equation (8) as well as related (unconditional) moments, including the expected dividend yield, return volatility, ex post equity premium, and Sharpe ratio. Our estimate of π is produced by finding the value of π that minimizes the distance between the collection of simulated moments (produced by the DK procedure) and the analogous sample moments (from the US experience over the last half century). The estimation of these expectations relies on the exact form of the conditional models for dividend growth rates and interest rates, that is, the parameters that characterize these models. A joint estimation of these models' parameters and π (*i.e.* minimizing the distance between simulated and sample moments by varying all the model's parameters and π at once) would be computationally very difficult. We utilize a two-step procedure in which first, for a given ex ante equity premium, we jointly estimate the parameters that characterize the evolution of dividend growth rates and interest rates. We use these models to simulate data to compare with realized S&P 500 data. Second, we do a grid search over values of the ex ante equity premium to find our SMM estimate of π.

It is helpful to consider some examples of estimators based on our simulation technique. The simplest estimator would have us considering only the ex ante equity premium moment, $\pi = E[R_t] - E[r_{f,t}]$, ignoring other potentially informative moments of the data, such as the dividend yield and return volatility. Exploiting the DK procedure, we would find that the π in Equation (8) which matches the ex post equity premium (the sample moment analogue of Equation (8)) is the sample estimate of the ex post equity premium, roughly 6%. That is, in this simplest case, when we minimize the distance between the sample moment and the simulated moment and find that the estimate of the ex ante equity premium is the ex post equity premium, we do so by construction. If the DK method is internally consistent, and if we are fitting only the ex post equity premium sample moment, then the difference must be zero at the value of π equal to the ex post equity premium. This DK estimator of π, considering only one moment of the data, would offer no advantage over the ex post equity premium, which is the traditional estimate of the ex ante equity premium. Adding a second moment to our estimation procedure, say the dividend yield, and minimizing the distance between the simulated and sample moments for the ex post equity premium and the dividend yield *jointly*, would likely lead to a somewhat different ex ante equity premium estimate. Furthermore, the estimate would be more precisely estimated (*i.e.,* with a smaller standard error) since two moments are exploited to estimate the ex ante equity premium, not just one moment, at least if the extra moment of the data provided some unique information about the value of the parameter π.

The DK method provides simulated dividend yields, ex post equity premia, and any other statistic that is derivative to returns and prices, such as return volatility, resulting in a broad collection of simulated moments with which to compare moments of the actual US data in order to derive an estimator. The large collection of available moments makes it likely that our analysis can provide a tighter bound on the value of the ex ante equity premium than has been achieved previously.

B The Simulation

To estimate the joint distribution of the financial quantities of interest, we consider models calibrated to the US economy. (We calibrate to US data over 1952 through 2004, with the starting year of 1952 motivated by the US Federal Reserve Board's adoption of a modern monetary policy regime in 1951.) We provide specific details on the nature of the models we consider and how we conduct our simulations in Appendices 1 and 2. Our entire procedure can be generally summarized in the following five steps:

Step 1: Specify assumptions about the ex ante equity premium demanded by investors. Is the premium constant or time-varying? If constant, what value does it take? If time-varying, how does the value change over time? Are there any structural breaks in the equity premium process over time? Pástor and Stambaugh (2001), among others, provide evidence that the equity premium has been trending downward over the sample period we study, finding a modest downward trend of roughly 0.80% in total since the early 1950s. Pástor and Stambaugh (2001) also find fairly strong support for there having been a structural break over the 1990s which led to a 0.5% drop in the equity premium.[5]

Once the process driving the ex ante equity premium is defined, we can specify the discount rate (which equals the risk-free rate plus the equity premium) that an investor would rationally apply to a forecasted dividend stream in order to calculate the present value of a dividend-paying stock. Note that if the equity premium varies over time, then the models generated in the next step are calibrated to mimic the degree of covariation between interest rates, dividend growth rates, and equity premia observed in the US data.

Step 2: Estimate econometric models for the time-series processes driving actual dividends and interest rates in the US economy, allowing for autocorrelation and covariation as observed in the US data. These models will later be used to Monte-Carlo simulate a variety of potential paths for US dividends and interest rates. The simulated dividend and interest rate paths are of course different in each of these simulated economies because different sequences of random innovations are applied to the common stochastic processes in each case. However, the key drivers of the simulated economies themselves are all still identical to those of the US economy since all economies share common stochastic processes fitted to US data.

Some of the models we consider assume that all cashflows received by investors come in the form of dividends (the standard assumption). Another set of models we consider embed higher cashflows and cashflow growth rates than observed in the US S&P 500 dividend data, to account for the observation of Bagwell and Shoven (1989), Fama and French (2002), and others, that dividends under-report total cashflows to shareholders. As reported by these authors, firms have been increasingly distributing cash to shareholders via share repurchases instead of via dividends, a phenomenon commonly known as disappearing dividends, a practice adopted widely beginning in the late 1970s. Fama and French find evidence that the disappearance of dividends is in part due to an increase in the inflow of new listing to US stock exchanges, representing mostly young companies

[5] A falling equity premium is thought to come from several sources, including the declining cost of diversifying through mutual funds over the last half century, the infeasibility before the advent of mutual funds to hold fully diversified portfolios (hence higher returns required by investors to hold relatively undiversified positions), and the broader pool of investors now participating in equity ownership, sharing in the market risk and presumably lowering the required rate of return to risky assets. See Siegel (1999) and Diamond (2000).

with the characteristics of firms that would not be expected to pay dividends, and in part due to a decline in the propensity of firms to pay dividends.

Thus, for some models in our simulations, we adopt higher cashflows than would be indicated by considering US dividend data alone. On a broad set of data, Grullon and Michaely (2002) find that total payouts to shareholders have remained fairly flat, not growing over the period we consider. To the extent that this is true of the S&P 500 data, the models we consider with upward-trending dividend growth are overly aggressive, but as we show below, the higher dividend growth rate only widens the range of plausible ex ante equity premia, meaning our estimate of the precision of our approach is conservative.

Step 3: Allow for the possibility of estimation error in the parameter values for the dividend growth rate, interest rate, and equity premium time-series models. That is, incorporate into the simulations uncertainty about the true parameter values. This allows for some models with more autocorrelation in the dividend growth, interest rate, and equity premium series, some with less, some with more correlation between the processes, some with less, some with a higher variance or mean of dividend growth and interest rates, some with less, and so on. This uncertainty is measured using the estimated covariance of the parameter estimates from our models generated in Steps 1 and 2, and the procedure to randomly select parameters from the estimated joint distribution of the parameters is detailed in Appendix 1. We also account for investor uncertainty about the true fundamental processes underlying prices and returns by performing tests insensitive to this uncertainty and its impact on prices and returns, as we describe below.

Further details about Steps 1 through 3 are contained in Appendix 1. Before continuing with summarizing Steps 4 and 5 of our methodology, it is worth identifying some models that emerge from various combinations of the assumptions embedded in Steps 1 through 3. The key models we consider in this paper are shown in Table 1. The first column of Table 1 indicates numbering that we assign to the models. The second column specifies the time-series process used to generate the interest rate and dividend growth rate series, corresponding to Step 2. The next three columns relate to Step 1 above, indicating whether or not the ex ante equity premium process incorporates a downward trend over time (and if so, how much the mean ex ante equity premium in 1952 differs from the value in 2004), whether or not there is a structural break (consisting of a 50 basis point drop) in the equity premium consistent with the findings of Pástor and Stambaugh (2001), and whether or not there is a break in the dividend growth rate process, consistent with the Bagwell and Shoven (1989) and Fama and French (2002) finding of an increase in share repurchases from the late 1970s onward.[6] The last column corresponds to Step 3, showing which models incorporate uncertainty in generating parameters. We consider a selection of 12 representative models, ranging from a simple model with no breaks or trends in the equity premium process (Model 1) to very complex models.[7] Each model is fully explored in the sections that follow. We now continue describing the two final steps of our basic methodology.

Table I goes about here.

Step 4: Calculate the fundamental stock returns (and hence ex post equity premia) that arise in each simulated economy, using a discounted-dividend-growth-rate model and based on assumptions about the ex ante equity premium from Step 1, the dividend growth rate and interest rate processes specified in Step 2, and the possible parameter uncertainty specified in Step 3. The model is rolled out to produce 53 annual observations of returns, prices, dividends, interest rates, and so on, mimicking the 53 years of annual US data available to us for comparison. Keep in mind the fact that the assumptions made in Steps 1 through 3 are the same for all simulated economies in a given experiment. That is, all economies in a given experiment have the same ex ante equity premium model (for instance a constant ex ante equity premium, or perhaps an ex ante equity premium that time-varies between a starting and ending value) and yet all economies in the set of simulations have different ex post equity premia. Given the returns and ex post equity premia for each economy, as well as the means of the interest rates and dividend growth rates produced for each economy, we are able to calculate various other important characteristics, including return volatility,

[6] In each case where we consider model specifications intended to capture real-world features like breaks and trends in rates and premia, we adopt parameterizations that bias our results to be more conservative (*i.e.* to produce a wider confidence interval for the ex ante equity premium). This allows us to avoid over-stating the gains in precision possible with our technique. For example, while Pástor and Stambaugh (2001) find evidence that there was a break in the equity premium process across several years in the 1990s, we concentrate the entire break into one year (1990). Allowing the break to be spread across several years would lead to a narrower bound on the ex ante equity premium than we find. See Appendix 1 for more details.

[7] For the sake of brevity, the Gordon (1962) constant dividend growth model is excluded from the set of models we explore in this paper. We did analyze the Gordon model and found it to perform very poorly. The model itself is rejected at every value of the ex ante equity premium, even more strongly than any other simple model considered in this paper is rejected.

dividend yields, and Sharpe ratios. There is nothing in our experimental design to exclude (rational) market crashes and dramatic price reversals. Indeed our simulations do produce such movements on occasion. The details of Step 4 are provided in Appendix 2.

Step 5: Examine the distributions of variables of interest, including ex post equity premia, Sharpe ratios, dividend yields, and regression coefficients (from estimating AR(1) and ARCH models for returns) that arise conditional on various mean values and various time-series characteristics of the ex ante equity premia. Comparing the performance of the US economy with various univariate and multivariate distributions of these quantities and conducting joint hypothesis tests allows us to determine a narrow range of equity premia consistent with the US market data. That is, only a small range of mean ex ante equity premia and time-varying equity premium models could have yielded the outcome of the past half century of high mean return and return standard deviation, low dividend yield, high ex post equity premium, *etc.*

A large literature makes use of similar techniques in many asset pricing applications, directly or indirectly simulating stock prices and dividends under various assumptions to investigate price and dividend behavior.[8] However, these studies typically employ restrictions on the dividend and discount rate processes in order to obtain prices from some variant of the Gordon (1962) model and/or some log-linear approximating framework. For instance, the present value (price, defined as P_0) of an infinite stream of expected discounted future dividends can be simplified under the Gordon model as

$$P_0 = D_1/(r - g), \tag{9}$$

where D_1 is the coming dividend, r is the constant discount rate, and g is the constant dividend growth rate. That is, by assuming constant r and g, one can analytically solve for the price. If, however, discount rates or dividend growth rates are in fact conditionally time-varying, then the infinite stream of expected discounted future dividends in Equation (5) cannot be simplified into Equation (9), and it is difficult or impossible to solve prices analytically without imposing other simplifying assumptions.

[8]See, for example, Scott (1985), Kleidon (1986), West (1988a,b), Campbell (1991), Gregory and Smith (1991), Mankiw, Romer, and Shapiro (1991), Hodrick (1992), Timmermann (1993, 1995), and Campbell and Shiller (1998).

Rather than employ approximations to solve our price calculations analytically, we instead simulate the dividend growth and discount rate processes directly, and evaluate the expectation through Monte Carlo integration techniques, adopting the DK method.[9] In the setting of time-varying dividend growth rates and interest rates which conditionally covary, this technique allows us to evaluate prices, returns, and other financial quantities without approximation error.[10] We also take extra care to calibrate our models to the time-series properties of actual market data. For example, annual dividend growth is strongly autocorrelated in the S&P 500 stock market data, counter to the assumption of a logarithmic random walk for dividends sometimes employed for tractability in other applications. Furthermore, interest rates are autocorrelated and cross-correlated with dividend growth rates. Thus we incorporate these properties in our 12 models (shown in Table I), which we use to produce our simulated dividend growth rates, interest rates, and, ultimately, our estimate of the ex ante equity premium.

We estimated each of the 12 models over a grid of discrete values of the ex ante equity premium, with the grid as fine as an eighth of a percent in the vicinity of a 3.5% equity premium, and no coarser than 100 basis points for equity premium values exceeding 5%. The entire exercise was conducted using distributed computing across a grid of 30 high-end, modern-generation computers over the course of a month. On a modern stand-alone computer, estimation of a single model for a single assumed value of the ex ante equity premium would take roughly one week to estimate (and, as stated above, we consider many values of the ex ante equity premium for each of our models).

II Univariate Conditional Distributions For Model 1

All of the results in this section of the paper are based on Model 1, as defined in Table I. Model 1 incorporates interest rates that follow an AR(1) process and dividend growth rates that follow a MA(1) process. The ex ante equity premium in Model 1 follows an AR(1) process (that emerges from Merton's (1980) conditional CAPM, as detailed in Appendix 1), with no trends or breaks in either the equity premium process or dividend growth rate process. We start with this "plain

[9] The Dondaldson and Kamstra (1996) method nests other fundamental dividend-discounting valuation methods as special cases. For instance, in a Gordon (1962) world of constant dividend growth rates and interest rates, the DK method produces the Gordon model price, albeit through numerical integration rather than analytically.

[10] There is still Monte Carlo simulation error, but that is random, unlike most types of approximation error, and it can also be measured explicitly and controlled to be very small, which we do, as explained in Appendix 2.

vanilla" model because it provides a good illustration of how well dividend-discounting models that incorporate time-varying autocorrelated dividend growth and discount rate processes can produce prices and returns that fit the experience of the last half century in the US. This model also provides a good starting point to contrast with models employing breaks and trends in equity premium and dividend growth processes. We consider more complex and arguably more realistic models incorporating trends and breaks later in the paper.

It is well known that the ex ante equity premium is estimated with error. See, for instance, Merton (1980), Gregory and Smith (1991), and Fama and French (1997). Any particular realization of the equity premium is drawn from a distribution, implying that given key information about the distribution (such as its mean and standard deviation), one can construct a confidence interval of statistically similar values and determine whether a particular estimate is outside the confidence interval. As mentioned above, an implication of this estimation error is that most studies have produced imprecise estimates of the mean equity premium. For instance, a typical study might yield an 800 basis point 95% confidence interval around the ex ante equity premium.[11] Studies including Fama and French (2002) have introduced innovations that make it possible to narrow the range. One of our goals is to further sharpen the estimate of the mean ex ante equity premium.

We first consider what we can learn by looking at the univariate statistics that emerge from our simulations. We can use the univariate distributions to place loose bounds on plausible values of the mean ex ante equity premium. While the analysis in this section based on univariate empirical distributions is somewhat casual, in Section III we conduct formal analysis based on χ^2 statistics and the joint distributions of the data, yielding very tight bounds on plausible values of the mean ex ante equity premium and identifying plausible models of the equity premium process, representing our main contributions.

Consider the following: conditional on a particular value of the ex ante equity premium, how unusual is an observed realization of the ex post equity premium? How unusual is an observed realization of the mean dividend yield? Each simulated economy produces a set of financial statistics based on the simulated *annual* time-series observations, and these financial statistics can be

[11] This particular range is based on the simple difference between mean realized equity returns and the average riskfree rate based on the last 130 years of data, as summarized in Table I of Fama and French (2002).

compared and contrasted with the US experience of the last half century. By considering not only the mean of a financial statistic across simulated economies, such as the mean ex post equity premium, but also conditional moments and higher moments including the standard deviation of excess returns produced in our simulations, we can determine with high refinement the ability of our simulated data to match characteristics of the US economy. For instance, market returns, to be discussed below, are volatile. Thus it is interesting to examine the degree to which our simulations are able to produce volatile returns and to look at the distribution of return variance as we vary the mean ex ante equity premium in our simulated economies.

We can compare any financial statistic from the last half century to our simulated economies provided the statistic is based on returns or dividends or prices, as these are data that the simulation produces. We could also consider moments based on interest rates or dividend growth rates, but since we calibrate our models to interest rates and dividend growth rates, all our simulations should (and do) fit these moments well by construction. We choose moments based on two considerations. First, the moments should be familiar and the significance of the moments to economic theory should be obvious. Second, the moments should be precisely estimated; if the moments are too "noisy," they will not help us narrow the range of ex ante equity premia. For instance, return skew and kurtosis are very imprecisely estimated with even 50 years of data, so that these moments are largely uninformative. The moments must also be well-defined; moments must be finite, for instance. The expected value of the price of equity is undefined, but we can use prices in concert with a cointegrated variable like lagged price (to form returns) or dividends (to form dividend yields).

Rather than presenting copious volumes of tabled results, we summarize the simulation results with concise plots of probability distributions of the simulated data for various interesting financial statistics. This permits us to determine if a particular ex ante equity premium produces financial statistics similar to what has been seen over the last half century in the US.

Figure 1 contains four panels, and in each panel we present the probability distribution function for one of various financial statistics (ex post equity premia, dividend yield, Sharpe ratio, and return volatility) based on each of four different ex ante equity premium settings. We also indicate the realized value for the actual US data. Comparison of the simulated distribution with realized values in these plots permits a very quick, if casual, first assessment of how well the realized US data agree with the simulated data, and which assumed values of the ex ante equity premium appear inconsistent with the experience of the last half century of US data.

Panels A through D of Figure 1 contain probability distribution functions (PDFs) corresponding to the mean ex post equity premium, the mean dividend yield, the Sharpe ratio, and return volatility respectively, based on assumed mean ex ante equity premia of 2.75%, 3.75%, 5%, and 8%. For the sake of clarity, the dotted lines depicting the PDFs in Figure 1 are thinnest for the 2.75% case and become progressively thicker for the 3.75%, 5%, and 8% cases. The actual US realized data is denoted in each panel with a solid vertical line.

The actual US mean equity premium, displayed in Panel A, is furthest in the right tail of the distribution corresponding to a 2.75% ex ante equity premium, and furthest in the left tail for the ex ante premium of 8%. The wide range of the distribution of the mean ex post equity premia for each assumed value of the ex ante equity premium is consistent with the experience of the last half century in the US, in which the mean ex post equity premium has a 95% confidence interval spanning plus or minus roughly 4% or 5%. The actual dividend yield of 3.4%, displayed in Panel B, is unusually low for the 5% and 8% ex ante equity premium cases, but it is near the center of the distribution for the ex ante premium values of 2.75% and 3.75%. In Panel C, only the Sharpe ratios generated with an ex ante equity premium of 8% appear inconsistent with the US experience of the last half century. The return volatility, displayed in Panel D, clearly indicates that the experience of the US over the last half century is somewhat unusual for all ex ante equity premia considered, though least unusual for the lowest ex ante equity premium. Casual observation, based on only the evidence in these univariate plots, implies that the ex ante equity premium which could have generated the actual high ex post equity premium and low dividend yield of the last half century of the US experience likely lies above 2.75% and below 5%.

Figure 1 goes about here.

We constructed similar plots for the mean return and for conditional moments, including the return first order autocorrelation coefficient estimate (the OLS parameter estimate from regressing returns on lagged returns and a constant, *i.e.*, the AR(1) coefficient), the return first order autoregressive conditional heteroskedasticity coefficient estimate (the OLS parameter estimate from regressing squared residuals on lagged squared residuals and a constant, *i.e.,* the ARCH(1) coefficient), and the price-dividend ratio's first order autocorrelation coefficient estimate (the OLS parameter estimate from regressing the price-dividend ratio on the lagged price-dividend ratio and a constant). The mean return distributions are similar to the ex post equity premium distributions shown in Figure 1, and all choices of the ex ante equity premium produce returns and price-dividend ratios that have conditional time-series properties matching the US data, so these results are not presented here.

Figure 1 has two central implications of interest to us. First, the financial variable statistics produced in our simulations are broadly consistent with what has been observed in the US economy over the past five decades. Most simulated statistics match the magnitudes of financial quantities from the actual US data, even though we do not calibrate to prices or returns.[12] Second, the results suggest that the 2.75% through 8% interval we present here likely contains the ex ante equity premium consistent with the US economy. Univariate results for Models 2 through 10 are qualitatively very similar to those presented for Model 1. Univariate results for Models 11 and 12, in contrast, are grossly rejected by the experience of the US economy. Detailed univariate results for Models 2 through 12 are omitted for the sake of brevity, but the poor performance of Models 11 and 12 will be evident in multivariate results reported below.

To narrow further the range of plausible ex ante equity premium values, we need to exploit the full power of our simulation procedure by considering the *joint distributions* of statistics that arise in our simulations and comparing them to empirical moments of the observed data. We consider the multivariate distributions of several moments of the data, including ex post equity premia, dividend yields, and return volatility. This exercise allows for inference that is not feasible with the univariate analysis conducted above, and it leads to a very precise estimate of the ex ante equity premium. We turn to this task in the next section, where we also broaden the class of models we consider.

[12] This in itself is noteworthy, as analytically tractable models, such as the Gordon (1962) growth model, typically imply constant or near-constant dividend yields and very little return volatility. In contrast, dividend yields observed in practice vary considerably over time and are strongly autocorrelated, and returns exhibit considerable volatility.

III Model Extensions, Multivariate Analysis, and Tests

The central focus in this section is on *joint* distributions of the financial statistics that emerge from our simulations: combinations of the returns, ex post equity premia, Sharpe ratios, dividend yields, *etc.*, and tests on the value of the ex ante equity premium using these joint distributions. We focus primarily on three moments of the data: the mean ex post equity premium, the excess return volatility, and the mean dividend yield. These three moments have the advantage of being the most precisely estimated and hence most informative for the value of the ex ante equity premium. Other moments that we could have considered are either largely redundant (such as the Sharpe Ratio which is a direct function of excess returns and the excess return standard deviation), or are so imprecisely estimated (for example, the ARCH(1) or AR(1) coefficients) that they would not help sharpen our estimates of the ex ante equity premium. Of course, we also do not consider the distributions of financial variables to which we calibrate our simulations (interest rates and dividend growth rates), as the simulated mean, variance, and covariance of these variables are, by construction, identical to the corresponding moments of the actual data to which we calibrate.

Our purpose in considering joint distributions is two-fold. First, multivariate tests are used to form a tight confidence bound on the true value of the ex ante equity premium. These tests strongly reject our models if the ex ante equity premium is outside of a narrow range around 3.5%. This range is not sensitive to even fairly substantial changes in the model specification, which suggests that the 3.5% finding is robust. Second, this analysis leads us to reject model specifications that fail to incorporate certain features, such as trends and breaks in the equity premium. Interestingly, even when a model specification is rejected, we find the most plausible ex ante equity premium still lies in the same range as the rest of our models, very near 3.5%.

Up to this point we have considered detailed results for Model 1 exclusively. The Model 1 simulation incorporates some appealing basic features, such as parameter uncertainty and calibrated time-series models for equity premia, interest rates, and dividend growth rates. It does not, however, incorporate some features of the equity premium process that have been indicated by other researchers. One omitted feature is a gradual downward trend in the equity premium, as documented in many studies, including Jagannathan, McGrattan, and Scherbina (2000), Pástor and

Stambaugh (2001), Bansal and Lundblad (2002), and Fama and French (2002). Another is a structural break in the equity premium process over the early 1990s, as shown by Pástor and Stambaugh (2001). An increase in the growth rate of cashflows (but not dividends) to investors starting in the late 1970s, as documented by Bagwell and Shoven (1989), Fama and French (2001) and others, is also a feature that Model 1 fails to incorporate. Therefore, in this section we consider models which incorporate one, two, or all three of these features, as well as different time-series models for interest rates and equity premia. We also consider stripped-down models to assess the marginal contribution of model features such as parameter uncertainty and the specification of the time-series process used to model dividend growth rates and interest rates.

In Figures 2 through 8 (to be fully discussed below), we present χ^2 test statistics for the null hypothesis that the US experience during 1952 through 2004 could have been a random draw from the simulated distribution of the mean ex post equity premium, the excess return volatility, and the mean dividend yield.[13]

A significant test statistic, in this context, suggests that the combination of financial statistics observed for the US economy is significantly unusual compared to the collection of simulated data, leading us to reject the null hypothesis that the given model and assumed ex ante equity premium value could have generated the US data of the last half century. It is possible to reject every ex ante equity premium value if we use models of the equity premium that are misspecified (the rejection of the null hypothesis can be interpreted as a rejection of the model). It is also possible that a very wide range of ex ante equity premium values are not rejected for a collection of models, thwarting our efforts to provide a precise estimate of the ex ante equity premium or a small range of allowable equity premium models.

As it happens, models that ignore breaks and trends in the equity premium are rejected for

[13]The χ^2 tests are based on joint normality of sample estimates of moments of the simulated data, which follow an asymptotic normal distribution based on a law of large numbers (see White, 1984, for details). In the case of the excess return volatility, we consider the *cube root* of the return variance, which is approximately normally distributed (see page 399 of Kendall and Stuart, 1977, for further details). We also estimate the probability of rejection using bootstrapped p-values, to guard against deviations from normality. These bootstrapped values are qualitatively identical to the asymptotic distribution p-values. Finally, when performing tests that include the dividend yield moment, if the simulation includes a break in dividends corresponding to an increase in cash payouts starting in 1978 in the US data (again, see Fama and French, 2001), we also adjust the US data to reflect the increase in mean payout levels. This makes for a small difference in the mean US payout ratio and no qualitative change to our results if ignored.

virtually every value of the ex ante equity premium we consider. But for a group of sophisticated models that incorporate trends and breaks in the equity premium, we cannot reject a narrow range of ex ante equity premia, roughly between 3% and 4%. We also find that models tend to be rejected if the impact on cashflows to shareholders from share repurchases are ignored. We begin with some simple models, then consider models that are arguably more realistic as they incorporate equity premium and cashflow trends and breaks, and finish by considering a host of related issues, including the impact of parameter estimation error and, separately, *investor* uncertainty about the fundamental value of equities.

A Simple (One-at-a-Time) Model Extensions

We now consider extensions to Model 1, each extension adding a single feature to the base model. Recall that the features of each model are summarized in Table I. For Model 2, an 80 basis point downward trend is incorporated in the equity premium process. For Model 3, a 50 basis point drop in year 39 of the simulation (corresponding to 1990 for the S&P 500 data) is incorporated in the equity premium process. For Model 4, the dividend growth rate process is shifted gradually upward a total of 100 basis points, starting in year 27 of the simulation (corresponding to 1978 for the S&P 500 data) and continuing for 20 years at a rate of 5 basis points per year. These one-at-a-time feature additions help us evaluate if one or another feature documented in the literature can markedly improve model performance over the simple base model.

Panel A of Figure 2 and Panel A of Figure 3 display plots of the value of joint χ^2 tests on three moments of the data, the mean ex post equity premium, the excess return volatility, and the mean dividend yield, for Models 1 though 4, and shows how the test statistic varies as the ex ante equity premium varies from 2.25% to 8% in increments as small as an eighth of a percent toward the lower end of that range. Panels B through D of Figures 2 and 3 display the univariate Student t-test statistics for each of these three moments of the data, again showing how the test statistic varies with the assumed value of the ex ante equity premium. The values of the ex ante equity premia indicated on the horizontal axis represent the *ending* values of the ex ante equity premium in each set of simulations. For models which incorporate a downward trend or a structural break in the equity premium, the ending value of the ex ante equity premium differs from the starting value.

So, for instance, Model 2 has a starting ex ante equity premium that is 80 basis points higher than that displayed in Figure 2, as Model 2 has an 80 basis point trend downward in the ex ante equity premium. For Model 1 the value of the ex ante equity premium is the same at the *end* of the 53-year simulation period as it is at the *start* of the 53-year period, as Model 1 does not incorporate a downward trend or structural break in the equity premium process. Critical values of the test statistics corresponding to statistical significance at the 10%, 5%, and 1% levels are indicated by thin dotted horizontal lines in each panel, with the lowest line indicating significance at the 10% level and the highest line the 1% significance level.

Figures 2 and 3 go about here.

Consider now specifically Panel A of Figures 2 and 3. (Note that we use a log scale for the vertical axis of the plots in Panel A of Figures 2 through 8 for clarity of presentation. Note as well that we postpone further discussion of Panels B through D until after we have introduced results for all the models, 1 through 12.) On the basis of Panel A of Figures 2 and 3, we see that only in the case of Model 4 do we observe χ^2 test statistics lower than the cutoff value implied by a 10% significance level (again, indicated by the lowest horizontal dotted line in the plot). The test statistics dip (barely) below the 10% cutoff line only for values of the ex ante equity premium within about 25 basis points of 4%. Models 1-3, in contrast, are rejected at the 10% level for *every* ex ante equity premium value. If we allow fairly substantial departures of the S&P 500 data from the expected distribution, say test statistics that are unusual at the 1% level of significance (the upper horizontal dotted line in the plot), then all the models indicate ranges of equity premia that are not rejected, in each case centered roughly between 3.5% and 4%. Recall that the equity premium plotted is the *ending* value, so if the model has a downward trend or decline because of a break in the equity premium, its ending value is below its average ex ante equity premium.

One conclusion to draw from the relative performance of these four competing models is that each additional feature over the base model, the dividend growth acceleration in the late 1970s and the trends and breaks in the equity premium, lead to better performance relative to the base model, but each in isolation is still inadequate. The model most easily rejected is clearly that which does not account for trends and breaks in the equity premium and cashflow processes.

B Further Model Extensions (Two or More at a Time)

We turn now to joint tests based on Models 5 though 10. These models incorporate the basic features of Model 1, including time-varying and dependent dividend growth and interest rates, parameter uncertainty, and, with the exception of Model 10, an equity premium process derived from the Merton (1980) conditional CAPM (detailed in Appendix 1). These models also permit trends and/or breaks in the equity premium and dividend growth rate processes two or more at-a-time and incorporate alternative time-series models for the interest rate and the equity premium processes. Models 1 through 4 demonstrate that it is not sufficient to model the equity premium as an autoregressive time-varying process, and that one-at-a-time augmentation with trends or breaks in the equity premium process is also not sufficient, though the augmentations do lead to improvements over the base model in our ability to match sample moments from the US experience of the last half century. Models 5 through 10 allow us to explore questions like: do we need a conditionally time-varying equity premium model built on the Merton conditional CAPM model, or is it sufficient to have an equity premium that simply trends downward with a break? If we have a break, a trend, and time-variation in the equity premium process, is it still essential to account for the disappearing dividends of the last 25 years? Are our results sensitive to the time-series model specifications we employ in our base model?

Model 5 is the base model, Model 1, augmented to include an 80 basis point gradual downward trend in the equity premium and a 100 basis point gradual upward trend in the dividend growth rate. Model 6 is the base model adjusted to incorporate a 30 basis point gradual downward trend in the equity premium, a 50 basis point abrupt decline in the equity premium, and a 100 basis point gradual upward trend in the dividend growth rate. Model 7 is the best model as indicated by the Bayesian Information Criterion (BIC),[14] augmenting the equity premium process with a 30 basis point gradual downward trend and a 50 basis point abrupt decline and adding a 100 basis point gradual upward trend in the dividend growth rate. Model 8 takes the second-best BIC model

[14]For Models 7 and 8 we employ the BIC to select the order of the ARMA model driving each of the interest rate, equity premium, and dividend growth rate processes. The order of each AR process and each MA process for each series is chosen over a (0, 1, 2) grid. The BIC has been shown by Hannan (1980) to provide consistent estimation of the order of linear ARMA models. We employ the BIC instead of alternative criteria because it delivers relatively parsimonious specifications and because it is widely used in the literature (*e.g.*, Nelson, 1991, uses the BIC to select EGARCH models).

and incorporates a 30 basis point gradual downward trend in the equity premium, a 50 basis point abrupt decline in the equity premium, and a 100 basis point gradual upward trend in the dividend growth rate. Model 9 is the base model adjusted to incorporate a 30 basis point gradual downward trend in the equity premium and a 50 basis point abrupt decline in the equity premium. Model 10 has the equity premium model following a deterministic downward trend with a 50 basis point structural break, interest rates following an AR(1), and dividend growth rates following an MA(1).

Given the existing evidence in support of a gradual downward trend in the equity premium, a structural break in the equity premium process over the early 1990s, and an increase in the growth rate of non-dividend cashflows to investors (such as share repurchases) starting in the late 1970s, we believe Models 6, 7, and 8 to be the best calibrated and therefore perhaps the most plausible among all the models we consider, and Model 5 to be a close alternative.

In Panel A of Figures 4, 5, and 6 we present plots of the χ^2 test statistics on three moments of the data, the mean ex post equity premium, the excess return volatility, and the mean dividend yield. Again, we consider Panels B through D later. We see in Panel A of Figures 4 and 5 that for Models 5 through 8 we cannot reject a range of ex ante equity premium values at the 5% level. These models produce test statistics that drop well below even the 10% critical value (recall that Panel A's scale is logarithmic, and thus compressed). These models all embed the increased cashflow feature and either an eighty basis point downward trend in the equity premium, or both a break and a trend in the equity premium, adding to an eighty basis point decline over the last half century. The range of ex ante equity premia supported (not rejected) is narrowest for Model 7 (the best model indicated by BIC) and Model 8 (the second best model indicated by BIC) with a range less than 75 basis points at the 10% level. The range is slightly wider for Models 5 and 6, roughly 75 to 100 basis points. In each case, the ex ante equity premium that yields the minimum joint test statistic, corresponding to our estimate of π, is centered between 3.25% and 3.75%.

For the models which exclude the cashflow increase, Models 9 and 10, displayed in Figure 6, we see that we can reject at the 10% level all ex ante equity premium values. Model 9 is best compared to Model 6, as it is equivalent to Model 6 with the sole difference of excluding the cashflow increase. We see from Panel A of Figures 4 and 6 that excluding the cashflow increase flattens the trough of the plot of χ^2 statistics, and approximately doubles the test statistic value, from a little over 3 for

Model 6 in Figure 4 to a little over 6 for Model 9 in Figure 6 (recall that the scale is compressed in Panel A as we use a log scale). Model 10 is identical to Model 9 apart from the sole difference that Model 10 excludes the Merton CAPM conditionally-varying equity premium process. Exclusion of this conditional time variation (modeled as a first order autoregressive process) worsens the ability of the model to match moments to the US experience at every value of the ex ante equity premium. The difference in performance leads us to reject a model excluding a conditionally-varying equity premium.

Figures 4, 5, and 6 go about here.

On the basis of our most plausible models, Models 6, 7, and 8, we can conservatively conclude that the ex ante equity premium is within 50 basis points of 3.5%. We can also conclude that models that allow for breaks and/or trends in the equity premium process are the only models that are not rejected by the data. Simple equity premium processes, those that rule out any one of a downward break and/or trend or a Merton (1980) CAPM conditionally-varying equity premium process, cannot easily account for the observed low dividend yields, high returns, and high return volatility. Ignoring the impact of share repurchases on cashflows to investors over the last 25 years also compromises our ability to match the experience of US prices and returns of the last half century.

C Is Sampling Variability (Uncertainty) in Generating Parameters Important?

All of the models we have considered so far, Models 1-10, incorporate parameter value uncertainty. This uncertainty is measured using the estimated covariance of the parameter estimates from our models. We generate model parameters by randomly drawing values from the joint distribution of the parameters, exploiting the asymptotic result that our full information maximum likelihood procedure produces parameter estimates that are jointly normally distributed, with an easily computed variance-covariance structure.

Now we consider two models that have no parameter sampling variability built into them, Models 11 and 12. In these models the point estimates from our ARMA estimation on the S&P 500 data are used for each and every simulation. Ignoring uncertainty about the true values for the parameters of the ARMA processes for interest rates, dividend growth rates, and the equity premium should dampen the variability of the generated financial statistics from these simulations, and potentially understate the range of ex ante equity premia supported by the last half century of US data. Model 11 is the base model augmented to incorporate a 30 basis point gradual downward trend in the equity premium, a 50 basis point abrupt decline in the equity premium, and a 100 basis point gradual upward trend in the dividend growth rate, with no parameter uncertainty. (Model 11 is identical to Model 6 apart from ignoring parameter uncertainty.) Model 12 is the base model, Model 1, with no parameter uncertainty.

Figure 7 goes about here.

In Panel A of Figure 7 we present plots of the χ^2 test statistics on three moments of the data, the mean ex post equity premium, the excess return volatility, and the mean dividend yield. Again, we consider Panels B through D later. We see in Panel A that both Models 11 and 12 are rejected for all values of the ex ante equity premium, though Model 11, which allows for trends and breaks, performs better than Model 12. The log scale for the vertical axis compresses the values, but the minimum χ^2 statistic for Model 12 is close to 30, indicating very strong rejection of the model, while the minimum χ^2 statistic for Model 11 is roughly 10. In each case, the ex ante equity premium that yields the minimum joint test statistic, corresponding to our estimate of π, is centered around 3%. It is apparent that parameter uncertainty is an important model feature. Ignoring parameter uncertainty leads to model rejection, even at the ex ante equity premium setting that corresponds to the minimum test statistic.

D The Moments That Matter

An interesting question that arises with regard to the joint tests is, where does the test power come from? That is, which variables give us the power to reject certain ranges of the ex ante equity premium in our joint χ^2 tests? An examination of the ranges of the ex ante equity premium consistent with the *individual* moments can shed some light on the source of the power of the joint tests. Panels B, C, and D of Figures 2 through 7 display plots of the univariate t-test statistics based on each of the variables we consider in the joint tests plotted in Panel A of these figures. Panel B of each figure plots t-test statistics on the ex post equity premium, Panel C of each figure plots t-test statistics on the excess return volatility, and Panel D of each figure plots t-test statistics on the price-dividend ratio.

Consider first Panel B of Figures 2 through 7. Virtually all of the models have a minimum t-test statistic at a point that is associated with an ex ante equity premium close to 6%.[15] Because our method involves minimizing the distance between the ex post equity premium based on the actual S&P 500 value (which is a little over 6%) and the ex post equity premium estimate based on the simulated data, it is not surprising that the minimum distance is achieved for models when they are set to have an ex ante equity premium close to 6%. The t-test on the mean ex post equity premium rises linearly as the ex ante equity premium setting departs from 6% for each model, but does not typically reject ex ante equity premium values at the 10% level until they deviate quite far from the ex ante value at which the minimum t-test is observed. For example, in Panel B of Figure 4 the ending ex ante equity premium must be as low as 2.25% or as high as 7% before we see a rejection at the 10% level. This wide range reflects the imprecision of the estimate of the ex post equity premium which is also evident in the actual S&P 500 data.

The t-tests on the excess return volatility, presented in Panel C of Figures 2 through 7, indicate that lower ex ante equity premium values lead to models that are better able to match the S&P 500 experience of volatile returns.[16] Note that as the ex ante equity premium decreases, the volatility of returns *increases*, so high ex ante equity premia lead to simulated return volatilities that are much lower than the actual S&P 500 return volatility we have witnessed over the last half century. The test statistic, however, rises slowly as the ex ante equity premium grows larger, in contrast to the joint test statistics plotted in Panel A of Figures 2 through 7, in which the χ^2 test statistic

[15] Recall that the ex ante equity premium values shown on the horizontal axes are *ending* values, so if the model has a downward trend or break in the equity premium process, its ending value is below the mean equity premium. For instance, Model 11 has a data generating process that incorporates trends and breaks that lead to an ending equity premium lower than the starting value. Accordingly, for this model we observe (in Panel B of Figure 7) a minimum t-test at an *ending* value of the ex ante equity premium which is below the 6% *average* equity premium. The coarseness of the grid of ex ante equity premium values around 6% prevents this feature from being more obvious for some of the other models.

[16] The intuition behind this result is easiest to see by making reference to the Gordon (1962) constant dividend growth model, shown above in Equation 9. As the discount rate, r, declines in magnitude, the Gordon price increases. The variable r equals the risk-free rate plus the equity premium in our simulations, so low values of the equity premium lead to values of the discount rate that are closer to the dividend growth rate, resulting in higher prices. When the value of the equity premium is low, small increases in the dividend growth rate or small decreases in the risk-free rate lead to large changes in the Gordon price. In our simulations (where the conditional mean dividend growth rate and conditional mean risk-free rate change over time), when the value of the equity premium is low, small changes in the conditional means of dividend growth rates or risk-free rates also lead to large prices changes, *i.e.* volatility.

rises *sharply* as the ex ante equity premium grows larger (recall that the Panel A vertical axis has a compressed log scale in Figures 2 through 7). Given these contrasting patterns, the return volatility moment is unlikely, *by itself*, to be causing the sharply rising joint test statistic.

Consider now the t-test statistics on the price-dividend ratio, plotted in Panel D of Figures 2 through 7. Notice that in all cases the t-test on the price-dividend ratio jumps up sharply as the ex ante equity premium rises above 3%. Thus the sharply increasing χ^2 statistics we saw in Panel A of the three figures are likely due in large part to information contained in the price-dividend ratio. However, return volatility reinforces and amplifies the sharp rejection of premia above 4% that the dividend yield also leads us to. In terms of the three moments we have considered in the joint χ^2 and univariate t-test statistics, it is evident that the upper range of ex ante equity premia consistent with the experience of the last half century in the US is limited by the high average S&P 500 price-dividend ratio (or equivalently, the low average S&P 500 dividend yield) together with the high volatility of returns. This result is invariant to the way we model dividend growth, interest rates, or the equity premium process. Even an ex ante equity premium of 5% produces economies with price-dividend ratios and return volatilities so low that they are greatly at odds with the high return volatility and high average price-dividend ratio observed over the past half century in the US.

D.1 Sensitivity to Declining Dividends Through Use of the Price-Dividend Ratio

To ensure that our results are not driven by a single moment of the data, in particular a moment of the data possibly impacted by declining dividend payments in the US, we perform two checks. First, in Models 4 through 8 we incorporate higher dividends and dividend growth rates than observed in US corporate dividends. This is to adjust for the practice, adopted widely beginning in the late 1970s, of US firms delivering cashflows to investors in ways (such as share repurchases) which are not recorded as corporate dividends. As we previously reported, Models 4 through 8 (the models that incorporate higher cashflows to investors than recorded by S&P 500 dividend payments, *i.e.*, the models that use cashflows *including* share repurchases) are best able to account for the observed US data. Reassuringly, the estimate of the equity premium emerging from Models 4 through 8 is virtually identical to that produced by the models that exclude share repurchases.

Our second check is to perform joint tests excluding the price-dividend ratio. Any sensitivity to mismeasurement of the price-dividend ratio should be mitigated if we consider joint test statistics that are based only the ex post equity premium and return volatility, excluding the price-dividend ratio. These (unreported) joint tests confirm two facts. First, when the joint tests exclude the price-dividend ratios, the value of the χ^2 statistic rises less sharply for values of the ex ante equity premium above 4%. Essentially, this indicates that using two moments of the data (excluding the price-dividend ratio) rather than all three makes it more difficult to identify the minimum test statistic value and thus more difficult to identify our estimate of the ex ante equity premium. This confirms our earlier intuition that the price-dividend ratio is instrumental in determining the steep rise of the joint test statistic in Panel A of Figures 2 through 7. Second, and most importantly, the minimum test statistic is still typically achieved for models with an ex ante equity premium value between 3% and 4%. For some of the models, the minimum test statistic is 25 or 50 basis points lower than that found when basing joint tests on the full set of three moments. For a few models, the minimum test statistic is 25 or 50 basis points higher. Again Models 1 through 3 are rejected for every value of the ex ante equity premium, and again for Models 4 through 8 the range of ex ante equity premia that are not rejected is narrow.

E Investors' Model Uncertainty

We have been careful to explore the impact of estimation uncertainty by simulating from the sampling distribution of our model parameters, and to explore the impact of model specification choice (and implicitly model *mis*specification) by looking at a variety of models for interest rates, dividend growth rates, and equity premium, ranging from constant rate models to various ARMA specifications, with and without trends and breaks in the equity premium and dividend growth rates. Comparing distributions of financial statistics emerging from this range of models to the outcome observed in the US over the last half century leads us to the conclusion that the range of true ex ante equity premia that could have generated the US experience is fairly narrow, under 100 basis points, centered roughly on 3.5%. We have not yet addressed, however, the impact of *investor* uncertainty regarding the true fundamental value of the assets being priced. Up to this point, all simulated prices and returns have been generated with knowledge of the (fundamental) processes generating interest rates and dividends.

It is impossible to be definitive in resolving the impact of investor uncertainty on prices and returns. To do so we would have to know what (incorrect) model of fundamental valuation investors are actually using. We can nonetheless focus our attention on procedures likely to be less affected by investor uncertainty than others. Up to this point, the joint tests we have used to identify the plausible range of ex ante equity premia have employed the observed return volatility over the last half century in the US and the volatility of returns produced in our simulated economies. However, investor uncertainty could cause market prices to over- and under-shoot fundamental prices, impacting return volatility, perhaps significantly. A joint test statistic based on only the mean equity premium and the mean price-dividend ratio, however, should be relatively immune to the impact of investor uncertainty. (In the absence of extended price bubbles, *mean* yields should not be impacted greatly by temporary pricing errors.) Thus we now consider the joint χ^2 test statistic based on only the mean return and the mean price-dividend ratio. Figure 8, Panel A plots the test statistics for Models 1, 2, and 3, Panel B plots the test statistics for Models 4, 5, and 6, Panel C plots the test statistics for Models 7, 8, and 9, and Panel D plots the test statistics for Models 10, 11, and 12, with a log scale for the vertical axis in all cases.

Figure 8 goes about here.

First consider results for Models 1 through 4, shown in Panels A and B of Figure 8. These are the base model with no trends or breaks, and models which incorporate only one feature (trend or break in the equity premium or dividend growth rate) at a time. We see again that Model 1 is rejected outright for every value of the ex ante equity premium, at the 10% level of significance, and we see again that adding trends or breaks, even one-at-a-time, improves performance. Now Model 2 (incorporating an 80 basis point downward trend in the equity premium) and Model 4 (incorporating the increased cashflow growth rate) are not rejected over narrow ranges at the 10% significance level. We find that Models 5, 6, 7, and 8, all incorporating trends and breaks in the equity premium and dividend growth rate processes and shown in Panels B and C of Figure 8, deliver a wide range of ex ante equity premia which cannot be rejected at any conventional level of statistical significance. We also see that Model 9 in Panel C, incorporating a trend (of 30 basis points) and a break (of 50 basis points) in the equity premium, performs similarly to Model 2, which has only a trend of 80 basis points (neither model incorporates a cashflow change). In Panel D we see Model 10 which has a deterministic equity premium with trends and breaks. This model's performance is also similar to Model 2, but slightly worse, rejected at the 10% level at every ex ante equity premium. Also in Panel D we see that Models 11 and 12, which do not incorporate parameter estimation uncertainty, are almost everywhere rejected. (In contrast to the joint test shown in Panel A of Figure 7, based on all three moments, we find that Model 11 is not rejected only for the 3% value of the ex ante equity premium.)

Overall, the value of the ex ante equity premium at which the joint test statistic is minimized (*i.e.*, our estimate of the ex ante equity premium) is not particularly affected by our having based the joint tests on two moments of the data rather than the original three, nor is our selection of plausible models for the equity premium process. Across the models, the highest estimate of the ex ante equity premium is roughly 4% (for Model 4) and the lowest is 3% (for Models 11 and 12). With the joint tests based on two moments, all models support (*i.e.*, do not reject) broader ranges of the ex ante equity premium, with the range widest for Models 4 through 8 (now spanning roughly 200 basis points for any given model, from ex ante equity premium values as low as 2.25% for Model 7 to values as high as 4.5% for Model 4). This widening of the range of plausible ex ante equity premia is consistent with a decline in the power of our joint test, presumably from omitting an important moment of the data, the return volatility. The widening of the range of plausible ex ante equity premia is also consistent with investors being uncertain about the true fundamental value of the assets being priced. The last half century of data from the US will be less informative as investor uncertainty about the processes governing fundamentals exaggerates the volatility of returns and hence reduces the precision of estimates of the ex ante equity premium.

To the extent that market prices are set in an efficient market dominated by participants with models of dividend growth rates and interest rates that reflect reality, these ranges of plausible ex ante equity premia based on only the two-moment joint test are overly wide. Still these ranges are useful for putting a loose bound on the likely range of the ex ante equity premium.

F Bootstrapped Test Statistics

Up to this point, all of our test statistics have relied on asymptotic distribution theory for critical values. The asymptotic distributions should be reliable both because we are looking at averages over independent events (our simulations are by construction independent) and because we have many simulations over which to average (2,000). Nonetheless, it is straightforward to use our simulated test statistics to bootstrap the distribution of the test statistics, thus we do so. While use of the bootstrap produces small quantitative changes to our results, our main findings remain unchanged. The best estimate of the mean ex ante equity premium and the range of plausible ex ante equity premia and equity premium models do not budge.

IV Conclusions

The equity premium of interest in theoretical models is the extra return investors anticipate when purchasing risky stock instead of risk-free debt. Unfortunately, we do not observe this ex ante equity premium in the data. We only observe the returns that investors actually receive ex post, after they purchase the stock and hold it over some period of time during which random economic shocks impact prices. US stocks have historically returned roughly 6% more than risk-free debt. Ex post estimates provided by recent papers suggest the US equity premium may be falling in recent years. However, all of these estimates are imprecise, and there is little consensus emerging about the true value of the ex ante equity premium. The imprecision and lack of consensus both hamper efforts to use equity premium estimates in practice, for instance to conduct valuation or to perform capital budgeting. The imprecision of equity premium estimates also complicates resolution of the equity premium puzzle and makes it difficult to determine if the equity premium changes over time.

In order to determine the most plausible value of the ex ante equity premium and the most plausible restrictions on how the equity premium evolves over time, we have exploited information not just on the ex post equity premium and the precision of this estimate, but also on related financial statistics that define the era in which this ex post equity premium was estimated. The idea of looking at related fundamental information in order to improve the estimate of the mean ex ante equity premium follows recent work on the equity premium which has also sought improvements through the use fundamental information like the dividend and earnings yields (Fama and French, 2002, and Jagannathan, McGrattan, and Scherbina, 2000), higher-order moments of the excess return distribution (Maheu and McCurdy, 2007) and return volatility and price movement directions (Pástor and Stambaugh, 2001).

Our central insight is that the knowledge that a low dividend yield, high ex post equity premium, high return volatility, and high Sharpe ratio all occurred together over the last five decades tells us something about the mean ex ante equity premium and the likelihood that the equity premium is time-varying with trends and breaks. Certainly, if sets of these financial statistics are considered together, we should be able to estimate the equity premium more accurately than if we were to look only at the ex post equity premium. This insight relies on the imposition of some structure from economic models, but our result is quite robust to a wide range of model structures, lending confidence to our conclusions.

We employ the simulated method of moments technique and build on the dividend discounting method of fundamental valuation of Donaldson and Kamstra (1996) to estimate the ex ante equity premium. We reject as inconsistent with the US experience all but a narrow range of values of the mean ex ante equity premium and all but a small number equity premium time-series models. We do so while incorporating model estimation uncertainty and allowing for investor uncertainty about the true state of the world. The range of ex ante equity premia that is most plausible is centered very close to 3.5% for virtually every model we consider. The models of the equity premium not rejected by our model specification tests – that is, consistent with the experience of the US over the last half century – incorporate substantial autocorrelation, a structural break, and/or a gradual downward trend in the equity premium process. For these models, the range of ex ante equity premia supported by our tests is very narrow, plus or minus 50 basis points around 3.5%. All together, our tests strongly support the notion that the equity premium process over the last half century in the US was very unlikely to have been constant, was likely to have demonstrated at least one sharp downward break, and was likely to have demonstrated a gradual downward trend.

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Appendices

Appendix 1: Models for Generating Data

In creating distributions of financial variables modeled on the US economy, we must generate the fundamental factors that drive asset prices: dividends and discount rates (where the discount rate is defined as the risk-free rate plus a possibly time-varying equity premium). Thus we must specify time-series models for dividend growth, interest rates, and ex ante equity premia so that our Monte Carlo simulations will generate dividends and discount rates that share key features with observed S&P 500 dividends and US discount rates. We consider a range of models to generate data in our simulations, as outlined in Table I. Each model incorporates specific characteristics that define the way we generate interest rates and dividend growth rates, and each model makes specific assumptions about the way the ex ante equity premium evolves over time, if indeed it does evolve over time. In providing further information about these defining aspects of our models, we consider each model feature from Table I in turn, starting with the time-series processes for interest rates, dividend growth rates, and the ex ante equity premium.

A1.1 Processes for the Interest Rate, Dividend Growth Rate and the Ex Ante Equity Premium

The interest rate and dividend growth rate series we generate are calibrated to the time-series properties of data observed in the US over the period 1952 to 2004. We considered the ability of various time-series models to eliminate residual autocorrelation and ARCH (evaluated with LM tests for residual autocorrelation and for ARCH, both using 5 lags), and we evaluated the log likelihood function and Bayesian Information Criterion (BIC) across models. Although we will describe the process of model selection one variable at-a-time, our final models were chosen using a Full Information Maximum Likelihood (FIML) systems equation estimation and a joint-system BIC optimization.

Economic theory admits a wide range of possible processes for the risk-free interest rate, from constant to autoregressive and highly non-linear heteroskedastic forms. We find that in practice, both AR(1) and ARMA(1,1) models of the logarithm of interest rates, based on the model of Hull (1993, page 408), perform well in capturing the time-series properties of observed interest rates. We also find the AR(1) and ARMA(1,1) specifications perform comparably to one another, markedly dominating the performance of other specifications including higher order models like ARMA(2,2). An attractive feature of modeling the log of interest rates is that doing so restricts nominal interest rates to be positive. Finally, we find standard tests for normality of the error term (and hence conditional log-normality of interest rates) do not reject the null of normality.

Since dividend growth rates have a minimum value of -100% and no theoretical maximum, a natural choice for their distribution is the log-normal. Thus we model the log of 1 plus the dividend growth rate, and we find that both a MA(1) and an AR(1) specification fit the data well, removing evidence of residual autocorrelation and ARCH at five lags. These specifications are preferred on the basis of the same criteria used to choose the specification for modeling interest rates. As with the interest rate data, we find standard tests for normality of the error term (and hence conditional log-normality of dividend growth rates) do not reject the null of normality.

Most of our models incorporate an ex ante equity premium that follows an ARMA process emerging from Merton's (1980) conditional CAPM. Merton's conditional CAPM is expressed in terms of returns in excess of the risk-free rate, or, in other words, the period-by-period equity premium. For the i^{th} asset,

$$E_t(r_{i,t}) \quad \lambda \; cov_{t-1}(r_{i,t} r_{m,t}), \tag{10}$$

where $r_{i,t}$ are excess returns on the asset, $r_{m,t}$ are excess returns on the market portfolio, cov_{t-1} is the time-varying conditional covariance between excess returns on the asset and on the market portfolio, and E_t is the conditional-expectations operator incorporating information available to the market up to but not including the beginning of period t. λ is a parameter of the model, described below.

For the expected excess market return, (10) becomes

$$E_t(r_{m,t}) = \lambda \; var_{t-1}(r_{m,t}) \tag{11}$$

where var_{t-1} is the market time-varying conditional variance. Merton (1980) argues that λ in (11) is the weighted sum of the reciprocal of each investor's coefficient of relative risk aversion, with the weight being related to the distribution of wealth among individuals.

Equation (11) defines a time-varying equity premium but has the equity premium varying only as a function of time-varying conditional variance. Following Bekaert and Harvey (1995), it is possible to allow λ in Equation (11) to vary over time by making it a parametric function of conditioning variables (indicated below as Z_{t-1}). The functional form Bekaert and Harvey employ (in Equation (12) of their paper) is exponential, restricting the price of risk to be positive:

$$\lambda_{t-1} = exp\left(\delta' Z_{t-1}\right). \tag{12}$$

Shiller (1984), Rozeff (1984), Campbell and Shiller (1988), Hodrick (1992), and Bekaert and Harvey (1995) all document the usefulness of dividend yields to predict returns, so we use lagged dividend yields as our conditioning variable. We make use of a simple ARCH specification to model $var_{t-1}(r_{m,t})$. Once again we calibrate to the S&P 500 over 1952 to 2004, estimating the following model:

$$r_{m,t} = \lambda_{t-1}\ var_{t-1}(r_{m,t}) + e_{m,t} \tag{13}$$

$$var_{t-1}(r_{m,t}) = \omega + \alpha e^2_{m,t-1} \tag{14}$$

$$\lambda_{t-1} = exp\left(\delta_0 + \delta_1 \frac{D_{t-1}}{P_{t-1}}\right). \tag{15}$$

The values of estimated parameters are $\delta_0 = -3.93$, $\delta_1 = 0.277$, $\omega = 0.0194$, and $\alpha = 0.542$. The R^2 of this model is 2.8%.

For our simulations, we model the time-series process of the ex ante time-varying equity premium (denoted π_t) by using the excess return as a proxy for the equity premium:

$$\hat{\pi}_t = \hat{\lambda}_{t-1} v\hat{a}r_{t-1}(r_{m,t}). \tag{16}$$

where $\hat{\lambda}_{t-1} = exp\left(-3.93 + 0.277\frac{D_{t-1}}{P_{t-1}}\right)$, $v\hat{a}r_{t-1}(r_{m,t}) = 0.0194 + 0.542\hat{e}^2_{m,t-1}$, and $\hat{e}_{m,t-1} = r_{m,t-1} - \hat{\pi}_{t-1}$. The time-varying equity premium we estimate here, $\hat{\pi}_t$, follows a strong AR(1) time-series process, similar to that of the risk-free interest rate,[17] so that when the equity premium is perturbated it reverts to its mean slowly. This permits slightly more volatile returns in our simulations than would otherwise be the case. The best way to see the impact of this slow mean reversion of the equity premium on our simulations is to compare Models 9 and 10. Model 9 has a conditionally time-varying equity premium (together with a trend and break in the premium) while Model 10 is identical except the equity premium does not conditionally vary. We find standard tests for normality of the error term (and hence conditional log-normality of the equity premium) show some evidence of non-normality when estimated as a single equation, but less or no evidence if estimated in a system of equations with the interest rate and dividend growth rate equations.

Hence we generate the ex ante equity premia, interest rate, and dividend growth rate series as autocorrelated series with jointly normal error terms, calibrated to the degree of autocorrelation observed in the US data. The processes we simulate also mimic the covariance structure between the residuals from the time-series models of equity premia, interest rates, and dividend growth rates as estimated using US data. We adjust the mean and the standard deviation of these log-normal processes to generate the desired level and variability for each when they are transformed back into levels. The coefficients and error covariance structure are estimated with FIML (very similar results are obtained using iterative GMM and Newey and West, 1987, heteroskedasticity and autocorrelation consistent covariance estimation).

To give a sense for what our estimated models for interest rates, dividend growth rates, and the equity premium look like, we present in Table A.I the estimated parameters of Model 1, which incorporates an AR(1) model for interest rates (r), a MA(1) model for dividend growth rates (g), and an AR(1) model for the ex ante equity premium (π).

[17] The mean of the estimated equity premium from this model is 5.8% and its standard deviation is 2.2%. An AR(1) model of the natural logarithm of the equity premium has a coefficient of 0.79 on the lagged equity premium, with a standard error of 0.050 and an R^2 of 0.83.

Table A.I
Estimated Parameters of Model 1

$log(r_t)$	=	-0.214	$+0.929\, log(r_{t-1})$	$+\epsilon_{r,t}$
		(0.262)	(0.086)	
$log(1+g_t)$	=	0.0516	$+0.454\, \epsilon_{g,t-1}$	$+\epsilon_{g,t}$
		(0.0063)	(0.084)	
$log(\hat{\pi}_t)$	=	-0.562	$+0.851\, log(\hat{\pi}_{t-1})$	$+\epsilon_{\pi,t}$
		(0.230)	(0.070)	

In Table A.I, standard errors of the estimated coefficients are shown in parentheses. The covariance of $\epsilon_{r,t}$ and $\epsilon_{g,t}$ equals 0.00240, the covariance of $\epsilon_{r,t}$ and $\epsilon_{\pi,t}$ equals -0.0117, and the covariance of $\epsilon_{g,t}$ and $\epsilon_{\pi,t}$ equals 0.0018. The variance of $\epsilon_{r,t}$ equals 0.0890, the variance of $\epsilon_{g,t}$ equals 0.000986, and the variance of $\epsilon_{\pi,t}$ equals 0.0648. The adjusted R^2 for the interest rate equation is 72.9%, the adjusted R^2 for the dividend growth rate equation is 30.0%, and the adjusted R^2 for the equity premium equation is 79.5%.

A1.2 Allowing a Downward Trend in the Ex Ante Equity Premium Process

Pástor and Stambaugh (2001), among others, provide evidence that the equity premium has been trending downward over the sample period we study, finding a modest downward trend of roughly 0.80% in total since the early 1950s, with much of the difference coming from a steep decline in the 1990s. Their study of the equity premium has the premium fluctuating between about 4% and 6% since 1834. Given this evidence and the fact that we calibrate to data starting in the 1950s, we investigate a 0.80% trend in the equity premium, and when modeling a trend with a break we limit ourselves to a 0.30% trend with an additional 50 basis point break, as discussed below. This is accomplished in conjunction with setting the ex ante equity premium to follow an AR(1) process.

A1.3 Allowing a Structural Break in the Equity Premium Process

Pástor and Stambaugh (2001) estimate the probability of a structural break in the equity premium over the last two centuries. They find fairly strong support for there having been a structural break over the 1990s which led to a 0.5% drop in the equity premium. An aggressive interpretation of their results would have the majority of the drop in the equity premium over the 1990s occurring at once. We decide to adopt a one-time-drop specification because doing so makes our results more conservative (*i.e.* produces a wider confidence interval for the ex ante equity premium). Spreading the drop in the premium across several years serves only to narrow the range of ex ante equity premium consistent with the US returns data over the last 50 years, which would only bolster our claims to provide a much tighter confidence interval about the estimate of the ex ante equity premium. Thus we incorporate an abrupt 50 basis point drop in the equity premium in some of the models we consider. We time the drop to coincide with 1990, 39 years into our simulation period. This feature of the equity premium process can be accomplished with or without incorporating other features discussed above.

A1.4 Allowing for Sampling Variability in Generating Parameters

Our experiments are motivated by the large sampling variability of the ex post equity premium, but when we produce our simulations we have to first estimate the parameter values for the time-series models of dividend growth rates, interest rates, and ex ante equity premia. These estimates themselves incorporate sampling variability. Fortunately, estimates of the sampling variability are available to us through the covariance matrix of our parameters, so we can incorporate uncertainty about the true values of these parameters into our simulations. We estimate our system of equations (the dividend growth rate, interest rate, and the ex ante equity premium equation) jointly with FIML, and generate for *each* simulation an independent set of parameters drawn randomly from the joint limiting normal distribution of these parameter estimates (including the variance and covariance of the equation residuals) subject to some technical considerations[18] and data consistency checks.[19] This process accounts for possible variability in the true state of the world that generates dividends, interest rates, and ex ante equity premia.

To illustrate, for Model 1 reported in Table A.I,

[18]The time-series models must exhibit stationarity, the growth rate of dividends must be strictly less than the discount rate, and the residual variances must be greater than zero.

[19]The parameters must generate mean interest rates, dividend growth rates, and ex post equity premia that lie within three standard deviations of the US data sample mean. Also, the limiting price-dividend ratio must be within 50 standard deviations of the mean US price-dividend ratio. This last consistency check rules out some extreme simulations generated when the random draw of parameters leads to near unit root behavior. The vast majority of simulations do not exhibit price-dividend ratios that are more than a few standard deviations from the mean of the US data.

$$
\begin{aligned}
log(r_t) &= \alpha_r + \rho_r \, log(r_{t-1}) + \epsilon_{r,t} \\
log(1+g_t) &= \alpha_g + \theta_g \, \epsilon_{g,t-1} + \epsilon_{g,t} \\
log(\hat{\pi}_t) & \quad \alpha_\pi + \rho_\pi \, log(\hat{\pi}_{t-1}) + \epsilon_{\pi,t},
\end{aligned}
$$

the estimated covariance matrix of the parameter estimates is shown in Table A.II.

Table A.II
Estimated Covariance Matrix for Model 1 Parameters

	α_r	ρ_r	α_g	θ_g	α_π	ρ_π
α_r	0.068705	0.022307	-.000051933	.000226443	-0.012165	-0.003511
ρ_r	0.022307	0.007436	-.000040346	.000114831	-0.004730	-0.001401
α_g	-0.000052	-0.000040	0.000039674	.000025651	0.000153	0.000031
θ_g	0.000226	0.000115	0.000025651	.007086714	0.001699	0.000454
α_π	-0.012165	-0.004730	0.000153376	.001699151	0.052664	0.015791
ρ_π	-0.003511	-0.001401	0.000031495	.000453874	0.015791	0.004844

The top-left element of Table A.II, equal to 0.068705, is the variance of the parameter estimate of α_r. The entry below the top-left element, equal to 0.022307, is the covariance between the estimate of α_r and ρ_r, and so on. The *estimated covariance matrix* of the equation residual variances is shown in Table A.III. (The variances themselves are reported in Section A1.1, as are the parameter estimates of the mean.)

Table A.III
Estimated Covariance Matrix of Model 1 Residual Variances

	ϵ_r^2	$\epsilon_r\epsilon_g$	$\epsilon_r\epsilon_\pi$	ϵ_g^2	$\epsilon_g\epsilon_\pi$	ϵ_π^2
ϵ_r^2	0.0000944	$1.9729 \cdot 10^{-6}$	$-8.351 \cdot 10^{-7}$	$-1.902 \cdot 10^{-7}$	$-1.564 \cdot 10^{-6}$	$-1.69 \cdot 10^{6}$
$\epsilon_r\epsilon_g$	$1.9729 \cdot 10^{-6}$	$8.5163 \cdot 10^{-7}$	$1.0437 \cdot 10^{-6}$	$4.3066 \cdot 10^{-8}$	$-1.602 \cdot 10^{-7}$	$9.1448 \cdot 10^{-7}$
$\epsilon_r\epsilon_\pi$	$-8.351 \cdot 10^{-7}$	$1.0437 \cdot 10^{-6}$	0.0000797	$1.8827 \cdot 10^{-7}$	$5.001 \cdot 10^{-6}$	-0.000044
ϵ_g^2	$-1.902 \cdot 10^{-7}$	$4.3066 \cdot 10^{-8}$	$1.8827 \cdot 10^{-7}$	$4.8337 \cdot 10^{-8}$	$9.6885 \cdot 10^{-8}$	$1.3458 \cdot 10^{-6}$
$\epsilon_g\epsilon_\pi$	$-1.564 \cdot 10^{-6}$	$-1.602 \cdot 10^{-7}$	$5.001 \cdot 10^{-6}$	$9.6885 \cdot 10^{-8}$	$3.5567 \cdot 10^{-6}$	0.0000203
ϵ_π^2	$-1.69 \cdot 10^{-6}$	$9.1448 \cdot 10^{-7}$	-0.000044	$1.3458 \cdot 10^{-6}$	0.0000203	0.0005009

The top-left element, equal to 0.0000944, is the variance of ϵ_r^2. The entry below the top-left element, equal to $-1.9729 \cdot 10^{-6}$, is the covariance between the estimate of ϵ_r^2 and the product of ϵ_r and ϵ_g, and so on.

Exploiting block diagonality of the parameters of the mean and variance, and asymptotic normality of all the estimated parameters, we generate two sets of normally distributed random variables.

Each set is independent of the other, the first set of six having the covariance matrix from Table A.II with means equal to the parameter estimates listed in Table A.I, and the second set of six having the covariance matrix from Table A.III, with means equal to the equation residual covariances listed in Section A1.1. This set of 12 random variables is then used to simulate interest rates, dividend growth rates, and equity premia, subject to the consistency checks footnoted earlier.

A1.5 Allowing for Disappearing Dividends

An issue with our calibration to dividends is the impact of declining dividend payments in the US. This phenomenon is a result of a practice adopted widely beginning in the late 1970s, whereby US firms have been increasingly delivering cashflows to investors in ways not recorded as corporate dividends, such as share repurchases. Fama and French (2001) document the widespread decline of regular dividend payments starting in 1978, consistent with evidence provided by Bagwell and Shoven (1989) and others. Fama and French find evidence that the disappearance of dividends is in part due to an increase in the inflow of new listing to US stock exchanges, representing mostly young companies with the characteristics of firms that would not be expected to pay dividends, and in part due to a decline in the propensity of firms to pay dividends. Fama and French find only a small decline in the probability to pay dividends among the firms that we calibrate to, those in the S&P 500 index.

Consistent with Fama and French, we find no evidence of a break in our data on dividend growth rates. Though dividend *yields* on the S&P 500 index have dropped dramatically over time, dividend growth rates have not. The decline in yields has been a function of prices rising faster than dividends since 1978, not dividends declining in any absolute sense. From 1952 through 1978, the year Fama and French document as the year of the structural break in dividend payments, dividend growth rates among the S&P 500 firms have averaged 4.9% with an annual standard deviation of 3.9%, and from 1979 to 2000 the dividend growth rates have averaged 5.5% with an annual standard deviation of 3.8%, virtually indistinguishable from the pre-1979 period. Time series properties pre- and post-1978 are also very similar across these two periods. Consistent with this stability of dividend growth pre- and post-1978 and Bagwell and Shoven's documentation of increased share repurchases in the 1980s, *earnings growth rates* of firms in the S&P 500 index have accelerated since the 1952-1978 period, from 6.8% pre-1979 to 7.8% post-1978. Similar to the dividend growth rate data, the time-series properties of the earnings growth rate data did not change.

In order to determine the sensitivity of our experiments to mismeasurement of cashflows to investors, we consider a dividend growth rate process with a structural break 27 years into the time series to correspond to a possible break in our dividend data for the S&P 500 data after 1978. We calibrate to the S&P 500 earnings data mean growth rate increase over 1979-2000, an upward shift of 100 basis points, to proxy for the increase in total cashflows to investors. That is, we increase the growth rate of dividends by 5 basis points a year for 20 years, starting in year 27 of the simulation (corresponding to 1978 for the S&P 500 data), to increase the mean growth rate of our dividend growth series 100 basis points, mimicking the proportional increase in earnings growth rates.

Appendix 2: Further Details on the Simulations

A2.1 Fundamentals

We define P_t as a stock's beginning-of-period-t price and E_t as the expectations operator conditional on information available up to but not including the beginning of period t. The discount rate (r_t, which equals the risk-free rate plus the equity premium) is the rate investors use to discount payments received during period t (*i.e.*, from the beginning of period t to the beginning of period $t+1$). Recall that investor rationality requires that the time t market price of a stock, which will pay a dividend D_{t+1} one period later and then sell for P_{t+1}, satisfy Equation (3):

$$P_t = E_t\left\{\frac{P_{t+1}+D_{t+1}}{1+r_t}\right\}. \tag{3}$$

Invoking the standard transversality condition that the expected present value of the stock price P_{t+i} falls to zero as i goes to infinity, and defining the growth rate of dividends during period t as $g_t \equiv (D_{t+1}-D_t)/D_t$, allows us rewrite Equation (3) as:

$$P_t = D_t E_t\left\{\sum_{i=0}^{\infty}\left(\Pi_{k=0}^{i}\left[\frac{1+g_{t+k}}{1+r_{t+k}}\right]\right)\right\}. \tag{5}$$

One attractive feature of expressing the present value stock price as in Equation (5), in terms of dividend growth rates and discount rates, is that this form highlights the irrelevance of inflation, at least to the extent that expected and actual inflation are the same. Notice that working with nominal growth rates and discount rates, as we do, is equivalent to working with deflated nominal rates (*i.e.*, real rates). That is, $\frac{1+([g_t-I_t]/[1+I_t])}{1+([r_t-I_t]/[1+I_t])} = \frac{(1+g_t)}{(1+r_t)}$, where I_t is inflation. Working with nominal values in our simulations removes a potential source of measurement error associated with attempts to estimate inflation.

Properties of prices and returns produced by Equation (5) depend in important ways on the modeling of the dynamics of the dividend growth, interest rate, and equity premium processes. For instance, the stock price would equal a constant multiple of the dividend level and returns would be very smooth over time if dividend growth and interest rates were set equal to constants plus independent innovations. However, using models that capture the serial dependence of dividend growth rates, interest rates, and equity premia observed in the data, as we do, would typically lead to time-varying price-dividend ratios and variable returns of the sort we observe in observed stock market data.

A2.2 Numerical Simulation

We now provide details on the numerical simulation which comprises Step 4 of the 5-step procedure outlined in Section I above. That is, we detail for the n^{th} economy the formation of the prices (P_t^n), returns (R_t^n), ex post equity premia ($\hat{\pi}^n$), *etc.* (where $n = 1, \cdots, N$ and $t = 1, \cdots, T$), given dividends, dividend growth rates, risk-free interest rates, and the equity premium of the n^{th} economy: D_t^n, g_{t-1}^n, and $r_{t-1}^n = r_{f,t-1}^n + \pi$.[20] For simplicity, we illustrate our methodology by assuming fixed parameters (no parameter uncertainty), a constant ex ante equity premium, and an AR(1) model for interest rates. Further, to illustrate the procedure required for a moving average error model, we assume a MA(1) process for dividend growth rates. Relaxing these assumptions (the assumptions to incorporate parameter uncertainty, ARMA(1,1) processes for interest rates and dividend growth rates, and a time-varying equity premium) complicates the procedure outlined below only slightly. Note that in our actual simulations we set the initial dividend growth rate and

[20]We set the number of economies, N, at 2,000. This is a sufficiently large number of replications to produce results with very small simulation error.

interest rate to their unconditional means, innovations to zero, and dividends to \$1, then simulate the economies out for 50 periods. At period 51 we start our calculation of market prices, returns, *etc.* (to avoid contaminating the simulations with the initial conditions). For simplicity, we do not include this detail in the description below but for concreteness we describe a similar prototypical simulation.

In terms of timing and information, recall that P_t^n is the stock's beginning-of-period-t price, r_t^n is the rate used to discount payments received during period t and is known at the beginning of period t, D_t^n is paid at the beginning of period t, g_t^n is defined as $(D_{t+1}^n - D_t^n)/D_t^n$ and is not known at the beginning of period t since it depends on D_{t+1}^n, and $E_t\{\cdot\}$ is the conditional expectation operator, with the conditioning information being the set of information available to investors up to but not including the beginning of period t. Finally, recall Equation (5), rewritten to correspond to the n^{th} economy:

$$P_t^n = D_t^n E_t\left\{\sum_{i=0}^{\infty}\left(\Pi_{k=0}^{i}\left[\frac{1+g_{t+k}^n}{1+r_{t+k}^n}\right]\right)\right\}. \tag{17}$$

Returns are constructed as $R_t^n = (P_{t+1}^n + D_{t+1}^n - P_t^n)/P_t^n$, and $\hat{\pi}^n = \overline{R}^n - \overline{r}_f^n$ where $\overline{R}^n = \frac{1}{T}\sum_{t=1}^{T} R_t^n$ and $\overline{r}_f^n = \frac{1}{T}\sum_{t=1}^{T} r_{f,t}^n$.

Based on Equation (17), we generate prices by generating a multitude of possible streams of dividends and discount rates, present-value discounting the dividends with the discount rates, and averaging the results, *i.e.*, by conducting a Monte Carlo integration.[21] Hence we produce prices (P_t^n), returns (R_t^n), ex post equity premia ($\hat{\pi}^n$), and a myriad of other financial quantities, utilizing only dividend growth rates and discount rates. The *exact* procedure by which we conduct this numerical simulation is described below and summarized in Figure A.1. (These steps, labeled Steps 4A through 4C, collectively constitute Step 4 of the 5-step procedure outlined in Section I above.)

[21] According to Equation (17), the stream of dividends and discount rates should be infinitely long, however truncating the stream at a sufficiently distant point in time denoted I leads to a very small approximation error. We discuss this point more fully below.

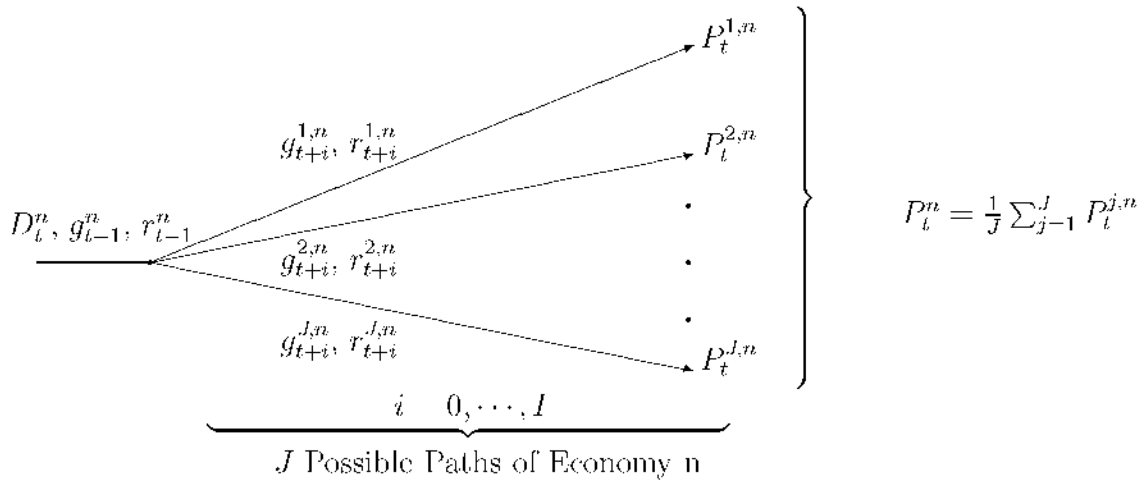


Figure A.1 *Diagram of a Simple Market Price Calculation for the* t^{th} *Observation of the* n^{th} *Economy (Steps 4A and 4B)*

Step 4A: In forming P_t^n, the most recent fundamental information available to an investor would be g_{t-1}^n, D_t^n, and r_{t-1}^n. Thus g_{t-1}^n, D_t^n, and r_{t-1}^n must be generated directly in our simulations, whereas P_t^n is calculated based on these g, D, and r. The objective of Steps 4A(i)-(iii) outlined below is to produce dividend growth and interest rates that replicate real-world dividend growth and interest rate data. That is, the simulated dividend growth and interest rates must have the same mean, variance, covariance, and autocorrelation structure as observed S&P 500 dividend growth rates and US interest rates. In terms of Figure A.1, Step 4A forms g_{t-1}^n, D_t^n, and r_{t-1}^n only.

Step 4A(i): Note that since, as described above, the logarithm of one plus the dividend growth rate is modeled as a MA(1) process, $log(1 + g_t^n)$ is a function of only innovations, labeled ϵ_g^n. Note also that since the logarithm of the interest rate is modeled as an AR(1) process, $log(r_{f,t}^n)$ is a function of $log(r_{f,t-1}^n)$ and an innovation labeled ϵ_r^n. Set the initial dividend, D_1^n, equal to the total S&P 500 dividend value for 1951 (observed at the end of 1951), and the lagged innovation of the logarithm of the dividend growth rates $\epsilon_{g,0}^n$ to 0. To match the real-world interest rate data, set $log(r_{f,0}^n) = -2.90$ (the mean value of log interest rates required to produce interest rates matching the mean of observed T-bill rates). Then generate two independent standard normal random numbers, η_1^n and ν_1^n (note that the subscript on these random numbers indicates time, t), and form two correlated random variables, $\epsilon_{r,1}^n = 0.319(0.25\eta_1^n + (1-.25^2)^{.5}\nu_1^n)$ and $\epsilon_{g,1}^n = 0.0311\eta_1^n$. These are the simulated innovations to the interest rate and dividend growth rate processes, formed to have standard deviations of 0.319 and 0.0311 respectively to match the data, and to be correlated with correlation coefficient 0.25 as we find in the S&P 500 return and T-bill rate data. Next, form

$log(1+g_1^n) = 0.049 + 0.64\epsilon_{g,0}^n + \epsilon_{g,1}^n$ and $log(r_{f,1}^n) = -0.35 + 0.88log(r_{f,0}^n) + \epsilon_{r,1}^n$ to match the parameters estimated on the S&P 500 index data 1952-2004 of these models (using Full Information Maximum Likelihood).[22] Also form $D_2^n = D_1^n(1+g_1^n)$.

Step 4A(ii): Produce two correlated normal random variables, $\epsilon_{r,2}^n$ and $\epsilon_{g,2}^n$ as in Step 4A(i) above, and conditioning on $\epsilon_{g,1}^n$ and $log(r_{f,1}^n)$ from Step 4A(i) produce $log(1+g_2^n) = 0.049 + 0.64\epsilon_{g,1}^n + \epsilon_{g,2}^n$, $log(r_{f,2}^n) = -0.35 + 0.88log(r_{f,1}^n) + \epsilon_{r,2}^n$, and $D_3^n = D_2^n(1+g_2^n)$.

Step 4A(iii): Repeat Step 4A(ii) to form $log(1+g_t^n)$, $log(r_{f,t}^n)$, and D_t^n for $t = 3, 4, 5, \cdots, T$ and for each economy $n = 1, 2, 3, \cdots, N$. Then calculate the dividend growth rate g_t^n and the discount rate r_t^n (which equals $r_{f,t}^n$ plus the ex ante equity premium).

Step 4B: For each time period $t = 1, 2, 3, \cdots, T$ and economy $n = 1, 2, 3, \cdots, N$ we calculate prices, P_t^n. In order to do this we must solve for the expectation of the infinite sum of discounted future dividends conditional on time $t-1$ information for economy n. That is, we must produce a set of possible paths of dividends and interest rates that might be observed in periods $t, t+1, t+2, \cdots$ given what is known at period $t-1$ and use these to solve the expectation of Equation (17). We use the superscript j to index the possible paths of future economies that could possibly evolve from the current state of the economy. In Step 4B(iv) below, we describe how we are able to solve for the expectation of an infinite sum using a *finite* stream of future dividends.

Step 4B(i): Set $\epsilon_{g,t-1}^{j,n} = \epsilon_{g,t-1}^n$ and $log(r_{f,t-1}^{j,n}) = log(r_{f,t-1}^n)$ for $j = 1, 2, 3, \cdots, J$.[23] Generate two independent standard normal random numbers, $\eta_t^{j,n}$ and $\nu_t^{j,n}$, and form two correlated random variables $\epsilon_{r,t}^{j,n} = 0.319(0.25\eta_t^{j,n} + (1-.25^2)^{.5}\nu_t^{j,n})$ and $\epsilon_{g,t}^{j,n} = 0.0311\eta_t^{j,n}$ for $j = 1, 2, 3, \cdots, J$.[24] These

[22]Note that by construction these parameters do not match those reported for the system reported in Appendix 1 as this system does not incorporate a time-varying equity premium.

[23]We choose J to lie between 1,000 and 100,000, as needed to ensure the Monte Carlo simulation error in calculating prices and returns is controlled to be less than 0.20%. For the typical case the simulation error is far less than 0.20%. To determine the simulation error, we conducted a simulation of the simulations. Unlike some Monte Carlo experiments (such as those estimating the size of a test statistic under the null) the standard error of the simulation error for most of our estimates (returns, prices, *etc.*) are themselves analytically intractable, and must be simulated. In order to estimate the standard error of the simulation error in estimating market prices, we estimated a single market price 2,000 times, each time independent of the other, and from this set of prices computed the mean and variance of the price estimate. If the experiment had no simulation error, each of the price estimates would be identical. With the number of possible paths, J, equal to no less than 1,000 we find that the standard deviation of the simulation error is less than 0.20% of the price, which is sufficiently small as not to be a source of concern for our study. The number of simulations has to be substantially greater than 1,000 for some cases depending on the model specification and the ex ante equity premium.

[24]For our random number generation we made use of a variance reduction technique, stratified sampling. This technique has us drawing pseudo-random numbers ensuring that $q\%$ of these draws come from the q^{th} percentile, so that our sampling does not weight any grouping of random draws too heavily.

are the simulated innovations to the interest rate and dividend growth rate processes, respectively. Form $log(1+g_t^{j,n}) = 0.049 + 0.64\epsilon_{g,t-1}^{j,n} + \epsilon_{g,t}^{j,n}$ and $log(r_{f,t}^{j,n}) = -0.35 + 0.88log(r_{f,t-1}^{j,n}) + \epsilon_{r,t}^{j,n}$.

Step 4B(ii): Produce two correlated normal random variables $\epsilon_{r,t+1}^{j,n}$ and $\epsilon_{g,t+1}^{j,n}$ as in Step 4B(i) above, and conditioning on $\epsilon_{g,t}^{j,n}$ and $log(r_{f,t}^{j,n})$ from Step 4B(i) produce $log(1+g_{t+1}^{j,n}) = 0.049 + 0.64\epsilon_{g,t}^{j,n} + \epsilon_{g,t+1}^{j,n}$ and $log(r_{f,t+1}^{j,n}) = -0.35 + 0.88log(r_{f,t}^{j,n}) + \epsilon_{r,t+1}^{j,n}$ for $j = 1, 2, 3, \cdots, J$.

Step 4B(iii): Repeat Step 4B(ii) to form $log(1+g_{t+i}^{j,n})$ and $log(r_{t+i}^{j,n})$ for $i = 2, 3, 4, \cdots, I$, $j = 1, 2, 3, \cdots, J$, and economies $n = 1, 2, 3, \cdots, N$.

Step 4B(iv): The discounted present value of each of the individual J streams of dividends is now taken in accordance with Equation (17), with the j^{th} present value price noted as $P_t^{j,n}$. Finally, the price for the n^{th} economy in period t is formed: $P_t^n = \frac{1}{J}\sum_{j=1}^{J} P_t^{j,n}$.

In considering these prices, note that according to Equation (17) the stream of discount rates and dividend growth rates should be infinitely long, while in our simulations we extend the stream for only a finite number of periods, I. Since the ratio of gross dividend growth rates to gross discount rates are less than unity in steady state, the individual product elements in the infinite sum in Equation (17) eventually converge to zero as I increases. (Indeed, this convergence to zero is exactly what is required for the standard transversality condition that the expected present value of the stock price P_{t+i} falls to zero as i goes to infinity.) We therefore set I large enough in our simulations so that the truncation does not materially effect our results. We find that setting $I = 1,000$ years is sufficient in all cases we studied. That is, the discounted present value of a dividend payment received 1,000 years in the future is essentially zero. Also note that the steps above are required to produce P_t^n, D_t^n, g_t^n, and r_t^n for $n = 1, \cdots, N$ and $t = 1, \cdots, T$; the intermediate terms superscripted with a j are required only to perform the numerical integration that yields P_t^N. Note that the length of the time series T is chosen to be 53 to imitate the 53 years of annual data we have available for the S&P 500 from 1952 to 2004.

Step 4C: After performing Steps 4A(i)-(iii) and 4B(i)-(iv) for $t = 1, \cdots, T$, rolling out N independent economies for T periods, we construct the market returns for each economy, $R_t^n = (P_{t+1}^n + D_{t+1}^n - P_t^n)/P_t^n$, and the ex post equity premium that agents in the n^{th} economy would observe, $\hat{\pi}^n$, estimated from Equation (1) as the mean difference in market returns and the risk-free rate.

Table I

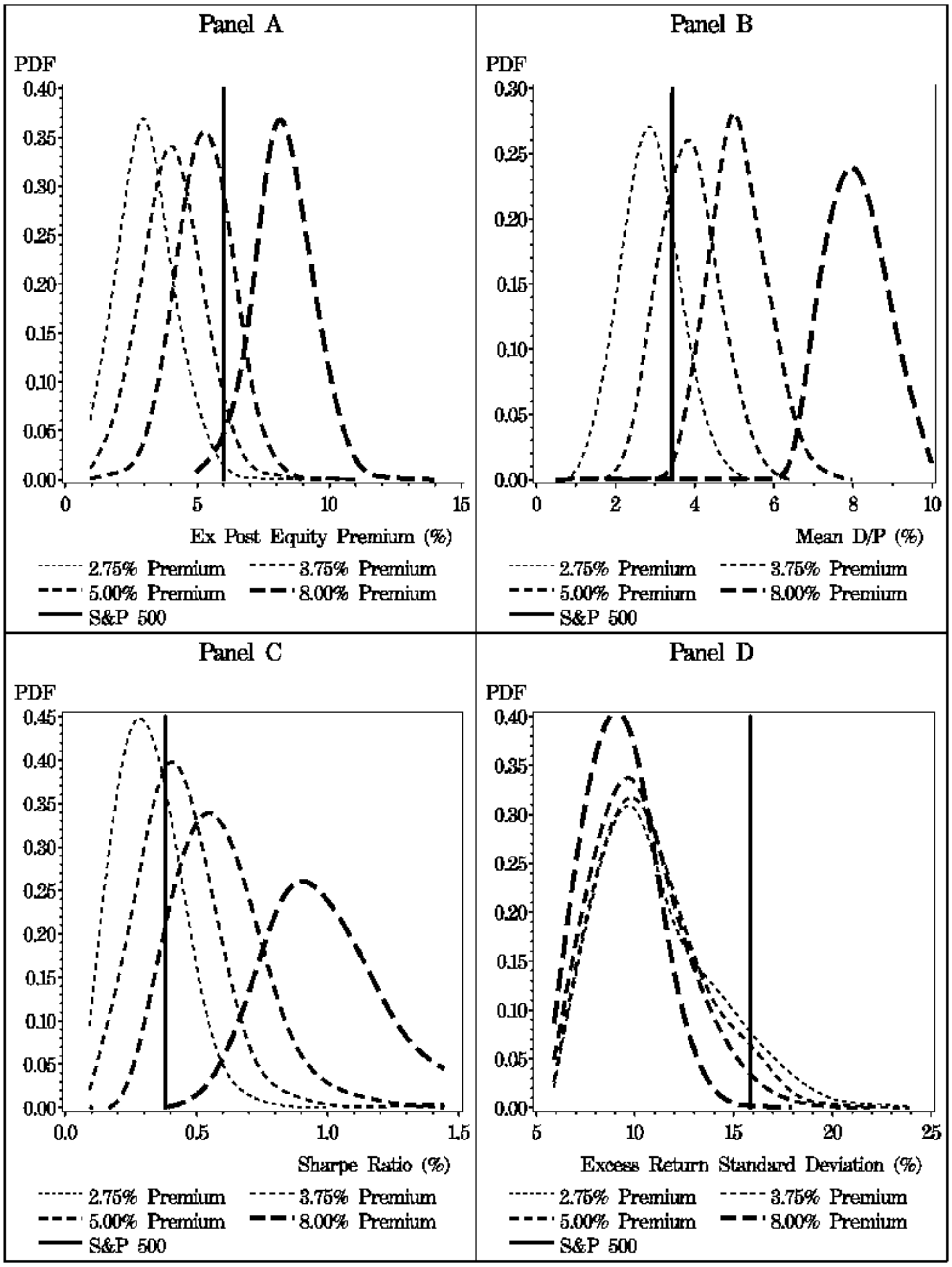
Characteristics of Simulated Models

Here we present the 12 models we consider, identifying the characteristics of their underlying data generating processes. The column titled "Processes for r, g, & π" indicates the nature of the time-series models used to generate the interest rates, dividend growth rates, and equity premium. See Appendix 1 for details on how this set of models was chosen and a description of how the equity premium series is produced. The column titled "Downward Trend in Equity Premium Process," identifies whether the ex ante equity premium trends downward over the course of the 53-year experiment, and if it does, provides the amount of the downward trend. The next column, "Structural Break in Equity Premium Process," indicates whether the model incorporates a sudden 50 basis point (bps) drop in the value of the ex ante equity premium. The column "Structural Break in Dividend Growth Process," indicates whether the model incorporates a gradual 100 basis point increase in the growth rate of the dividend growth rate. The final column indicates that all the models except Models 11 and 12 incorporate sampling variability in generating parameters. Additional model details are as follows. Parsimonious Model: interest rates follow an AR(1), dividend growth rates follow a MA(1), the equity premium follows an AR(1). Deterministic π Model: interest rates follow an AR(1), dividend growth rates follow a MA(1), the equity premium follows a deterministic downward trend with a 50 bps structural break. Best BIC Model:† interest rates follow an ARMA(1,1), dividend growth rates follow a MA(1), the equity premium follows an AR(1). Second-Best BIC Model:† interest rates follow an ARMA(1,1), dividend growth rates follow a MA(1), the equity premium follows an ARMA(1,1). Further details about each model feature are provided in Appendix 1.

Model	Processes for r, g, & π	Downward Trend in Equity Premium Process	Structural Break in Equity Premium Process	Structural Break in Dividend Growth Process	Sampling Variability in Generating Parameters
1	Parsimonious Model	No	No	No	Yes
2	Parsimonious Model with π Trend	Yes (80 bps)	No	No	Yes
3	Parsimonious Model with π Break	No	Yes (50 bps)	No	Yes
4	Parsimonious Model with Dividend Growth Trend	No	No	Yes	Yes
5	Parsimonious Model with π Trend and Dividend Growth Trend	Yes (80 bps)	No	Yes	Yes
6	Parsimonious Model with π Break, π Trend, and Dividend Growth Trend	Yes (30 bps)	Yes (50 bps)	Yes	Yes
7	Best BIC Model† with π Break, π Trend, and Dividend Growth Trend	Yes (30 bps)	Yes (50 bps)	Yes	Yes
8	Second-Best BIC Model† with π Break, π Trend, and Dividend Growth Trend	Yes (30 bps)	Yes (50 bps)	Yes	Yes
9	Parsimonious Model with π Break and π Trend	Yes (30 bps)	Yes (50 bps)	No	Yes
10	Deterministic π Model with π Break and π Trend	Yes (30 bps)	Yes (50 bps)	No	Yes
11	Parsimonious Model with Constant Parameters π Break, π Trend, and Dividend Growth Trend	Yes (30 bps)	Yes (50 bps)	Yes	No
12	Parsimonious Model with Constant Parameters	No	No	No	No

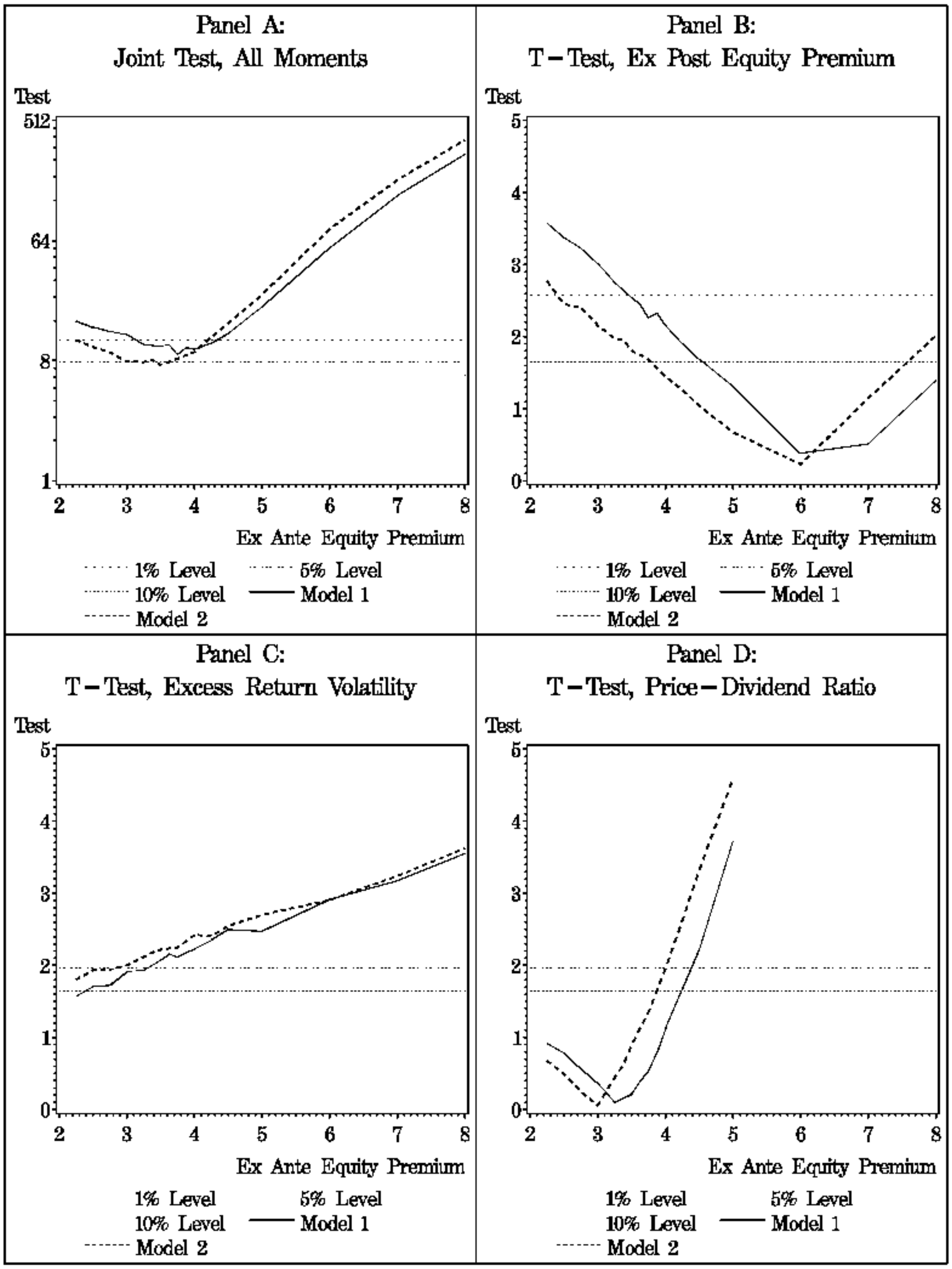
† For Models 7 and 8 we employ the Bayesian Information Criterion (BIC) to select the order of the ARMA model driving each of the interest rate, equity premium, and dividend growth rate processes. The order of each AR process and each MA process for each series is chosen over a (0, 1, 2) grid.

Figure 1: Probability Distribution Functions of Simulated Ex Post Equity Premia, Dividend Yields, Sharpe Ratios, and Return Standard Deviations



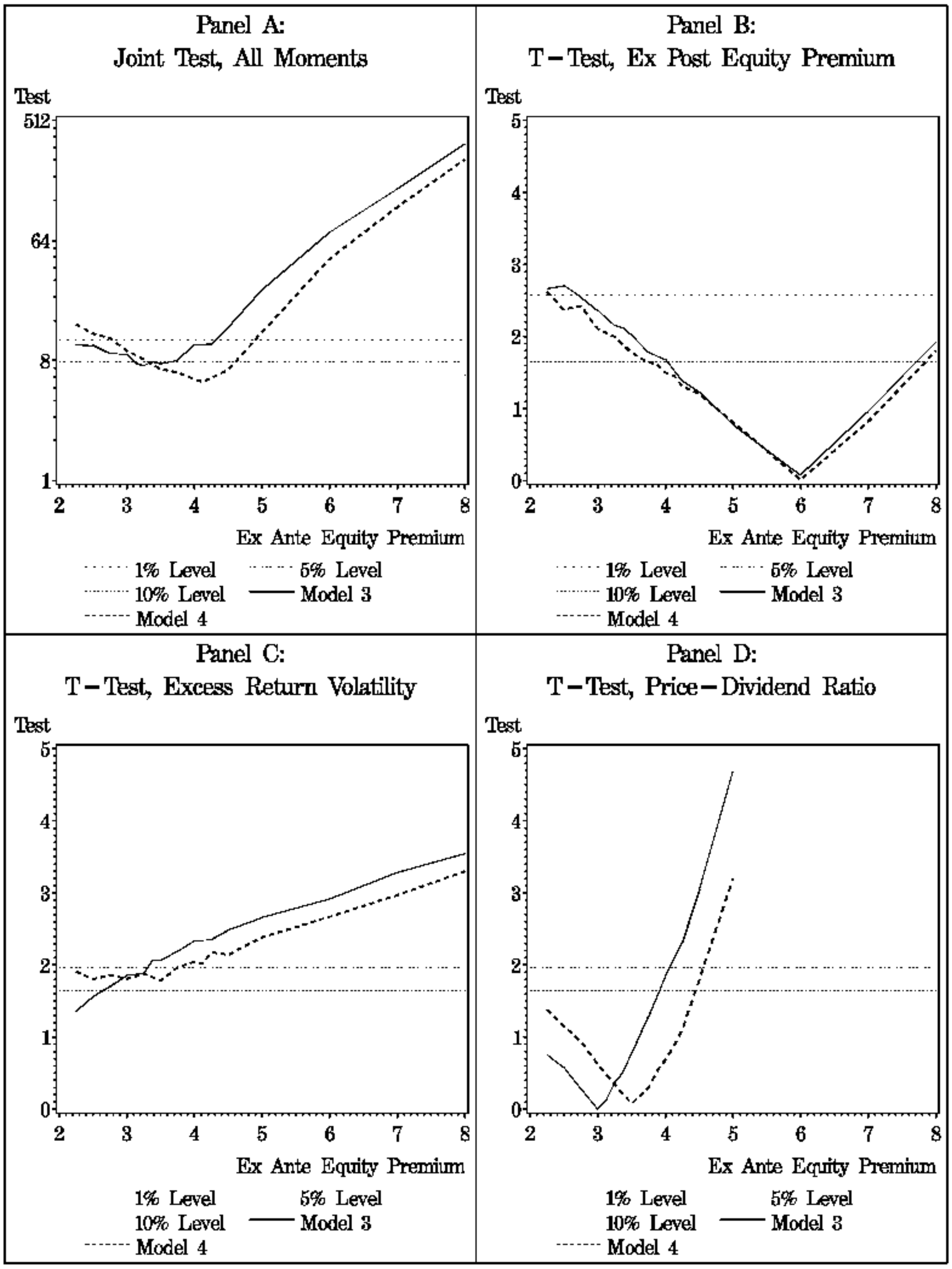
This figure contains probability distribution functions (PDFs) for various financial statistics generated in 2,000 simulated economies based on Model 1 from Table I. Each panel contains a PDF for each of four different assumed values of the ex ante equity premium: 2.75%, 3.75%, 5%, and 8%. Panel A shows the distribution of the ex post equity premium (mean return minus mean interest rate), Panel B shows the mean dividend yield distribution (dividend divided by price), Panel C shows the Sharpe ratio distribution (excess return divided by the standard deviation of the excess return), and Panel D shows the distribution of the standard deviation of excess returns. In each panel, a vertical line indicates the US data realized over 1952-2004, the value of the estimated ex post equity premium, mean dividend yield, mean Sharpe ratio, and excess return standard deviation, respectively. The simulated statistics are estimated on 53 years of generated data for each economy, mimicking the data period we used to estimate the actual US results.

Figure 2: Joint and Individual Tests Statistics for Models 1 and 2



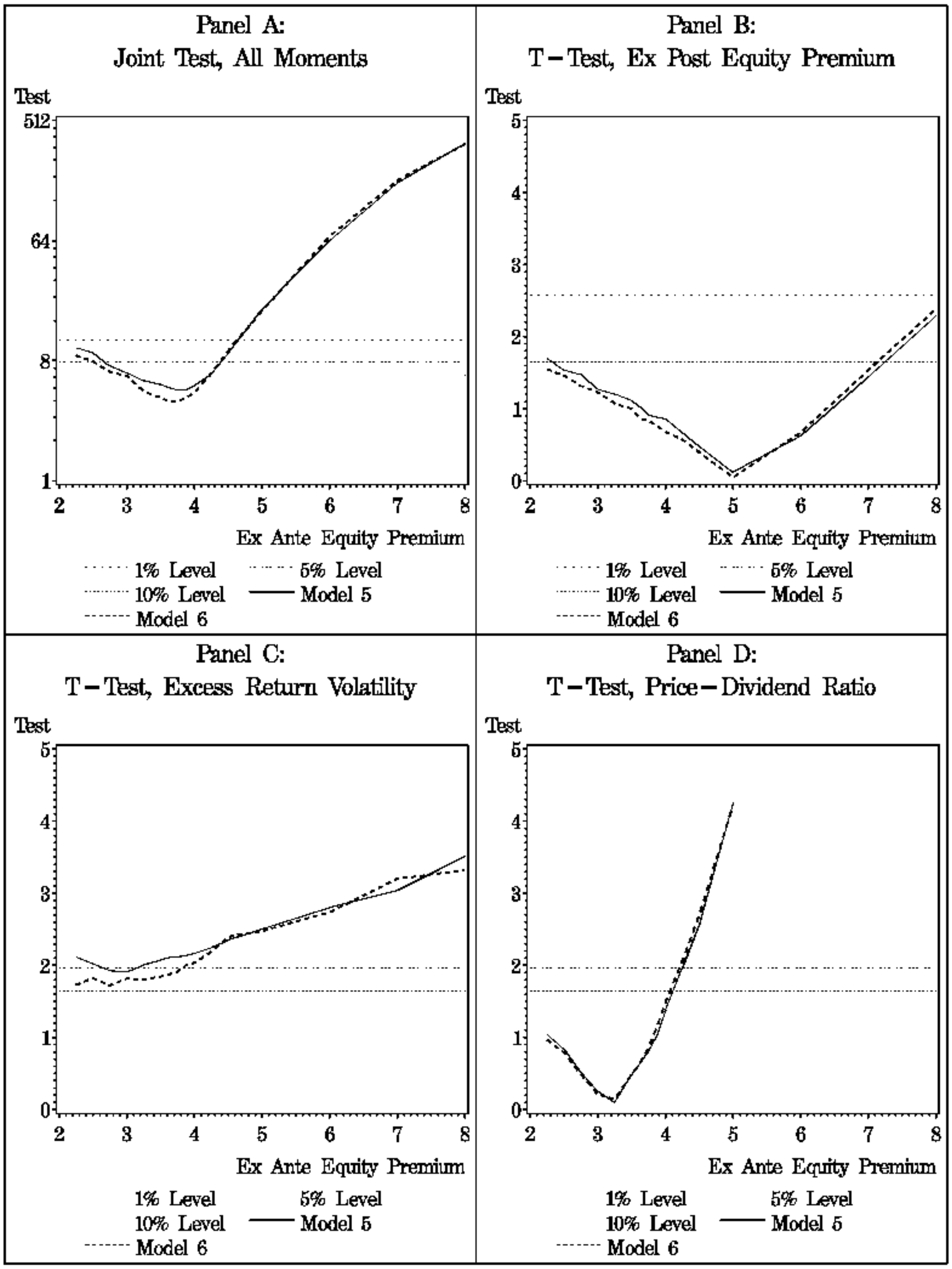
This figure contains plots of test statistics for Models 1 and 2. Panel A plots joint χ^2 tests based on a set of three variables (the ex post equity premium, the mean dividend yield, and the excess return volatility) for various ending values of the ex ante equity premium for each model. In Panel A the vertical axis is plotted on a log scale. The remaining panels contains t-test values corresponding to tests on the individual variables for each of the models: the ex post equity premium in Panel B, the excess return volatility in Panel C, and price-dividend ratio in Panel D. In each panel the critical values of the test statistics corresponding to test significance at the 10%, 5%, and 1% levels are indicated by horizontal lines.

Figure 3: Joint and Individual Tests Statistics for Models 3 and 4



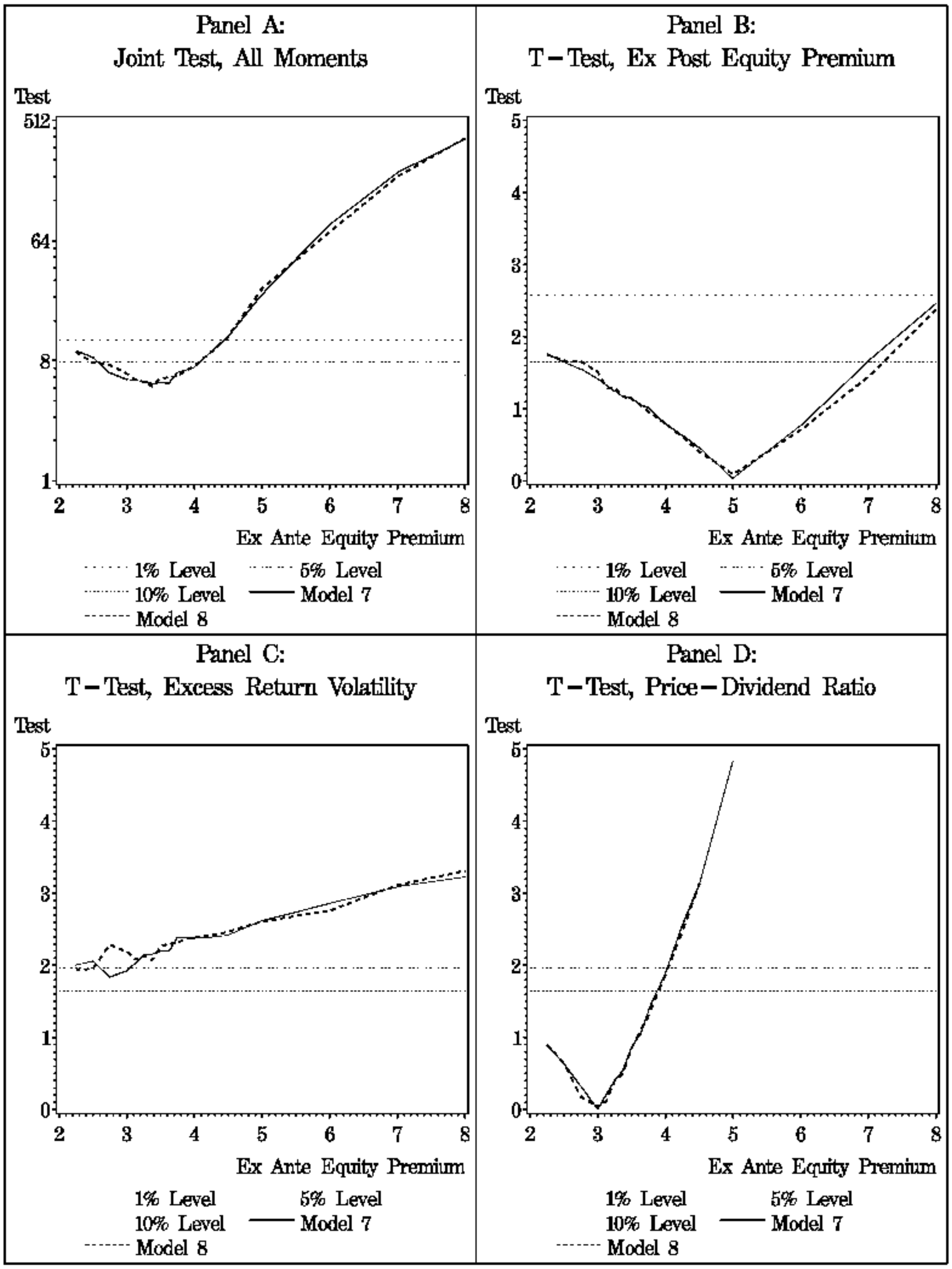
This figure contains plots of test statistics for Models 3 and 4. Panel A plots joint χ^2 tests based on a set of three variables (the ex post equity premium, the mean dividend yield, and the excess return volatility) for various ending values of the ex ante equity premium for each model. In Panel A the vertical axis is plotted on a log scale. The remaining panels contains t-test values corresponding to tests on the individual variables for each of the models: the ex post equity premium in Panel B, the excess return volatility in Panel C, and price-dividend ratio in Panel D. In each panel the critical values of the test statistics corresponding to test significance at the 10%, 5%, and 1% levels are indicated by horizontal lines.

Figure 4: Joint and Individual Tests Statistics for Models 5 and 6



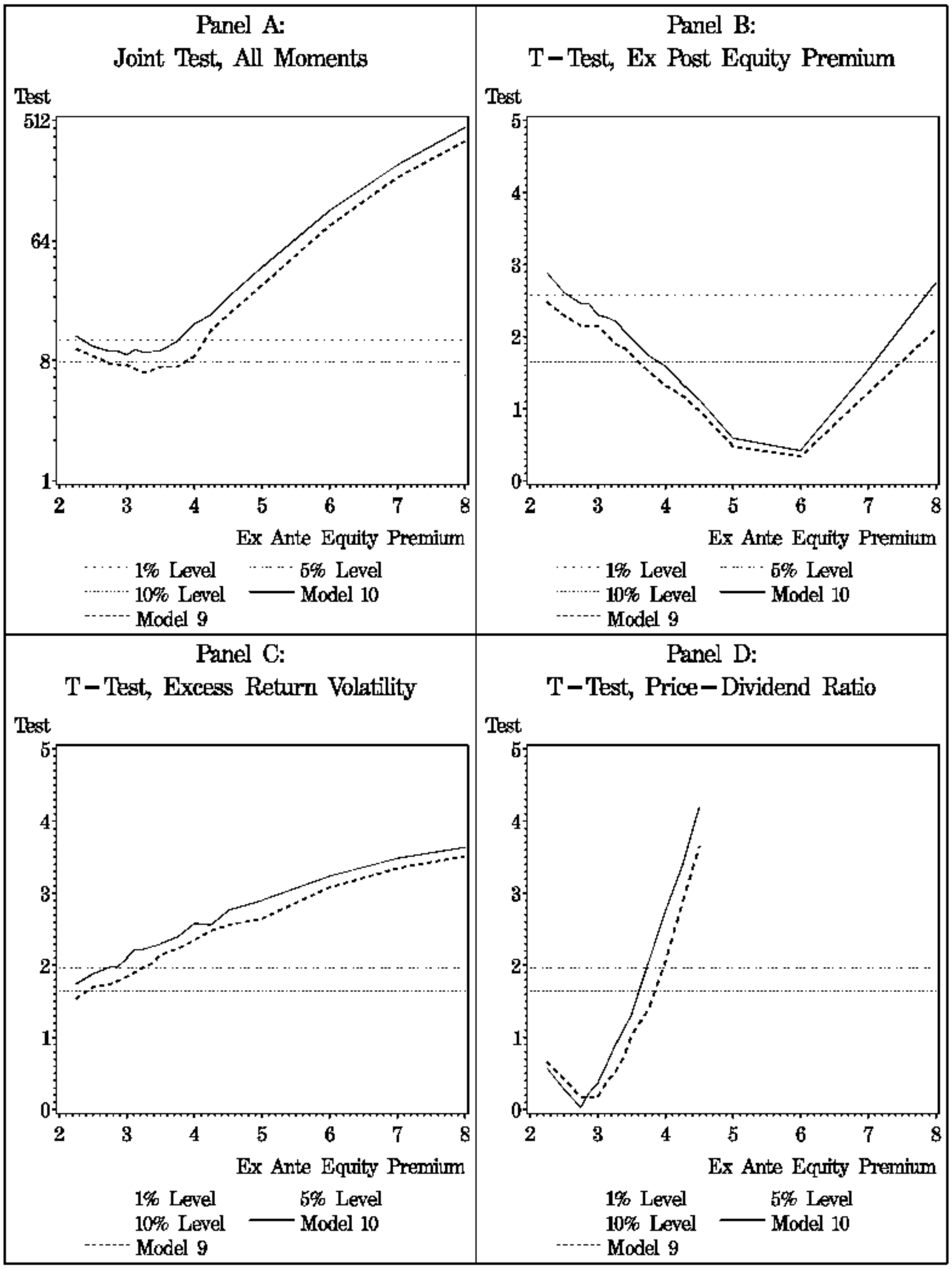
This figure contains plots of test statistics for Models 5 and 6. Panel A plots joint χ^2 tests based on a set of three variables (the ex post equity premium, the mean dividend yield, and the excess return volatility) for various ending values of the ex ante equity premium for each model. In Panel A the vertical axis is plotted on a log scale. The remaining panels contains t-test values corresponding to tests on the individual variables for each of the models: the ex post equity premium in Panel B, the excess return volatility in Panel C, and price-dividend ratio in Panel D. In each panel the critical values of the test statistics corresponding to test significance at the 10%, 5%, and 1% levels are indicated by horizontal lines.

Figure 5: Joint and Individual Tests Statistics for Models 7 and 8



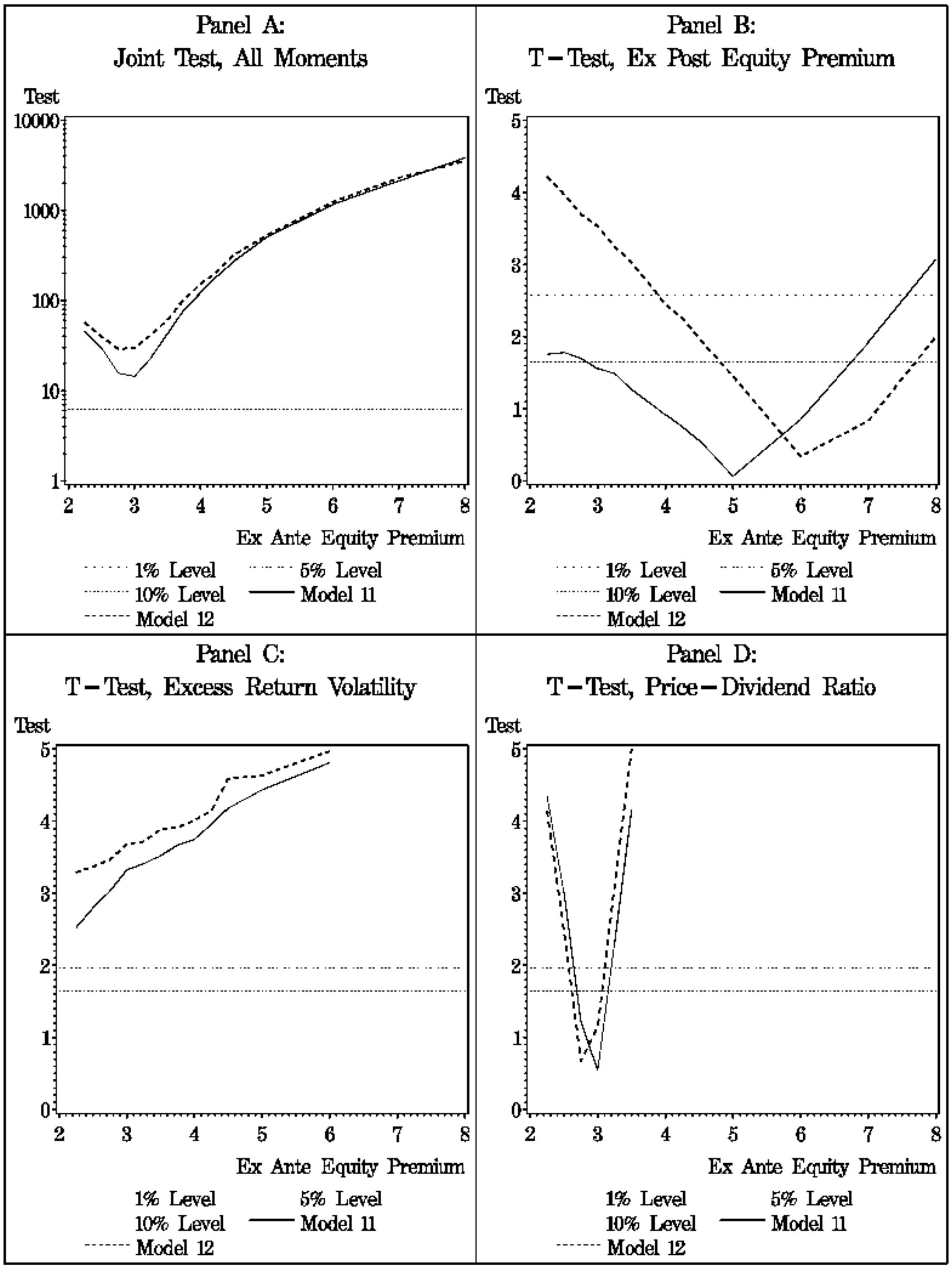
This figure contains plots of test statistics for Models 7 and 8. Panel A plots joint χ^2 tests based on a set of three variables (the ex post equity premium, the mean dividend yield, and the excess return volatility) for various ending values of the ex ante equity premium for each model. In Panel A the vertical axis is plotted on a log scale. The remaining panels contains t-test values corresponding to tests on the individual variables for each of the models: the ex post equity premium in Panel B, the excess return volatility in Panel C, and price-dividend ratio in Panel D. In each panel the critical values of the test statistics corresponding to test significance at the 10%, 5%, and 1% levels are indicated by horizontal lines.

Figure 6: Joint and Individual Tests Statistics for Models 9 and 10



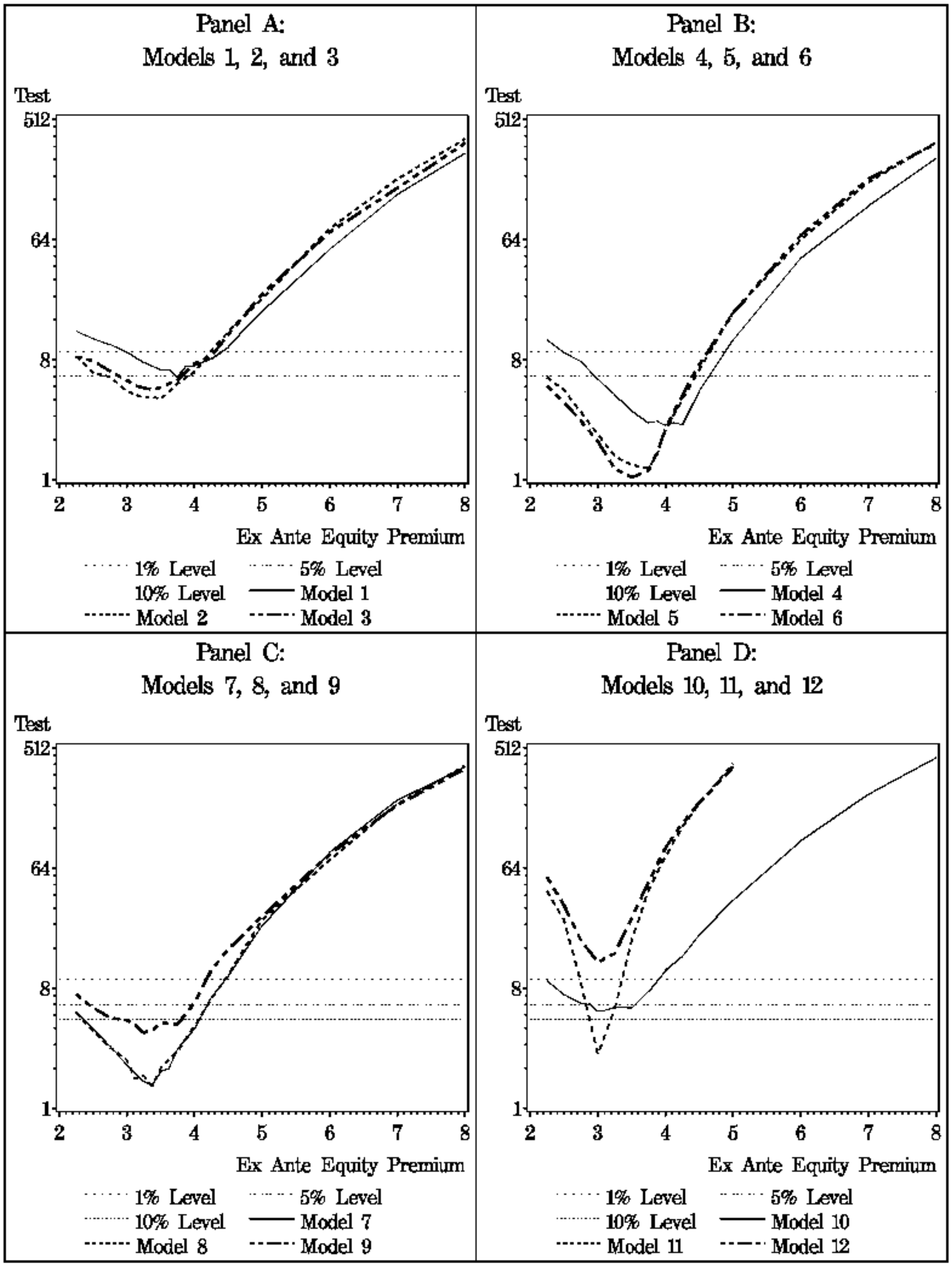
This figure contains plots of test statistics for Models 9 and 10. Panel A plots joint χ^2 tests based on a set of three variables (the ex post equity premium, the mean dividend yield, and the excess return volatility) for various ending values of the ex ante equity premium for each model. In Panel A the vertical axis is plotted on a log scale. The remaining panels contains t-test values corresponding to tests on the individual variables for each of the models: the ex post equity premium in Panel B, the excess return volatility in Panel C, and price-dividend ratio in Panel D. In each panel the critical values of the test statistics corresponding to test significance at the 10%, 5%, and 1% levels are indicated by horizontal lines.

Figure 7: Parameter Estimation Certainty: Joint and Individual Tests Statistics for Models 11 and 12



This figure contains plots of test statistics for Models 11 and 12. Panel A plots joint χ^2 tests based on a set of three variables (the ex post equity premium, the mean dividend yield, and the excess return volatility) for various ending values of the ex ante equity premium for each model. In Panel A the vertical axis is plotted on a log scale. The remaining panels contains t-test values corresponding to tests on the individual variables for each of the models: the ex post equity premium in Panel B, the excess return volatility in Panel C, and price-dividend ratio in Panel D. In each panel the critical values of the test statistics corresponding to test significance at the 10%, 5%, and 1% levels are indicated by horizontal lines.

Figure 8: Investors' Model Uncertainty
Joint Tests Based on a Subset of Moments for Models 1-12



This figure contains plots of joint χ^2 tests based on a set of two variables, the ex post equity premium and the mean dividend yield, for various ending values of the ex ante equity premium for each model. Panel A presents the test statistics for Models 1, 2, and 3, Panel B presents the test statistics for Models 4, 5, and 6, Panel C presents the test statistics for Models 7, 8, and 9, and Panel D presents the test statistics for Models 10, 11, and 12. The vertical axis of each plot is on a log scale. In each panel the critical values of the test statistics corresponding to test significance at the 10%, 5%, and 1% levels are indicated by horizontal lines.

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The Equity Risk Premium: A Review of Models

Fernando Duarte
Carlo Rosa

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Abstract

We estimate the equity risk premium (ERP) by combining information from twenty models. The ERP in 2012 and 2013 reached heightened levels—of around 12 percent—not seen since the 1970s. We conclude that the high ERP was caused by unusually low Treasury yields.

Key words: equity premium, stock returns

Duarte, Rosa: Federal Reserve Bank of New York (e-mail: fernando.duarte@ny.frb.org, carlo.rosa@ny.frb.org). The authors thank Tobias Adrian and James Egelhof for helpful comments on earlier drafts. This article is an update of, and a more comprehensive and rigorous treatment than, our blog post in Liberty Street Economics (http://libertystreeteconomics.newyorkfed.org/2013/05/are-stocks-cheap-a-review-of-the-evidence.html). The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

1. Introduction

The equity risk premium —the expected return on stocks in excess of the risk-free rate— is a fundamental quantity in all of asset pricing, both for theoretical and practical reasons. It is a key measure of aggregate risk-aversion and an important determinant of the cost of capital for corporations, savings decisions of individuals and budgeting plans for governments. Recently, the equity risk premium (ERP) has also returned to the forefront as a leading indicator of the evolution of the economy, a potential explanation for jobless recoveries and a gauge of financial stability[3].

In this article, we estimate the ERP by combining information from twenty prominent models used by practitioners and featured in the academic literature. Our main finding is that the ERP has reached heightened levels. The first principal component of all models –a linear combination that explains as much of the variance of the underlying data as possible– places the one-year-ahead ERP in June 2012 at 12.2 percent, above the 10.5 percent that was reached during the financial crisis in 2009 and at levels similar to those in the mid and late 1970s. Since June 2012 and until the end of our sample in June 2013, the ERP has remained little changed, despite substantial positive realized returns. It is worth keeping in mind, however, that there is considerable uncertainty around these estimates. In fact, the issue of whether stock returns are predictable is still an active area of research.[4] Nevertheless, we find that the dispersion in estimates across models, while quite large, has been shrinking, potentially signaling increased agreement

[3] As an indicator of future activity, a high ERP at short horizons tends to be followed by higher GDP growth, higher inflation and lower unemployment. See, for example, Piazzesi and Schneider (2007), Stock and Watson (2003), and Damodaran (2012). Bloom (2009) and Duarte, Kogan and Livdan (2013) study connections between the ERP and real aggregate investment. As a potential explanation of the jobless recovery, Hall (2014) and Kuehn, Petrosky-Nadeau and Zhang (2012) propose that increased risk-aversion has prevented firms from hiring as much as would be expected in the post-crisis macroeconomic environment. Among many others, Adrian, Covitz and Liang (2013) analyze the role of equity and other asset prices in monitoring financial stability.
[4] A few important references among a vast literature are Ang and Bekaert (2007), Goyal and Welch (2008), Campbell and Thompson (2008), Kelly and Pruitt (2013), Chen, Da and Zhao (2013), Neely, Rapach, Tu and Zhou (2014).

even when the models are substantially different from each other and use more than one hundred different economic variables.

In addition to estimating the level of the ERP, we investigate the reasons behind its recent behavior. Because the ERP is the difference between expected stock returns and the risk-free rate, a high estimate can be due to expected stock returns being high or risk-free rates being low. We conclude the ERP is high because Treasury yields are unusually low. Current and expected future dividend and earnings growth play a smaller role. In fact, expected stock returns are close to their long-run mean. One implication of a bond-yield-driven ERP is that traditional indicators of the ERP like the price-dividend or price-earnings ratios, which do not use data from the term structure of risk-free rates, may not be as good a guide to future excess returns as they have been in the past.

As a second contribution, we present a concise and coherent taxonomy of ERP models. We categorize the twenty models into five groups: predictors that use historical mean returns only, dividend-discount models, cross-sectional regressions, time-series regressions and surveys. We explain the methodological and practical differences among these classes of models, including the assumptions and data sources that each require.

2. The Equity Risk Premium: Definition

Conceptually, the ERP is the compensation investors require to make them indifferent at the margin between holding the risky market portfolio and a risk-free bond. Because this compensation depends on the future performance of stocks, the ERP incorporates expectations of future stock market returns, which are not directly observable. At the end of the day, any model of the ERP is a model of investor expectations. One challenge in estimating the ERP is that it is not clear what truly constitutes the market return and the risk-free rate in the real world. In practice, the most common measures of total market returns are based on broad stock market indices, such as the S&P 500 or the Dow Jones Industrial

Average, but those indices do not include the whole universe of traded stocks and miss several other components of wealth such as housing, private equity and non-tradable human capital. Even if we restricted ourselves to all traded stocks, we still have several choices to make, such as whether to use value or equal-weighted indices, and whether to exclude penny or infrequently traded stocks. A similar problem arises with the risk-free rate. While we almost always use Treasury yields as measures of risk-free rates, they are not completely riskless since nominal Treasuries are exposed to inflation[5] and liquidity risks even if we were to assume there is no prospect of outright default. In this paper, we want to focus on how expectations are estimated in different models, and not on measurement issues regarding market returns and the risk-free rate. Thus, we follow common practice and always use the S&P 500 as a measure of stock market prices and either nominal or real Treasury yields as risk-free rates so that our models are comparable with each other and with most of the literature.

While implementing the concept of the ERP in practice has its challenges, we can precisely define the ERP mathematically. First, we decompose stock returns[6] into an expected component and a random component:

$$R_{t+k} = E_t[R_{t+k}] + error_{t+k}. \tag{1}$$

In equation (1), R_{t+k} are *realized* returns between t and $t{-}k$, and $E_t[R_{t+k}]$ are the returns that were expected from t to $t + k$ using information available at time t. The variable $error_{t+k}$ is a random variable that is unknown at time t and realized at $t + k$. Under rational expectations, $error_{t+k}$ has a mean of zero and is orthogonal to $E_t[R_{t+k}]$. We keep the discussion as general as possible and do not assume rational

[5] Note that inflation risk in an otherwise risk-free nominal asset does not invalidate its usefulness to compute the ERP. If stock returns and the risk-free rate are expressed in nominal terms, their difference has little or no inflation risk. This follows from the following formula, which holds exactly in continuous time and to a first order approximation in discrete time: real stock returns – real risk-free rate = (nominal stock returns – expected inflation) – (nominal risk-free rate – expected inflation) = nominal stock returns– nominal risk-free rate. Hence, there is no distinction between a nominal and a real ERP.
[6] Throughout this article, all returns are *net* returns. For example, a five percent return corresponds to a net return of 0.05 as opposed to a *gross* return of 1.05.