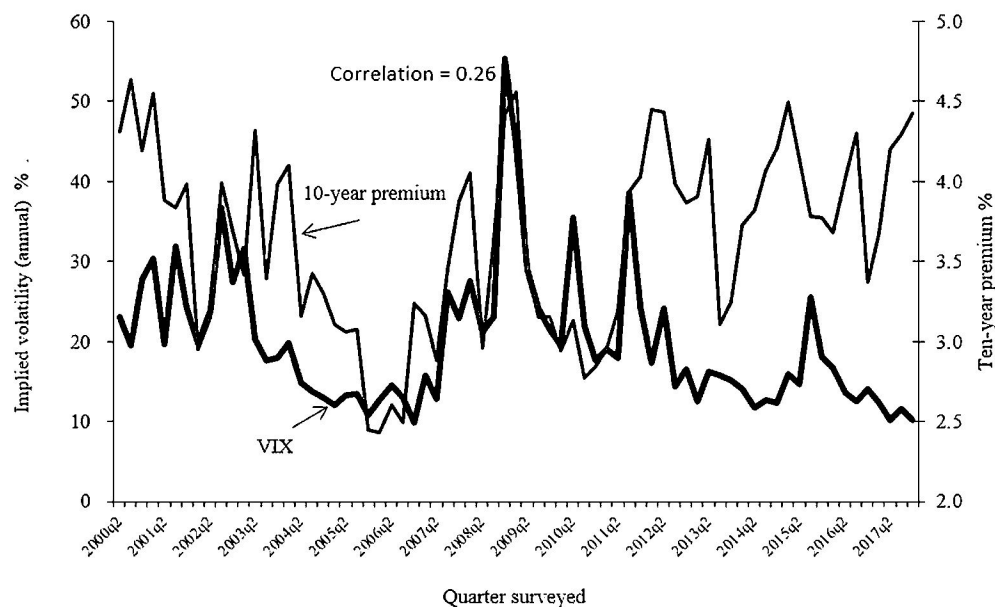


demanded by CFOs and the VIX value (see figure 19 below), though the correlation has dropped over the last decade (from 0.64 to 0.26):

Figure 19: Volatility Index (VIX) and Survey Risk Premiums



Santa-Clara and Yan (2006) use options on the S&P 500 to estimate the ex-ante risk assessed by investors from 1996 and 2002 and back out an implied equity risk premium on that basis.¹⁴⁵ To estimate the ex-ante risk, they allow for both continuous and discontinuous (or jump) risk in stocks, and use the option prices to estimate the probabilities of both types of risk. They then assume that investors share a specific utility function (power utility) and back out a risk premium that would compensate for this risk. Based on their estimates, investors should have demanded an equity risk premium of 11.8% for their perceived risk and that the perceived risk was about 70% higher than the realized risk over this period. Ross (2015) uses the implied volatilities in calls and puts on the S&P 500 to extract not only equity risk premiums but to also estimate the probabilities of catastrophic events embedded in stock prices.¹⁴⁶

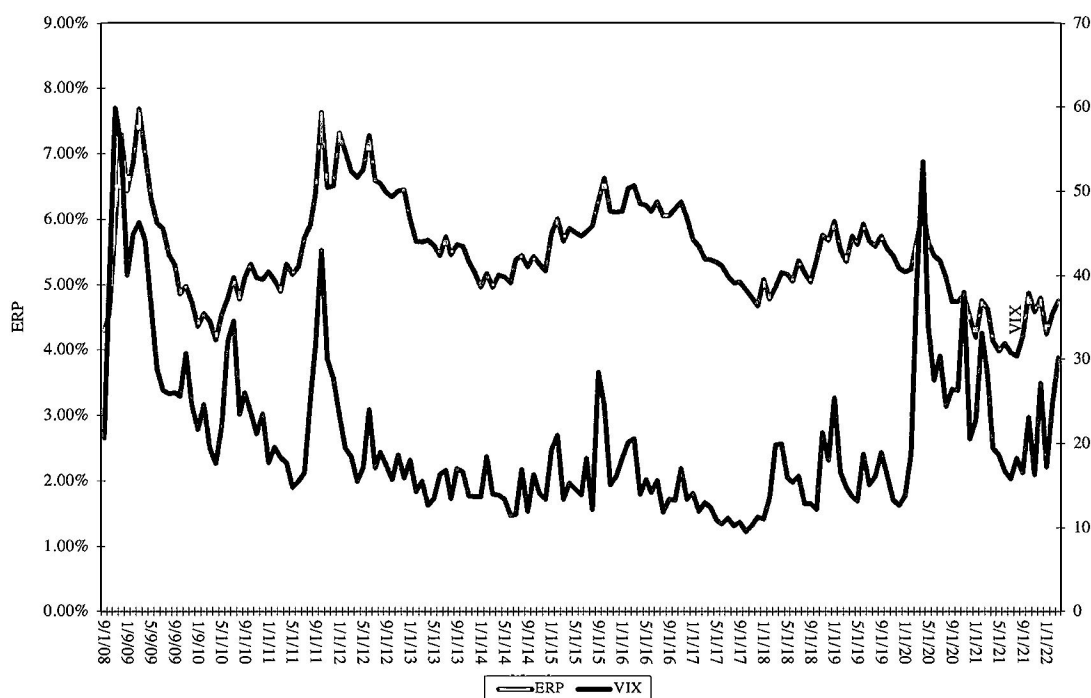
The link between equity market volatility and the equity risk premium also became clearer during the market meltdown in the last quarter of 2008. Earlier in the paper, we noted the dramatic shifts in the equity risk premiums, especially in the last year, as the

¹⁴⁵ Santa-Clara, P. and S. Yan, 2006, *Crashes, Volatility, and the Equity Premium: Lessons from S&P 500 Options*, Review of Economics and Statistics, v92, pg 435-451.

¹⁴⁶ Ross, S.M., 2015, *The Recovery Theorem*, Journal of Finance, v 70, 615-648.

financial crisis has unfolded. In Figure 20, we look at the implied equity risk premium at the start of each month from September 2008 to March 2022 and the volatility index (VIX), at the same point in time, for the S&P 500:

Figure 20: ERP versus VIX



Note that the surge in equity risk premiums between September 2008 and December 2008 coincided with a jump in the volatility index and that both numbers have declined in the years since the crisis. The drop in the VIX between September 2011 and March 2012 was not accompanied by a decrease in the implied equity risk premium, but equity risk premiums drifted down in the year after. While the VIX stayed low for much of 2014, equity risk premiums climbed through the course of the year. In the last few months of 2015, the VIX spiked again on global market crises and the equity risk premium also went up. In 2020, the VIX and the equity risk premium spiked in February and March, as markets melted down, but the spike subsided entirely in the equity risk premium and mostly in the VIX, by year end.

In a paper referenced earlier, Bollerslev, Tauchen and Zhou (2009) take a different tack and argue that it is not the implied volatility per se, but the variance risk, i.e., the difference between the implied variance (in option prices) and the actual variance, that

drives expected equity returns.¹⁴⁷ Thus, if the realized variance in a period is far higher (lower) than the implied variance, you should expect to see higher (lower) equity risk premiums demanded for subsequent periods. While they find evidence to back this proposition, they also note the relationship is strongest for short term returns (next quarter) and are weaker for longer-term returns. Bekaert and Hoerova (2013) decomposed the squared VIX into two components, a conditional variance of the stock market and an equity variance premium, and conclude that while the latter is a significant predictor of stock returns but the former is not.¹⁴⁸

Choosing an Equity Risk Premium

We have looked at three different approaches to estimating risk premiums, the survey approach, where the answer seems to depend on who you ask and what you ask them, the historical premium approach, with wildly different results depending on how you slice and dice historical data and the implied premium approach, where the final number is a function of the model you use and the assumptions you make about the future. Ultimately, though, we have to choose a number to use in analysis and that number has consequences. In this section, we consider why the approaches give you different numbers and a pathway to use to devise which number is best for you.

Why do the approaches yield different values?

The different ways of estimating equity risk premium provide cover for analysts by providing justification for almost any number they choose to use in practice. No matter what the premium used by an analyst, whether it be 3% or 12%, there is back-up evidence offered that the premium is appropriate. While this may suffice as a legal defense, it does not pass muster on common sense grounds since not all risk premiums are equally justifiable. To provide a measure of how the numbers vary, the values that we have

¹⁴⁷ Bollerslev, T. G. Tauchen and H. Zhou, 2009, *Expected Stock Returns and Variance Risk Premia*, Review of Financial Studies, v22, 4463-4492.

¹⁴⁸ Bekaert, G. and M. Hoerova, 2013, *The VIX, Variance Premium and Stock Market Volatility*, SSRN Working Paper, http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2342200.

attached to the US equity risk premium, using different approaches, in January 2022 are summarized in table 25.

Table 25: Equity Risk Premium (ERP) for the United States – January 2022

<i>Approach Used</i>	<i>ERP</i>	<i>Additional information</i>
Survey: CFOs	4.42%	Campbell and Harvey survey of CFOs (2018); Average estimate. Median was 3.63%.
Survey: Global Fund Managers	4.60%	Merrill Lynch (January 2014) survey of global managers
Historical - US	5.13%	Geometric average - Stocks minus T.Bonds: 1928-2018
Historical – Multiple Equity Markets	3.20%	Average premium across 20 markets from 1900-2017: Dimson, Marsh and Staunton (2018)
Current Implied premium	4.24%	From S&P 500 – January 1, 2022
Average Implied premium (1960-2021)	4.21%	Average of implied equity risk premium
Average Implied premium (2012-2021)	5.35%	Average of implied equity risk premium
Default spread based premium	3.62%	Baa Default Spread on 1/1/22 * Median value of (ERP/ Default Spread)

The equity risk premiums, using the different approaches, yield a range, with the lowest value being 3.20% and the highest being 5.35%. Note that the range would have been larger if we used other measures of historical risk premiums: different time periods, arithmetic instead of geometric averages.

There are several reasons why the approaches yield different answers much of time and why they converge sometimes.

1. When stock prices enter an extended phase of upward (downward) movement, the historical risk premium will climb (drop) to reflect past returns. Implied premiums will tend to move in the opposite direction, since higher (lower) stock prices

generally translate into lower (higher) premiums. In 1999, for instance, after the technology induced stock price boom of the 1990s, the implied premium was 2% but the historical risk premium was almost 6%.

2. Survey premiums reflect historical data more than expectations. When stocks are going up, investors tend to become more optimistic about future returns and survey premiums reflect this optimism. In fact, the evidence that human beings overweight recent history (when making judgments) and overreact to information can lead to survey premiums overshooting historical premiums in both good and bad times. In good times, survey premiums are even higher than historical premiums, which, in turn, are higher than implied premiums; in bad times, the reverse occurs.
3. When the fundamentals of a market change, either because the economy becomes more volatile or investors get more risk averse, historical risk premiums will not change but implied premiums will. Shocks to the market are likely to cause the two numbers to deviate. After the attack on the World Trade Center in September 2001, for instance, implied equity risk premiums jumped almost 0.50% but historical premiums were unchanged (at least until the next update).

In summary, we should not be surprised to see large differences in equity risk premiums as we move from one approach to another, and even within an approach, as we change estimation parameters.

Which approach is the “best” approach?

If the approaches yield different numbers for the equity risk premium, and we have to choose one of these numbers, how do we decide which one is the “best” estimate? The answer to this question will depend upon several factors:

- a. Predictive Power: In corporate finance and valuation, what we ultimately care about is the equity risk premium for the future. Consequently, the approach that has the best predictive power, i.e. yields forecasts of the risk premium that are closer to realized premiums, should be given more weight. So, which of the approaches does best on this count?

Campbell and Shiller (1988) suggested that the dividend yield, a simplistic measure of the implied equity risk premium, had significant predictive power for future

returns.¹⁴⁹ However, Goyal and Welch (2007) examined many of the measures suggested as predictors of the equity risk premium in the literature, including the dividend yield and the earnings to price ratio, and find them all wanting.¹⁵⁰ Using data from 1926 to 2005, they conclude that while the measures do reasonably well in sample, they perform poorly out of sample, suggesting that the relationships in the literature are either spurious or unstable. Campbell and Thompson (2008) disagree, noting that putting simple restrictions on the predictive regressions improve out of sample performance for many predictive variables.¹⁵¹ Jagannathan and Liu (2019) also dissent, noting that using a latent model for dividends not only helps forecast future dividend growth, but that the learning from dividend dynamics can help predict future stock returns.¹⁵²

To answer this question, we looked at the implied equity risk premiums from 1960 to 2021 and considered four predictors of this premium – the historical risk premium through the end of the prior year, the implied equity risk premium at the end of the prior year, the average implied equity risk premium over the previous five years and the premium implied by the Baa default spread. Since the survey data does not go back very far, we could not test the efficacy of the survey premium. Our results are summarized in table 26:

Table 26: Predictive Power of different estimates- 1960 – 2021

<i>Predictor</i>	<i>Correlation with implied premium next year</i>	<i>Correlation with actual return- next 5 years</i>	<i>Correlation with actual return – next 10 years¹⁵³</i>
Earnings Yield	0.476**	0.194	0.420**
Dividend Yield	0.203	0.217	0.360**

¹⁴⁹ Campbell, J. Y. and R. J. Shiller. 1988, *The Dividend-Price Ratio And Expectations Of Future Dividends And Discount Factors*, Review of Financial Studies, v1(3), 195-228.

¹⁵⁰ Goyal, A. and I. Welch, 2007, *A Comprehensive Look at the Empirical Performance of Equity Premium Prediction*, Review of Financial Studies, v21, 1455-1508.

¹⁵¹ Campbell, J.Y., and S.B. Thompson, 2008, *Predictive Excess Stock Returns Out of Sample: Can Anything Beat the Historical Average?* Review of Financial Studies, v21, 150-9-1531.

¹⁵² Jagannathan, R. and B. Liu, 2019, *Dividend Dynamics, Learning and Expected Stock Returns*, Journal of Finance v74, pg 401-448.

¹⁵³ I computed the compounded average return on stocks in the following five (ten) years and netted out the compounded return earned on T.Bonds over the following five (ten) years. This was a switch from the simple arithmetic average of returns over the next 10 years that I was using until last year's survey.

Current implied premium	0.763**	0.471**	0.608**
Average implied premium: Last 5 years	0.718**	0.386**	0.537**
Historical Premium	-0.497**	-0.467**	-0.597**
Default Spread based premium	0.046	0.142	0.228

** Significant at 5% level

Over this period, the implied equity risk premium at the end of the prior period was the best predictor of the implied equity risk premium in the next period, whereas historical risk premiums did worst. If we extend our analysis to make forecasts of the actual return premium earned by stocks over bonds for the next five or ten years, the current implied premium remains the best predictor, though the earnings yield does well for ten-year returns. Historical risk premiums perform even worse as forecasts of actual risk premiums over the next 5 or 10 years; in fact, they operate as good contra indicators, with a high historical risk premium forecasting lowered actual returns in the future. If predictive power were the only test, historical premiums clearly fail the test.

- b. Beliefs about markets: Implicit in the use of each approach are assumptions about market efficiency or lack thereof. If you believe that markets are efficient in the aggregate, or at least that you cannot forecast the direction of overall market movements, the current implied equity premium is the most logical choice, since it is estimated from the current level of the index. If you believe that markets, in the aggregate, can be significantly overvalued or undervalued, the historical risk premium or the average implied equity risk premium over long periods becomes a better choice. If you have absolutely no faith in markets, survey premiums will be the choice.
- c. Purpose of the analysis: Notwithstanding your beliefs about market efficiency, the task for which you are using equity risk premiums may determine the right risk premium to use. In acquisition valuations and equity research, for instance, you are asked to assess the value of an individual company and not take a view on the level of the overall

market. This will require you to use the current implied equity risk premium, since using any other number will bring your market views into the valuation. To see why, assume that the current implied premium is 4% and you decide to use a historical premium of 6% in your company valuation. Odds are that you will find the company to be over valued, but a big reason for your conclusion is that you started off with the assumption that the market itself is over valued by about 25-30%.¹⁵⁴ To make yourself market neutral, you will have to stick with the current implied premium. In corporate finance, where the equity risk premium is used to come up with a cost of capital, which in turn determines the long-term investments of the company, it may be more prudent to build in a long-term average (historical or implied) premium.

In conclusion, there is no one approach to estimating equity risk premiums that will work for all analyses. If predictive power is critical or if market neutrality is a pre-requisite, the current implied equity risk premium is the best choice. For those more skeptical about markets, the choices are broader, with the average implied equity risk premium over a long time period having the strongest predictive power. Historical risk premiums are very poor predictors of both short-term movements in implied premiums or long-term returns on stocks.

As a final note, there are papers that report consensus premiums, often estimated by averaging across approaches. I remain skeptical about these estimates, since the approaches vary not only in terms of accuracy and predictive power but also in their philosophy. Averaging a historical risk premium with an implied premium may give an analyst a false sense of security but it really makes no sense since they represent different views of the world and push in different directions.

Five myths about equity risk premiums

There are widely held misconceptions about equity risk premiums that we would like to dispel in this section.

1. Estimation services “know” the risk premium: When Ibbotson and Sinquefeld put together the first database of historical returns on stocks, bonds and bills in the 1970s,

¹⁵⁴ If the current implied premium is 4%, using a 6% premium on the market will reduce the value of the index by about 25-30%.

the data that they used was unique and not easily replicable, even for professional money managers. The niche they created, based on proprietary data, has led some to believe that Ibbotson Associates, and data services like them, have the capacity to read the historical data better than the rest of us, and therefore come up with better estimates. Now that the access to data has been democratized, and we face a much more even playing field, there is no reason to believe that any service has an advantage over any other, when it comes to historical premiums. Analysts should no longer be allowed to hide behind the defense that the equity risk premiums they use come from a reputable service and are thus beyond questioning.

2. There is no right risk premium: The flip side of the “services know it best” argument is that the data is so noisy that no one knows what the right risk premium is, and that any risk premium within a wide range is therefore defensible. As we have noted in this paper, it is indeed possible to arrive at outlandishly high or low premiums, but only if you use estimation approaches that do not hold up to scrutiny. The arithmetic average premium from 2012 to 2021 for stocks over treasury bonds is an equity risk premium estimate, but it is not a good one.
3. The equity risk premium does not change much over time: Equity risk premiums reflect both economic fundamentals and investor risk aversion and they do change over time, sometimes over very short intervals, as evidenced by what happened in the last quarter of 2008. Shocks to the system – a collapse of a large company or sovereign entity or a terrorist attack – can cause premiums to shoot up overnight. A failure to recognize this reality will lead to analyses that lag reality.
4. Using the same premium is more important than using the right premium: Within many investment banks, corporations and consulting firms, the view seems to be that getting all analysts to use the same number as the risk premium is more important than testing to see whether that number makes sense. Thus, if all equity research analysts use 5% as the equity risk premium, the argument is that they are all being consistent. There are two problems with this argument. The first is that using a premium that is too high or low will lead to systematic errors in valuation. For instance, using a 5% risk premium across the board, when the implied premium is 4%, will lead you to find that most stocks are overvalued. The second is that the impact of using too high a premium can

vary across stocks, with growth stocks being affected more negatively than mature companies. A portfolio manager who followed the recommendations of these analysts would then be over invested in mature companies and under invested in growth companies.

5. If you adjust the cash flows for risk, there is no need for a risk premium: While statement is technically correct, adjusting cash flows for risk has to go beyond reflecting the likelihood of negative scenarios in the expected cash flow. The risk adjustment to expected cash flows to make them certainty equivalent cash flows requires us to answer exactly the same questions that we deal with when adjusting discount rates for risk.

Summary

The risk premium is a fundamental and critical component in portfolio management, corporate finance and valuation. Given its importance, it is surprising that more attention has not been paid in practical terms to estimation issues. In this paper, we began by looking at the determinants of equity risk premiums including macroeconomic volatility, investor risk aversion and behavioral components. We then looked at the three basic approaches used to estimate equity risk premiums – the survey approach, where investors or managers are asked to provide estimates of the equity risk premium for the future, the historical return approach, where the premium is based upon how well equities have done in the past and the implied approach, where we use future cash flows or observed bond default spreads to estimate the current equity risk premium.

The premiums that we estimate can vary widely across approaches, and we considered two questions towards the end of the paper. The first is why the numbers vary across approaches and the second is how to choose the “right” number to use in analysis. For the latter question, we argued that the choice of a premium will depend upon the forecast period, whether you believe markets are efficient and whether you are required to be market neutral in your analysis.

Appendix 1: Historical Returns on Stocks, Bonds and Bills – United States

The historical returns on stocks include dividends each year and the historical returns on T.Bonds are computed for a constant-maturity 10-year treasury bond and include both price change and coupon each year.

	<i>Annual Return</i>			<i>Excess Return</i>		<i>Average Return</i>	
<i>Year</i>	<i>S&P 500</i>	<i>3-month T.Bill</i>	<i>10-year T. Bond</i>	<i>Stocks - Bills</i>	<i>Stocks - Bonds</i>	<i>Arithmetic Average: Stocks minus T.Bonds</i>	<i>Geometric Average: Stocks minus T. Bonds</i>
1928	43.81%	3.08%	0.84%	40.73%	42.98%	42.98%	42.98%
1929	-8.30%	3.16%	4.20%	-11.46%	-12.50%	15.24%	12.33%
1930	-25.12%	4.55%	4.54%	-29.67%	-29.66%	0.27%	-3.60%
1931	-43.84%	2.31%	-2.56%	-46.15%	-41.28%	-10.12%	-15.42%
1932	-8.64%	1.07%	8.79%	-9.71%	-17.43%	-11.58%	-15.81%
1933	49.98%	0.96%	1.86%	49.02%	48.13%	-1.63%	-7.36%
1934	-1.19%	0.32%	7.96%	-1.51%	-9.15%	-2.70%	-7.61%
1935	46.74%	0.18%	4.47%	46.57%	42.27%	2.92%	-2.49%
1936	31.94%	0.17%	5.02%	31.77%	26.93%	5.59%	0.40%
1937	-35.34%	0.30%	1.38%	-35.64%	-36.72%	1.36%	-4.22%
1938	29.28%	0.08%	4.21%	29.21%	25.07%	3.51%	-1.87%
1939	-1.10%	0.04%	4.41%	-1.14%	-5.51%	2.76%	-2.17%
1940	-10.67%	0.03%	5.40%	-10.70%	-16.08%	1.31%	-3.30%
1941	-12.77%	0.08%	-2.02%	-12.85%	-10.75%	0.45%	-3.88%
1942	19.17%	0.34%	2.29%	18.84%	16.88%	1.54%	-2.61%
1943	25.06%	0.38%	2.49%	24.68%	22.57%	2.86%	-1.18%
1944	19.03%	0.38%	2.58%	18.65%	16.45%	3.66%	-0.21%
1945	35.82%	0.38%	3.80%	35.44%	32.02%	5.23%	1.35%
1946	-8.43%	0.38%	3.13%	-8.81%	-11.56%	4.35%	0.63%
1947	5.20%	0.57%	0.92%	4.63%	4.28%	4.35%	0.81%
1948	5.70%	1.02%	1.95%	4.68%	3.75%	4.32%	0.95%
1949	18.30%	1.10%	4.66%	17.20%	13.64%	4.74%	1.49%
1950	30.81%	1.17%	0.43%	29.63%	30.38%	5.86%	2.63%
1951	23.68%	1.48%	-0.30%	22.20%	23.97%	6.61%	3.46%
1952	18.15%	1.67%	2.27%	16.48%	15.88%	6.98%	3.94%
1953	-1.21%	1.89%	4.14%	-3.10%	-5.35%	6.51%	3.57%
1954	52.56%	0.96%	3.29%	51.60%	49.27%	8.09%	4.98%
1955	32.60%	1.66%	-1.34%	30.94%	33.93%	9.01%	5.93%
1956	7.44%	2.56%	-2.26%	4.88%	9.70%	9.04%	6.07%
1957	-10.46%	3.23%	6.80%	-13.69%	-17.25%	8.16%	5.23%
1958	43.72%	1.78%	-2.10%	41.94%	45.82%	9.38%	6.39%

1959	12.06%	3.26%	-2.65%	8.80%	14.70%	9.54%	6.66%
1960	0.34%	3.05%	11.64%	-2.71%	-11.30%	8.91%	6.11%
1961	26.64%	2.27%	2.06%	24.37%	24.58%	9.37%	6.62%
1962	-8.81%	2.78%	5.69%	-11.59%	-14.51%	8.69%	5.97%
1963	22.61%	3.11%	1.68%	19.50%	20.93%	9.03%	6.36%
1964	16.42%	3.51%	3.73%	12.91%	12.69%	9.13%	6.53%
1965	12.40%	3.90%	0.72%	8.50%	11.68%	9.20%	6.66%
1966	-9.97%	4.84%	2.91%	-14.81%	-12.88%	8.63%	6.11%
1967	23.80%	4.33%	-1.58%	19.47%	25.38%	9.05%	6.57%
1968	10.81%	5.26%	3.27%	5.55%	7.54%	9.01%	6.60%
1969	-8.24%	6.56%	-5.01%	-14.80%	-3.23%	8.72%	6.33%
1970	3.56%	6.69%	16.75%	-3.12%	-13.19%	8.21%	5.90%
1971	14.22%	4.54%	9.79%	9.68%	4.43%	8.12%	5.87%
1972	18.76%	3.95%	2.82%	14.80%	15.94%	8.30%	6.08%
1973	-14.31%	6.73%	3.66%	-21.03%	-17.97%	7.73%	5.50%
1974	-25.90%	7.78%	1.99%	-33.68%	-27.89%	6.97%	4.64%
1975	37.00%	5.99%	3.61%	31.01%	33.39%	7.52%	5.17%
1976	23.83%	4.97%	15.98%	18.86%	7.85%	7.53%	5.22%
1977	-6.98%	5.13%	1.29%	-12.11%	-8.27%	7.21%	4.93%
1978	6.51%	6.93%	-0.78%	-0.42%	7.29%	7.21%	4.97%
1979	18.52%	9.94%	0.67%	8.58%	17.85%	7.42%	5.21%
1980	31.74%	11.22%	-2.99%	20.52%	34.72%	7.93%	5.73%
1981	-4.70%	14.30%	8.20%	-19.00%	-12.90%	7.55%	5.37%
1982	20.42%	11.01%	32.81%	9.41%	-12.40%	7.18%	5.10%
1983	22.34%	8.45%	3.20%	13.89%	19.14%	7.40%	5.34%
1984	6.15%	9.61%	13.73%	-3.47%	-7.59%	7.13%	5.12%
1985	31.24%	7.49%	25.71%	23.75%	5.52%	7.11%	5.13%
1986	18.49%	6.04%	24.28%	12.46%	-5.79%	6.89%	4.97%
1987	5.81%	5.72%	-4.96%	0.09%	10.77%	6.95%	5.07%
1988	16.54%	6.45%	8.22%	10.09%	8.31%	6.98%	5.12%
1989	31.48%	8.11%	17.69%	23.37%	13.78%	7.08%	5.24%
1990	-3.06%	7.55%	6.24%	-10.61%	-9.30%	6.82%	5.00%
1991	30.23%	5.61%	15.00%	24.62%	15.23%	6.96%	5.14%
1992	7.49%	3.41%	9.36%	4.09%	-1.87%	6.82%	5.03%
1993	9.97%	2.98%	14.21%	6.98%	-4.24%	6.65%	4.90%
1994	1.33%	3.99%	-8.04%	-2.66%	9.36%	6.69%	4.97%
1995	37.20%	5.52%	23.48%	31.68%	13.71%	6.80%	5.08%
1996	22.68%	5.02%	1.43%	17.66%	21.25%	7.01%	5.30%
1997	33.10%	5.05%	9.94%	28.05%	23.16%	7.24%	5.53%

1998	28.34%	4.73%	14.92%	23.61%	13.42%	7.32%	5.63%
1999	20.89%	4.51%	-8.25%	16.38%	29.14%	7.63%	5.96%
2000	-9.03%	5.76%	16.66%	-14.79%	-25.69%	7.17%	5.51%
2001	-11.85%	3.67%	5.57%	-15.52%	-17.42%	6.84%	5.17%
2002	-21.97%	1.66%	15.12%	-23.62%	-37.08%	6.25%	4.53%
2003	28.36%	1.03%	0.38%	27.33%	27.98%	6.54%	4.82%
2004	10.74%	1.23%	4.49%	9.52%	6.25%	6.53%	4.84%
2005	4.83%	3.01%	2.87%	1.82%	1.97%	6.48%	4.80%
2006	15.61%	4.68%	1.96%	10.94%	13.65%	6.57%	4.91%
2007	5.48%	4.64%	10.21%	0.84%	-4.73%	6.43%	4.79%
2008	-36.55%	1.59%	20.10%	-38.14%	-56.65%	5.65%	3.88%
2009	25.94%	0.14%	-11.12%	25.80%	37.05%	6.03%	4.29%
2010	14.82%	0.13%	8.46%	14.69%	6.36%	6.03%	4.31%
2011	2.10%	0.03%	16.04%	2.07%	-13.94%	5.80%	4.10%
2012	15.89%	0.05%	2.97%	15.84%	12.92%	5.88%	4.20%
2013	32.15%	0.07%	-9.10%	32.08%	41.25%	6.29%	4.62%
2014	13.52%	0.05%	10.75%	13.47%	2.78%	6.25%	4.60%
2015	1.36%	0.21%	1.28%	1.15%	0.08%	6.18%	4.54%
2016	11.77%	0.51%	0.69%	11.26%	11.08%	6.24%	4.62%
2017	21.61%	1.39%	2.80%	20.22%	18.80%	6.38%	4.77%
2018	-4.23%	2.37%	-0.02%	-6.17%	-4.21%	6.26%	4.66%
2019	31.22%	1.55%	9.64%	29.66%	21.58%	6.43%	4.83%
2020	18.01%	0.09%	11.33%	17.93%	6.69%	6.43%	4.84%
2021	28.47%	0.06%	-4.42%	28.41%	32.88%	6.71%	5.13%

Appendix 2: Moody's Sovereign Ratings by Country- January 2022 (FC = Foreign Currency, LC = Local Currency)

Country	FC	LC	Country	FC	LC	Country	FC	LC	Country	FC	LC
Abu Dhabi	Aa2	Aa2	Denmark	Aaa	Aaa	Latvia	A3	A3	Saudi Arabia	A1	A1
Albania	B1	B1	Dominican Republic	Ba3	Ba3	Lebanon	C	C	Senegal	Ba3	Ba3
Angola	B3	B3	Ecuador	Caa3	-	Liechtenstein	-	-	Serbia	Ba2	Ba2
Argentina	Ca	Ca	Egypt	B2	B2	Lithuania	A2	A2	Sharjah	Baa3	Baa3
Armenia	Ba3	Ba3	El Salvador	Caa1	-	Luxembourg	Aaa	Aaa	Singapore	Aaa	Aaa
Australia	Aaa	Aaa	Estonia	A1	A1	Macao SAR, China	Aa3	Aa3	Slovakia	A2	A2
Austria	Aa1	Aa1	eSwatini	B3	B3	Malaysia	A3	A3	Slovenia	A3	A3
Azerbaijan	Ba2	Ba2	Ethiopia	Caa2	Caa2	Makives	Caa1	Caa1	Solomon Isl	Caa1	Caa1
Bahamas	Ba3	Ba3	Fiji	B1	B1	Mali	Caa1	Caa1	South Africa	Ba2	Ba2
Bahamas-Offshore Banks	-	-	Finland	Aa1	Aa1	Mali	A2	A2	Spain	Baa1	Baa1
Bahrain	B2	B2	France	Aa2	Aa2	Mauritius	Baa2	Baa2	Sri Lanka	Caa2	-
Bahrain-Offshore Banks (1)	-	-	Gabon	Caa1	Caa1	Mexico	Baa1	Baa1	Sint Maarten	Ba2	Ba2
Bangladesh	Ba3	Ba3	Georgia	Ba2	Ba2	Moldova	B3	B3	St. Vincent &	B3	B3
Barbados	Caa1	Caa1	Germany	Aaa	Aaa	Mongolia	B3	B3	Suriname	Caa3	Caa3
Belarus	B3	B3	Ghana	B3	B3	Montenegro	B1	-	Sweden	Aaa	Aaa
Belgium	Aa3	Aa3	Greece	Ba3	Ba3	Morocco	Ba1	Ba1	Switzerland	Aaa	Aaa
Belize	Caa3	Caa3	Guatemala	Ba1	Ba1	Mozambique	Caa2	Caa2	Taiwan, China	Aa3	Aa3
Benin	B1	B1	Guernsey (Channel Islands)	-	-	Namibia	Ba3	Ba3	Tajikistan	B3	B3
Bermuda	A2	A2	Honduras	B1	B1	Netherlands	Aaa	Aaa	Tanzania	B2	B2
Bolivia	B2	B2	Hong Kong SAR, China	Aa3	Aa3	New Zealand	Aaa	Aaa	Thailand	Baa1	Baa1
Bosnia and Herzegovina	B3	B3	Hungary	Baa2	Baa2	Nicaragua	B3	B3	Togo	B3	B3
Botswana	A3	A3	Iceland	A2	A2	Niger	B3	B3	Trinidad and	Ba2	Ba2
Brazil	Ba2	Ba2	India	Baa3	Baa3	Nigeria	B2	B2	Tunisia	Caa1	Caa1
Bulgaria	Baa1	Baa1	Indonesia	Baa2	Baa2	Norway	Aaa	Aaa	Turkey	B2	B2
Cambodia	B2	B2	Iraq	Caa1	Caa1	Oman	Ba3	Ba3	Uganda	B2	B2
Cameroon	B2	B2	Ireland	A2	A2	Pakistan	B3	B3	Ukraine	B3	B3
Canada	Aaa	Aaa	Isle of Man	Aa3	Aa3	Panama	Baa2	-	United Arab E	Aa2	Aa2
Cayman Islands	Aa3	Aa3	Israel	A1	A1	Panama-Offshore Banks	-	-	United Kingd	Aa3	Aa3
Cayman Islands-Offshore Banks	-	-	Italy	Baa3	Baa3	Papua New Guinea	B2	B2	United States	Aaa	Aaa
Chile	A1	A1	Jamaica	B2	B2	Paraguay	Ba1	Ba1	Uzbekistan	B1	B1
China	A1	A1	Japan	A1	A1	Peru	Baa1	Baa1	Uruguay	Baa2	Baa2
Colombia	Baa2	Baa2	Jersey (Channel Islands)	-	-	Philippines	Baa2	Baa2	Venezuela	C	C
Costa Rica	B2	B2	Jordan	B1	B1	Poland	A2	A2	Vietnam	Ba3	Ba3
Cote d'Ivoire	Ba3	Ba3	Kazakhstan	Baa2	Baa2	Portugal	Baa2	Baa2	Zambia	Ca	Ca
Croatia	Ba1	Ba1	Kenya	B2	B2	Qatar	Aa3	Aa3			
Cuba	Ca	Ca	Korea	Aa2	Aa2	Republic of the Congo	Caa2	Caa2			
Cyprus	Ba1	Ba1	Kuwait	A1	A1	Romania	Baa3	Baa3			
Czech Republic	Aa3	Aa3	Kyrgyz Republic	B2	B2	Russia	Baa3	Baa3			
Democratic Republic of the Congo	Caa1	Caa1	Laos	Caa2	Caa2	Rwanda	B2	B2			

Appendix 3: Country Risk Scores from the PRS Group – January 2022

Political Risk Services (PRS) is a risk estimation service that estimates country risk on multiple dimensions. The risk scores reported in this table are composite risk scores for each country, with lower numbers indicating higher risk.

<i>Country</i>	<i>PRS Score</i>	<i>Country</i>	<i>PRS Score</i>	<i>Country</i>	<i>PRS Score</i>	<i>Country</i>	<i>PRS Score</i>
Albania	69.3	El Salvador	68.3	Liberia	59.0	Serbia	69.5
Algeria	62.3	Estonia	73.3	Libya	66.3	Sierra Leone	57.0
Angola	62.8	Ethiopia	56.8	Lithuania	74.5	Singapore	85.5
Argentina	67.0	Finland	81.8	Luxembourg	86.3	Slovakia	73.3
Armenia	64.8	France	74.0	Madagascar	63.5	Slovenia	72.0
Australia	80.0	Gabon	65.3	Malawi	59.8	Somalia	51.5
Austria	78.0	Gambia	65.8	Malaysia	74.0	South Africa	69.5
Azerbaijan	72.0	Germany	82.5	Mali	59.3	Spain	73.0
Bahamas	72.0	Ghana	67.0	Malta	76.3	Sri Lanka	61.0
Bahrain	68.3	Greece	69.3	Mexico	70.0	Sudan	36.3
Bangladesh	66.8	Guatemala	71.5	Moldova	68.3	Suriname	55.0
Belarus	65.0	Guinea	57.5	Mongolia	66.3	Sweden	83.5
Belgium	77.5	Guinea-Bissau	62.8	Morocco	68.3	Switzerland	86.8
Bolivia	66.5	Guyana	66.3	Mozambique	50.5	Syria	45.5
Botswana	76.5	Haiti	56.3	Myanmar	53.0	Taiwan	86.0
Brazil	69.5	Honduras	68.0	Namibia	72.0	Tanzania	64.8
Brunei	79.0	Hong Kong	76.8	Netherlands	80.3	Thailand	65.5
Bulgaria	74.8	Hungary	74.0	New Zealand	79.5	Togo	63.8
Burkina Faso	61.3	Iceland	81.0	Nicaragua	66.8	Trinidad & Tobago	76.5
Cameroon	61.3	India	72.5	Niger	58.0	Tunisia	62.5
Canada	81.3	Indonesia	69.3	Nigeria	60.5	Turkey	58.3
Chile	73.8	Iran	63.8	Norway	88.0	Uganda	59.5
China, Peoples' Rep.	71.8	Iraq	65.3	Oman	71.8	Ukraine	66.5
Colombia	64.5	Ireland	82.3	Pakistan	57.3	United Arab Emirates	78.5
Congo, Dem. Republic	57.5	Israel	75.8	Panama	74.0	United Kingdom	76.0
Congo, Republic	66.5	Italy	76.5	Papua New Guinea	67.5	United States	72.3
Costa Rica	73.0	Jamaica	73.8	Paraguay	68.8	Uruguay	74.5
Cote d'Ivoire	60.5	Japan	80.3	Peru	70.8	Uzbekistan	75.5
Croatia	75.5	Jordan	64.8	Philippines	69.8	Venezuela	41.8
Cuba	55.3	Kazakhstan	72.8	Poland	74.5	Vietnam	71.0
Cyprus	71.3	Kenya	63.0	Portugal	77.3	Yemen, Republic	52.8
Czech Republic	77.8	Korea, D.P.R.	51.5	Qatar	78.0	Zambia	63.0
Denmark	85.5	Korea, Republic	80.8	Romania	70.5	Zimbabwe	61.0
Dominican Republic	73.0	Kuwait	72.8	Russia	73.5		
Ecuador	68.5	Latvia	73.5	Saudi Arabia	79.3		
Egypt	64.5	Lebanon	51.5	Senegal	61.8		

**Appendix 4: Equity Market volatility, relative to S&P 500: Total Equity Risk Premiums
and Country Risk Premiums (Daily returns from 1/21 – 1/22)**

The standard deviation in stocks is computed using the primary index for each country, using 260 days of returns. The ERP for the US is 4.24%.

<i>Country</i>	<i>Std deviation in Equities</i>	<i>Relative Equity Volatility</i>	ERP	CRP
Argentina	31.72%	2.41	10.20%	5.96%
Bahrain	8.01%	0.61	2.58%	-1.66%
Bangladesh	15.04%	1.14	4.84%	0.60%
Bosnia	25.03%	1.90	8.05%	3.81%
Botswana	2.43%	0.18	0.78%	-3.46%
Brazil	19.95%	1.51	6.42%	2.18%
Bulgaria	15.12%	1.15	4.86%	0.62%
Chile	24.79%	1.88	7.97%	3.73%
China	19.74%	1.50	6.35%	2.11%
Colombia	17.88%	1.36	5.75%	1.51%
Costa Rica	5.65%	0.43	1.82%	-2.42%
Croatia	12.28%	0.93	3.95%	-0.29%
Cyprus	14.43%	1.09	4.64%	0.40%
Czech Republic	15.61%	1.18	5.02%	0.78%
Egypt	15.53%	1.18	5.00%	0.76%
Estonia	20.79%	1.58	6.69%	2.45%
Greece	20.44%	1.55	6.58%	2.34%
Hungary	25.51%	1.94	8.21%	3.97%
India	16.67%	1.26	5.36%	1.12%
Indonesia	12.56%	0.95	4.04%	-0.20%
Israel	13.95%	1.06	4.49%	0.25%
Italy	21.10%	1.60	6.79%	2.55%
Jamaica	15.31%	1.16	4.93%	0.69%
Jordan	10.03%	0.76	3.23%	-1.01%
Kazakhstan	14.79%	1.12	4.76%	0.52%
Kenya	15.26%	1.16	4.91%	0.67%
Kuwait	8.87%	0.67	2.85%	-1.39%
Laos	18.16%	1.38	5.84%	1.60%
Latvia	18.18%	1.38	5.85%	1.61%
Lebanon	22.59%	1.71	7.27%	3.03%

Lithuania	14.15%	1.07	4.55%	0.31%
Macedonia	13.98%	1.06	4.50%	0.26%
Malaysia	11.08%	0.84	3.56%	-0.68%
Malta	11.85%	0.90	3.81%	-0.43%
Mauritius	9.03%	0.69	2.90%	-1.34%
Mexico	14.24%	1.08	4.58%	0.34%
Mongolia	25.53%	1.94	8.21%	3.97%
Morocco	9.47%	0.72	3.05%	-1.19%
Namibia	23.11%	1.75	7.43%	3.19%
Nigeria	9.66%	0.73	3.11%	-1.13%
Oman	7.59%	0.58	2.44%	-1.80%
Pakistan	15.38%	1.17	4.95%	0.71%
Palestine	7.57%	0.57	2.44%	-1.80%
Panama	4.47%	0.34	1.44%	-2.80%
Peru	25.53%	1.94	8.21%	3.97%
Philippines	19.80%	1.50	6.37%	2.13%
Qatar	9.71%	0.74	3.12%	-1.12%
Romania	17.08%	1.30	5.49%	1.25%
Russia	32.39%	2.46	10.42%	6.18%
Saudi Arabia	12.43%	0.94	4.00%	-0.24%
Serbia	9.68%	0.73	3.11%	-1.13%
Singapore	12.13%	0.92	3.90%	-0.34%
Slovakia	12.08%	0.92	3.89%	-0.35%
Slovenia	16.15%	1.23	5.20%	0.96%
South Africa	18.53%	1.41	5.96%	1.72%
Sri Lanka	24.22%	1.84	7.79%	3.55%
Taiwan	17.14%	1.30	5.51%	1.27%
Tanzania	12.84%	0.97	4.13%	-0.11%
Thailand	12.00%	0.91	3.86%	-0.38%
Tunisia	5.51%	0.42	1.77%	-2.47%
Turkey	29.06%	2.20	9.35%	5.11%
UAE	14.57%	1.11	4.69%	0.45%
Ukraine	35.21%	2.67	11.33%	7.09%
US	13.18%	1.00	4.24%	0.00%
Venezuela	41.63%	3.16	13.39%	9.15%

Appendix 5: Equity Volatility versus Bond/CDS volatility- January 2022

Standard deviation in equity index (σ_{Equity}) and government bond price (σ_{Bond}) was computed, using the last 260 trading days, where available. To compute the σ_{CDS} , we first computed the standard deviation of the CDS in basis points over the the last 260 trading days and then divided by the level of the CDS to get a coefficient of variation.

Country	Std deviation in Equities	σ_{Bond}	$\sigma_{\text{Equity}} / \sigma_{\text{Bond}}$	$\sigma (\text{CDS})$	CDS	CV(CDS)	$\sigma_{\text{Equity}} / \sigma_{\text{CDS}}$
Algeria	NA	NA	NA	0.23%	1.10%	20.91%	NA
Angola	NA	NA	NA	0.53%	5.94%	8.92%	NA
Argentina	31.72%	NA	NA	0.70%	23.32%	3.00%	10.57
Bahrain	8.01%	NA	NA	0.21%	3.40%	6.18%	1.30
Bangladesh	15.04%	NA	NA	NA	NA	NA	NA
Bosnia	25.03%	NA	NA	NA	NA	NA	NA
Botswana	2.43%	NA	NA	Na	NA	NA	NA
Brazil	19.95%	14.29%	1.40	0.23%	2.91%	7.90%	2.52
Bulgaria	15.12%	7.12%	2.12	0.36%	0.81%	44.44%	0.34
Chile	24.79%	24.46%	1.01	0.26%	1.25%	20.80%	1.19
China	19.74%	9.77%	2.02	0.20%	0.74%	27.03%	0.73
Colombia	17.88%	10.06%	1.78	0.24%	2.77%	8.66%	2.06
Costa Rica	5.65%	NA	NA	0.72%	3.92%	18.37%	0.31
Croatia	12.28%	11.33	0.01	0.38%	1.11%	34.23%	0.36
Cyprus	14.43%	5.64%	2.56	0.21%	0.74%	28.38%	0.51
Czech Republic	15.61%	8.64%	1.81	0.19%	0.47%	40.43%	0.39
Egypt	15.53%	NA	NA	0.67%	5.74%	11.67%	1.33
El Salvador	NA	NA	NA	0.45%	18.33%	2.45%	NA
Estonia	20.79%	NA	NA	0.37%	0.85%	43.53%	0.48
Ghana	NA	NA	NA	0.82%	12.54%	6.54%	NA
Greece	20.44%	11.81%	1.73	0.35%	1.69%	20.71%	0.99
Guatemala	NA	NA	NA	0.46%	2.30%	20.00%	NA
Hungary	25.51%	15.28%	1.67	0.15%	0.69%	21.74%	1.17
India	16.67%	10.51%	1.59	0.23%	1.44%	15.97%	1.04
Indonesia	12.56%	7.19%	1.75	0.43%	1.36%	31.62%	0.40
Iraq	NA	NA	NA	0.43%	5.63%	7.64%	NA

Israel	13.95%	5.86%	2.38	0.39%	0.72%	54.17%	0.26
Italy	21.10%	NA	NA	0.24%	1.41%	17.02%	1.24
Jamaica	15.31%	NA	NA	NA	NA	NA	NA
Jordan	10.03%	NA	NA	NA	NA	NA	NA
Kazakhstan	14.79%	NA	NA	0.49%	2.46%	19.92%	0.74
Kenya	15.26%	NA	NA	0.19%	4.44%	4.28%	3.57
Kuwait	8.87%	NA	NA	0.21%	0.86%	24.42%	0.36
Laos	18.16%	NA	NA	NA	NA	NA	NA
Latvia	18.18%	4.38%	4.15	0.25%	0.74%	33.78%	0.54
Lebanon	22.59%	NA	NA	NA	NA	NA	NA
Lithuania	14.15%	4.18%	3.39	0.19%	0.79%	24.05%	0.59
Macedonia	13.98%	NA	NA	NA	NA	NA	NA
Malaysia	11.08%	6.94%	1.60	0.28%	0.81%	34.57%	0.32
Malta	11.85%	NA	NA	NA	NA	NA	NA
Mauritius	9.03%	NA	NA	NA	NA	NA	NA
Mexico	14.24%	12.65%	1.13	0.29%	1.58%	18.35%	0.78
Mongolia	25.53%	NA	NA	0.78%	4.37%	17.85%	1.43
Morocco	9.47%	NA	NA	0.21%	1.32%	15.91%	0.60
Namibia	23.11%	NA	NA	0.38%	2.80%	13.57%	1.70
Nigeria	9.66%	18.35%	0.53	0.41%	5.53%	7.41%	1.30
Oman	7.59%	NA	NA	0.43%	3.19%	13.48%	0.56
Pakistan	15.38%	8.55%	1.80	0.48%	3.67%	13.08%	1.18
Palestine	7.57%	NA	NA	NA	NA	NA	NA
Panama	4.47%	NA	NA	0.24%	1.26%	19.05%	0.23
Peru	25.53%	15.60%	1.64	0.21%	1.31%	16.03%	1.59
Philippines	19.80%	7.14%	2.77	0.35%	0.92%	38.04%	0.52
Qatar	9.71%	NA	NA	0.33%	0.74%	44.59%	0.22
Romania	17.08%	8.65%	1.97	0.30%	1.24%	24.19%	0.71
Russia	32.39%	18.35%	1.77	0.35%	1.70%	20.59%	1.57
Rwanda	NA	NA	NA	0.25%	3.36%	7.44%	NA
Saudi Arabia	12.43%	5.52%	2.25	0.35%	0.88%	39.77%	0.31
Senegal	NA	NA	NA	0.35%	2.66%	13.16%	NA
Serbia	9.68%	NA	NA	0.22%	1.37%	16.06%	0.60

Singapore	12.13%	6.73%	1.80	NA	NA	NA	NA
Slovakia	12.08%	6.35	0.02	0.15%	0.63%	23.81%	0.51
Slovenia	16.15%	9.41%	1.72	0.25%	0.87%	28.74%	0.56
South Africa	18.53%	15.16%	1.22	0.35%	2.85%	12.28%	1.51
Sri Lanka	24.22%	NA	NA	0.37%	19.69%	1.88%	12.89
Taiwan	17.14%	NA	NA	NA	NA	NA	NA
Tanzania	12.84%	NA	NA	NA	NA	NA	NA
Thailand	12.00%	3.78%	3.17	0.25%	0.52%	48.08%	0.25
Tunisia	5.51%	NA	NA	0.45%	8.82%	5.10%	1.08
Turkey	29.06%	18.34%	1.58	0.45%	5.51%	8.17%	3.56
UAE	14.57%	NA	NA	NA	NA	NA	NA
Ukraine	35.21%	25.90%	1.36	0.50%	6.17%	8.10%	4.34
Uruguay	NA	NA	NA	0.33%	1.46%	22.60%	NA
Venezuela	41.63%	49.70%	0.84	NA	NA	NA	NA
Vietnam	25.33%	NA	NA	0.32%	1.56%	20.51%	1.23
Zambia	NA	NA	NA	NA	NA	NA	NA
Average			1.77				1.46
Median			1.74				0.73

Appendix 6: Year-end Implied Equity Risk Premiums: 1961-2021

These estimates of equity risk premium for the S&P 500 are forward looking and are computed based on the index level at the end of each year and the expected cash flows on the index for the future. The cash flows are computed as dividends plus stock buybacks in each year.

<i>Year</i>	<i>S&P 500</i>	<i>Earnings*</i>	<i>Dividends*</i>	<i>T.Bond Rate</i>	<i>Estimated Growth</i>	<i>Implied ERP</i>
1961	71.55	3.37	2.04	2.35%	2.41%	2.92%
1962	63.1	3.67	2.15	3.85%	4.05%	3.56%
1963	75.02	4.13	2.35	4.14%	4.96%	3.38%
1964	84.75	4.76	2.58	4.21%	5.13%	3.31%
1965	92.43	5.30	2.83	4.65%	5.46%	3.32%
1966	80.33	5.41	2.88	4.64%	4.19%	3.68%
1967	96.47	5.46	2.98	5.70%	5.25%	3.20%
1968	103.86	5.72	3.04	6.16%	5.32%	3.00%
1969	92.06	6.10	3.24	7.88%	7.55%	3.74%
1970	92.15	5.51	3.19	6.50%	4.78%	3.41%
1971	102.09	5.57	3.16	5.89%	4.57%	3.09%
1972	118.05	6.17	3.19	6.41%	5.21%	2.72%
1973	97.55	7.96	3.61	6.90%	8.30%	4.30%
1974	68.56	9.35	3.72	7.40%	6.42%	5.59%
1975	90.19	7.71	3.73	7.76%	5.99%	4.13%
1976	107.46	9.75	4.22	6.81%	8.19%	4.55%
1977	95.1	10.87	4.86	7.78%	9.52%	5.92%
1978	96.11	11.64	5.18	9.15%	8.48%	5.72%
1979	107.94	14.55	5.97	10.33%	11.70%	6.45%
1980	135.76	14.99	6.44	12.43%	11.01%	5.03%
1981	122.55	15.18	6.83	13.98%	11.42%	5.73%
1982	140.64	13.82	6.93	10.47%	7.96%	4.90%
1983	164.93	13.29	7.12	11.80%	9.09%	4.31%
1984	167.24	16.84	7.83	11.51%	11.02%	5.11%
1985	211.28	15.68	8.20	8.99%	7.89%	3.84%
1986	242.17	14.43	8.19	7.22%	5.54%	3.58%
1987	247.08	16.04	9.17	8.86%	9.66%	3.99%
1988	277.72	24.12	10.22	9.14%	9.76%	3.77%
1989	353.4	24.32	11.73	7.93%	9.58%	3.51%
1990	330.22	22.65	12.35	8.07%	7.39%	3.89%
1991	417.09	19.30	12.97	6.70%	6.34%	3.48%
1992	435.71	20.87	12.64	6.68%	4.67%	3.55%

1993	466.45	26.90	12.69	5.79%	4.73%	3.17%
1994	459.27	31.75	13.36	7.82%	7.23%	3.55%
1995	615.93	37.70	14.17	5.57%	5.65%	3.29%
1996	740.74	40.63	14.89	6.41%	6.13%	3.20%
1997	970.43	44.09	15.52	5.74%	5.45%	2.73%
1998	1229.23	44.27	16.20	4.65%	4.60%	2.26%
1999	1469.25	51.68	16.71	6.44%	5.75%	2.05%
2000	1320.28	56.13	16.27	5.11%	3.71%	2.87%
2001	1148.09	38.85	15.74	5.05%	3.56%	3.62%
2002	879.82	46.04	16.08	3.81%	3.57%	4.10%
2003	1111.91	54.69	17.88	4.25%	5.35%	3.69%
2004	1211.92	67.68	19.407	4.22%	4.90%	3.65%
2005	1248.29	76.45	22.38	4.39%	6.16%	4.08%
2006	1418.3	87.72	25.05	4.70%	5.93%	4.16%
2007	1468.36	82.54	27.73	4.02%	5.03%	4.37%
2008	903.25	65.39	28.05	2.21%	2.11%	6.43%
2009	1115.10	59.65	22.31	3.84%	0.28%	4.36%
2010	1257.64	83.66	23.12	3.29%	3.33%	5.20%
2011	1257.60	97.05	26.02	1.88%	2.75%	6.01%
2012	1426.19	102.47	30.44	1.76%	2.93%	5.78%
2013	1848.36	107.45	36.28	3.04%	5.01%	4.96%
2014	2058.90	113.01	39.44	2.17%	2.77%	5.78%
2015	2043.94	106.32	43.16	2.27%	2.96%	6.12%
2016	2238.83	108.86	45.03	2.45%	2.64%	5.69%
2017	2673.61	124.94	49.73	2.41%	3.22%	5.08%
2018	2506.85	148.34	53.61	2.68%	3.24%	5.96%
2019	3230.78	162.35	58.80	1.92%	2.57%	5.20%
2020	3756.07	139.76	56.70	0.93%	0.74%	4.72%
2021	4766.18	206.38	59.20	1.51%	1.71%	4.24%

^a The earnings and dividend numbers for the S&P 500 represent the estimates that would have been available at the start of each of the years and thus may not match up to the actual numbers for the year. For instance, in January 2022, the estimated earnings for the S&P 500 index included actual earnings for three quarters of 2021 and the estimated earnings for the last quarter of 2020. The actual earnings for the last quarter would not have been available until April 2022.

Implied Equity Premiums

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- Let's start with a general proposition. If you know the price paid for an asset and have estimates of the expected cash flows on the asset, you can estimate the IRR of these cash flows. If you paid the price, this is what you have priced the asset to earn (as an expected return).
- If you assume that stocks are correctly priced in the aggregate and you can estimate the expected cashflows from buying stocks, you can estimate the expected rate of return on stocks by finding that discount rate that makes the present value equal to the price paid. Subtracting out the riskfree rate should yield an implied equity risk premium.
- This implied equity premium is a forward looking number and can be updated as often as you want (every minute of every day, if you are so inclined).

Implied Equity Premiums: January 2008

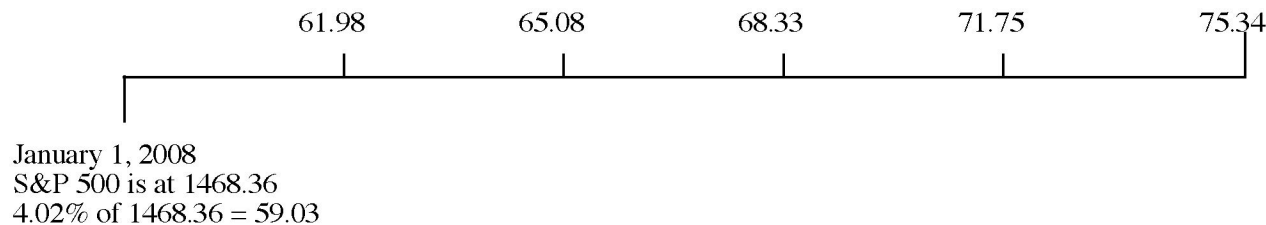
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- We can use the information in stock prices to back out how risk averse the market is and how much of a risk premium it is demanding.

Between 2001 and 2007 dividends and stock buybacks averaged 4.02% of the index each year.

Analysts expect earnings to grow 5% a year for the next 5 years. We will assume that dividends & buybacks will keep pace..
Last year's cashflow (59.03) growing at 5% a year

After year 5, we will assume that earnings on the index will grow at 4.02%, the same rate as the entire economy (= riskfree rate).



- If you pay the current level of the index, you can expect to make a return of 8.39% on stocks (which is obtained by solving for r in the following equation)

$$1468.36 = \frac{61.98}{(1+r)} + \frac{65.08}{(1+r)^2} + \frac{68.33}{(1+r)^3} + \frac{71.75}{(1+r)^4} + \frac{75.34}{(1+r)^5} + \frac{75.35(1.0402)}{(r - .0402)(1+r)^5}$$

- Implied Equity risk premium = Expected return on stocks - Treasury bond rate = 8.39% - 4.02% = 4.37%

Implied Risk Premium Dynamics

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- Assume that the index jumps 10% on January 2 and that nothing else changes. What will happen to the implied equity risk premium?
 - a. Implied equity risk premium will increase
 - b. Implied equity risk premium will decrease
- Assume that the earnings jump 10% on January 2 and that nothing else changes. What will happen to the implied equity risk premium?
 - a. Implied equity risk premium will increase
 - b. Implied equity risk premium will decrease
- Assume that the riskfree rate increases to 5% on January 2 and that nothing else changes. What will happen to the implied equity risk premium?
 - a. Implied equity risk premium will increase
 - b. Implied equity risk premium will decrease

A year that made a difference.. The implied premium in January 2009

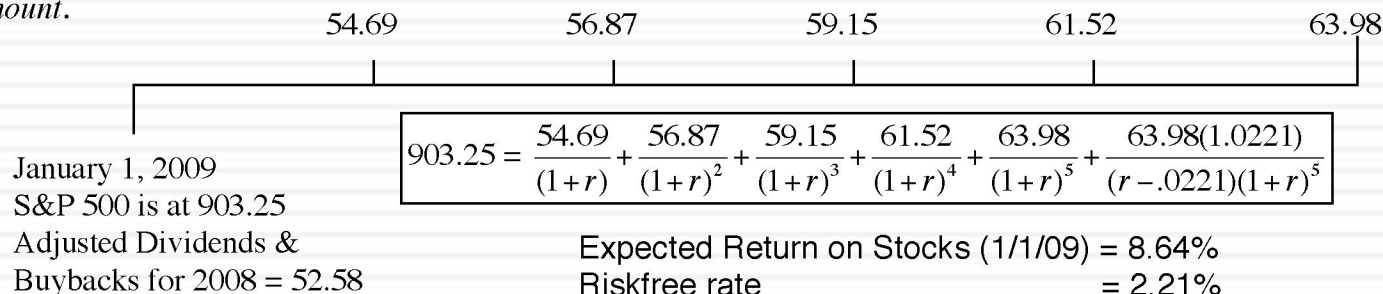
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Year	Market value of index	Dividends	Buybacks	Cash to equity	Dividend yield	Buyback yield	Total yield
2001	1148.09	15.74	14.34	30.08	1.37%	1.25%	2.62%
2002	879.82	15.96	13.87	29.83	1.81%	1.58%	3.39%
2003	1111.91	17.88	13.70	31.58	1.61%	1.23%	2.84%
2004	1211.92	19.01	21.59	40.60	1.57%	1.78%	3.35%
2005	1248.29	22.34	38.82	61.17	1.79%	3.11%	4.90%
2006	1418.30	25.04	48.12	73.16	1.77%	3.39%	5.16%
2007	1468.36	28.14	67.22	95.36	1.92%	4.58%	6.49%
2008	903.25	28.47	40.25	68.72	3.15%	4.61%	7.77%
Normalized	903.25	28.47	24.11	52.584	3.15%	2.67%	5.82%

In 2008, the actual cash returned to stockholders was 68.72. However, there was a 41% dropoff in buybacks in Q4. We reduced the total buybacks for the year by that amount.

Analysts expect earnings to grow 4% a year for the next 5 years. We will assume that dividends & buybacks will keep pace..
Last year's cashflow (52.58) growing at 4% a year

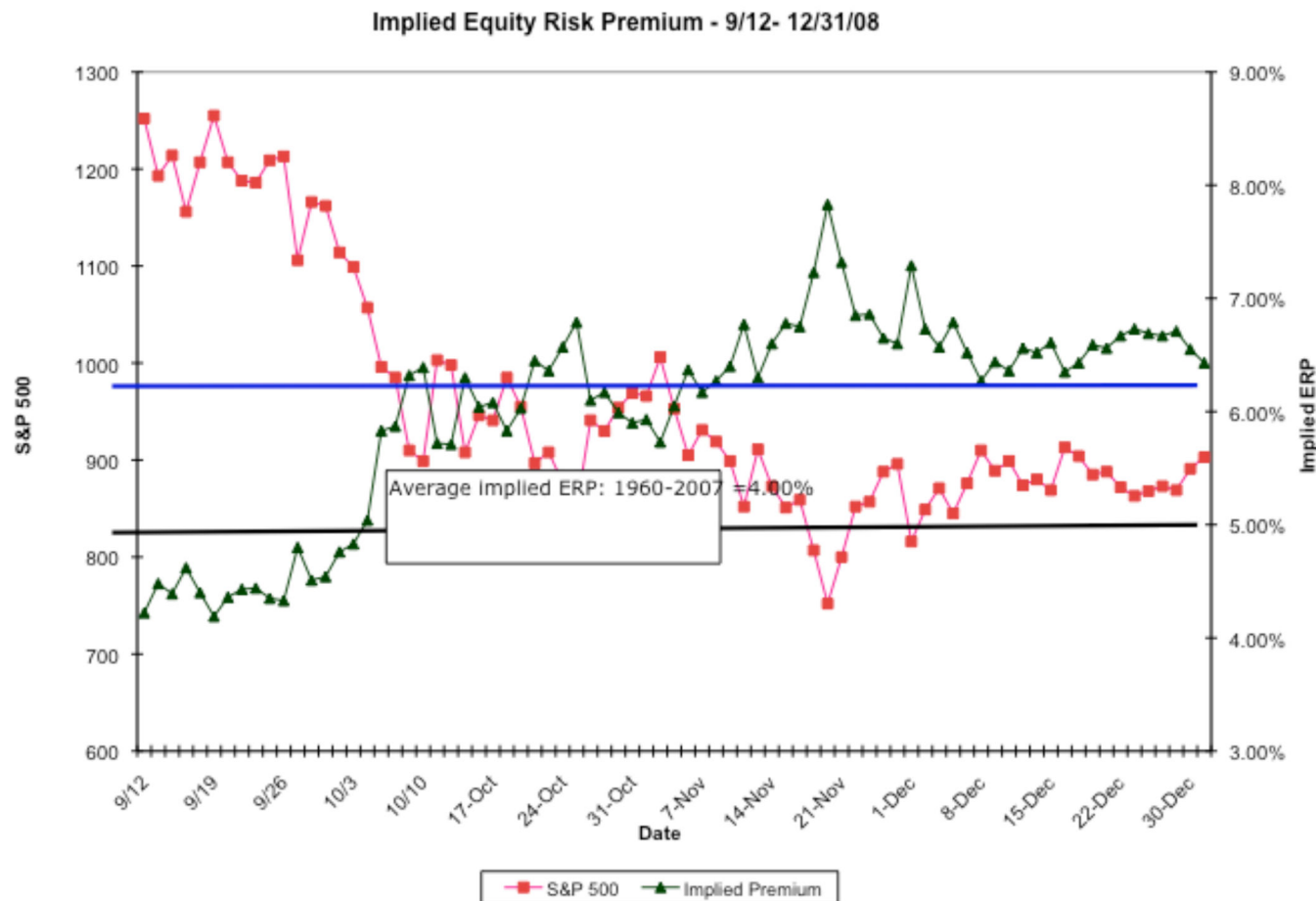
After year 5, we will assume that earnings on the index will grow at 2.21%, the same rate as the entire economy (= riskfree rate).



Aswath Damodaran

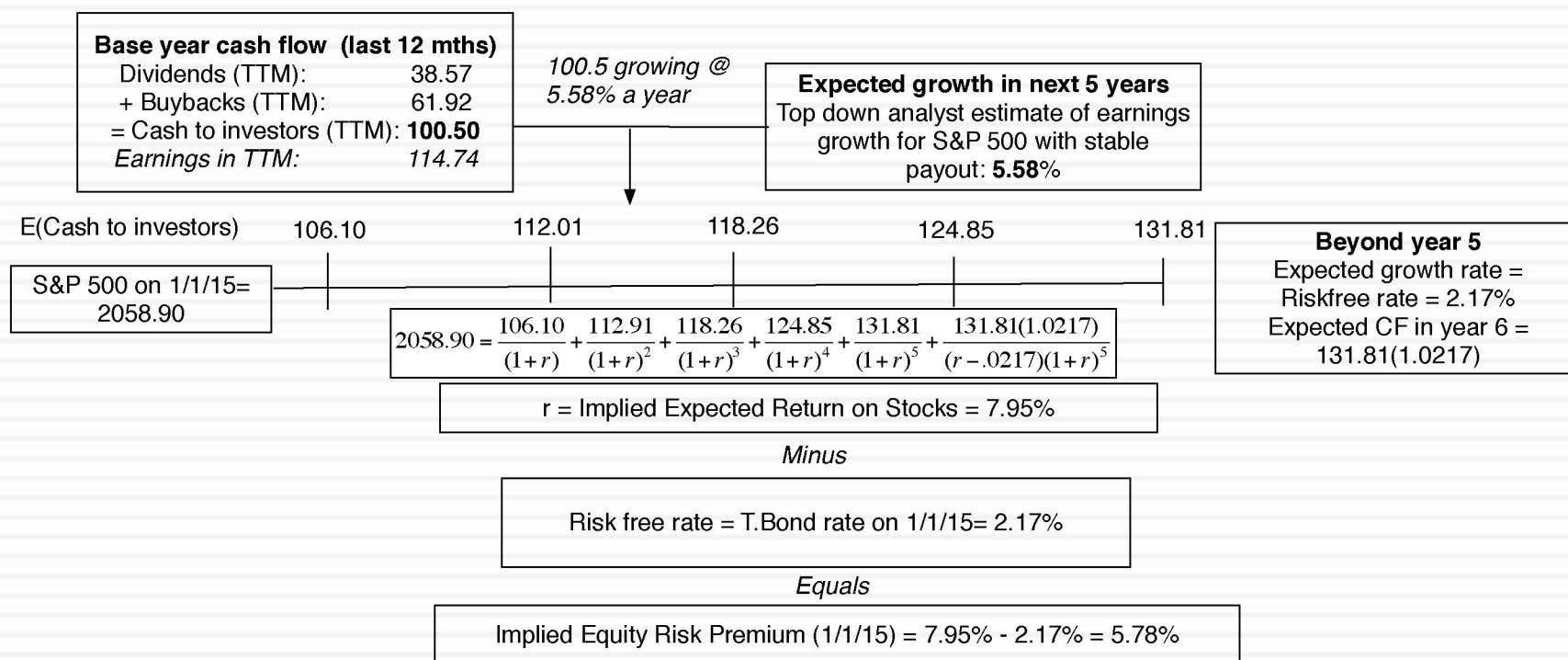
The Anatomy of a Crisis: Implied ERP from September 12, 2008 to January 1, 2009

64



An Updated Equity Risk Premium: January 2015

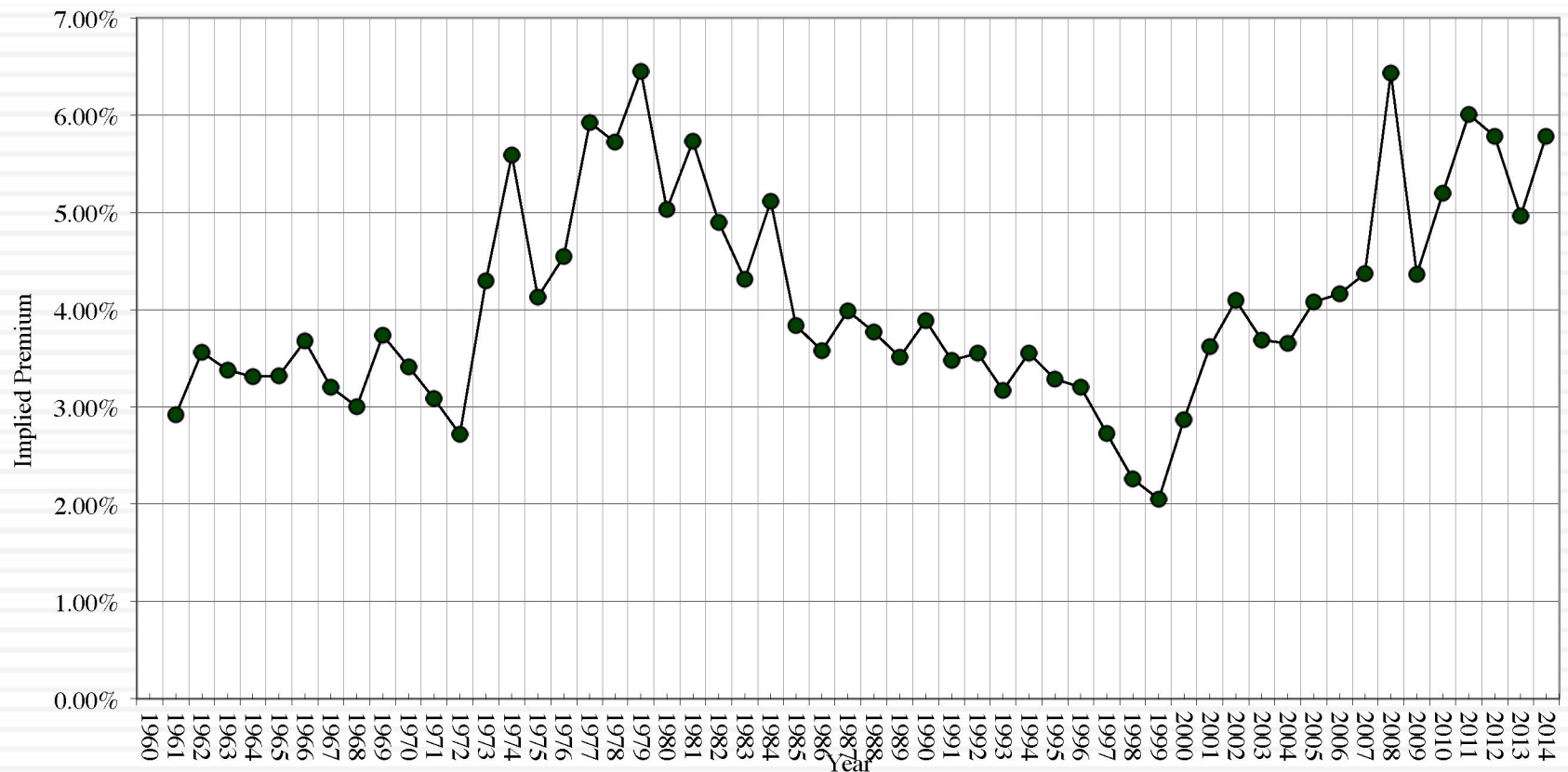
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Implied Premiums in the US: 1960-2014

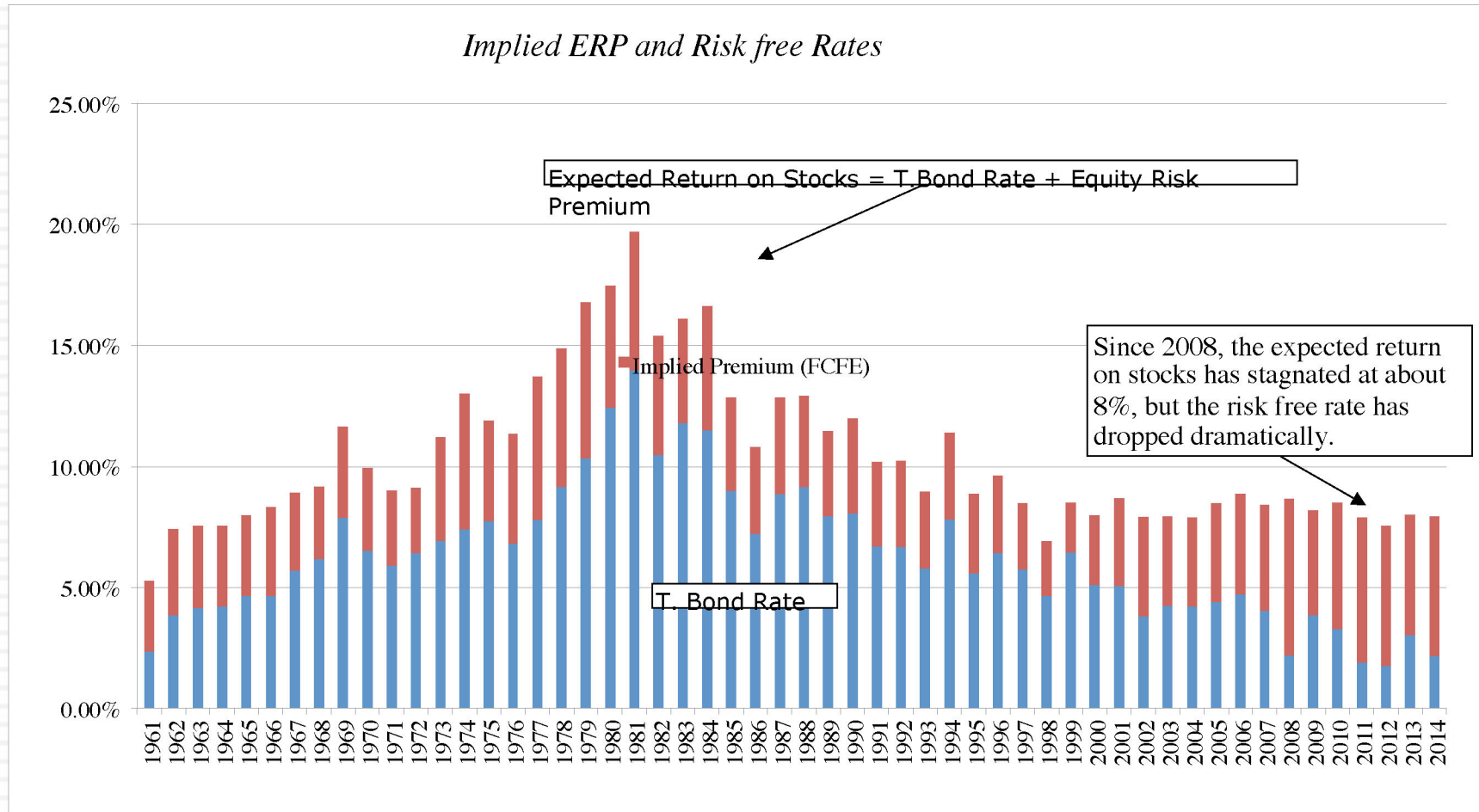
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Implied Premium for US Equity Market: 1960-2014



Implied Premium versus Risk Free Rate

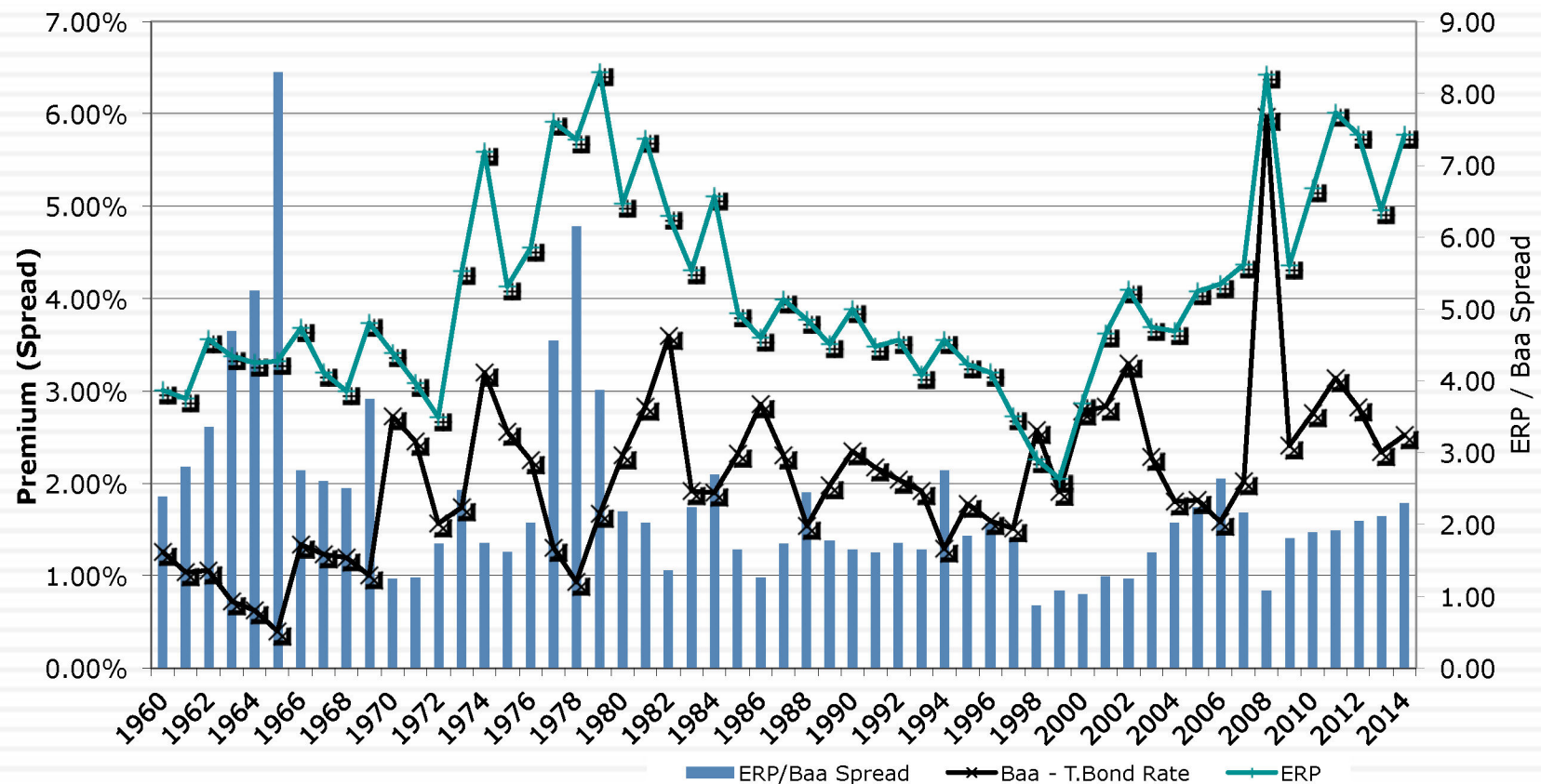
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Equity Risk Premiums and Bond Default Spreads

68

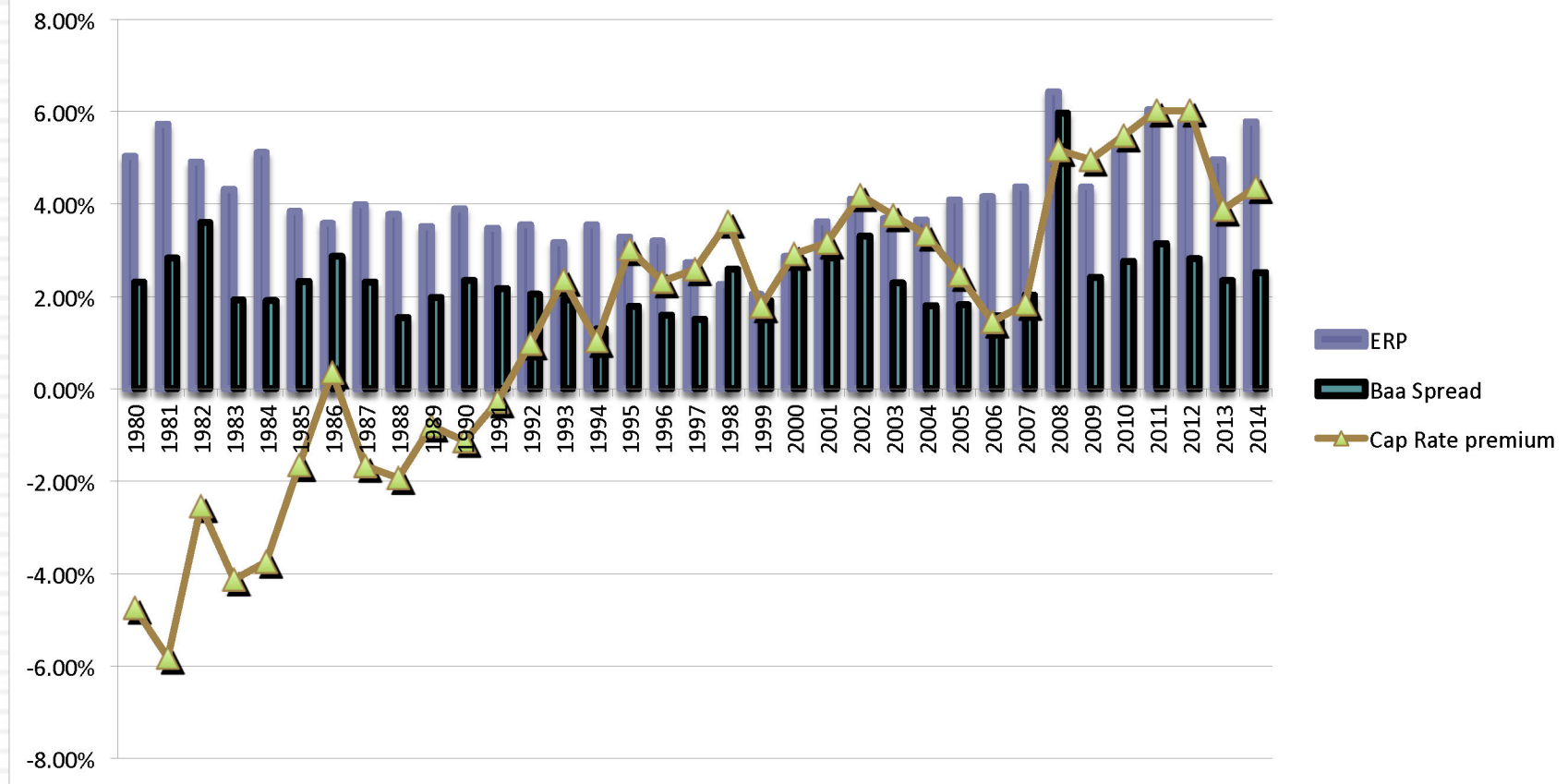
Figure 16: Equity Risk Premiums and Bond Default Spreads



Equity Risk Premiums and Cap Rates (Real Estate)

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Figure 17: Equity Risk Premiums, Cap Rates and Bond Spreads



Why implied premiums matter?

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- In many investment banks, it is common practice (especially in corporate finance departments) to use historical risk premiums (and arithmetic averages at that) as risk premiums to compute cost of equity. If all analysts in the department used the arithmetic average premium (for stocks over T.Bills) for 1928-2014 of 8% to value stocks in January 2014, given the implied premium of 5.75%, what are they likely to find?
 - a. The values they obtain will be too low (most stocks will look overvalued)
 - b. The values they obtain will be too high (most stocks will look under valued)
 - c. There should be no systematic bias as long as they use the same premium to value all stocks.

Which equity risk premium should you use?

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If you assume this

Premiums revert back to historical norms and your time period yields these norms

Market is correct in the aggregate or that your valuation should be market neutral

Market makes mistakes even in the aggregate but is correct over time

Premium to use

Historical risk premium

Current implied equity risk premium

Average implied equity risk premium over time.

<i>Predictor</i>	<i>Correlation with implied premium next year</i>	<i>Correlation with actual risk premium – next 10 years</i>
Current implied premium	0.712	0.424
Average implied premium: Last 5 years	0.646	0.360
Historical Premium	-0.394	-0.486
Default Spread based premium	0.059	0.174

And the approach can be extended to emerging markets

Implied premium for the Sensex (September 2007)

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- Inputs for the computation
 - ▣ Sensex on 9/5/07 = 15446
 - ▣ Dividend yield on index = 3.05%
 - ▣ Expected growth rate - next 5 years = 14%
 - ▣ Growth rate beyond year 5 = 6.76% (set equal to riskfree rate)
- Solving for the expected return:

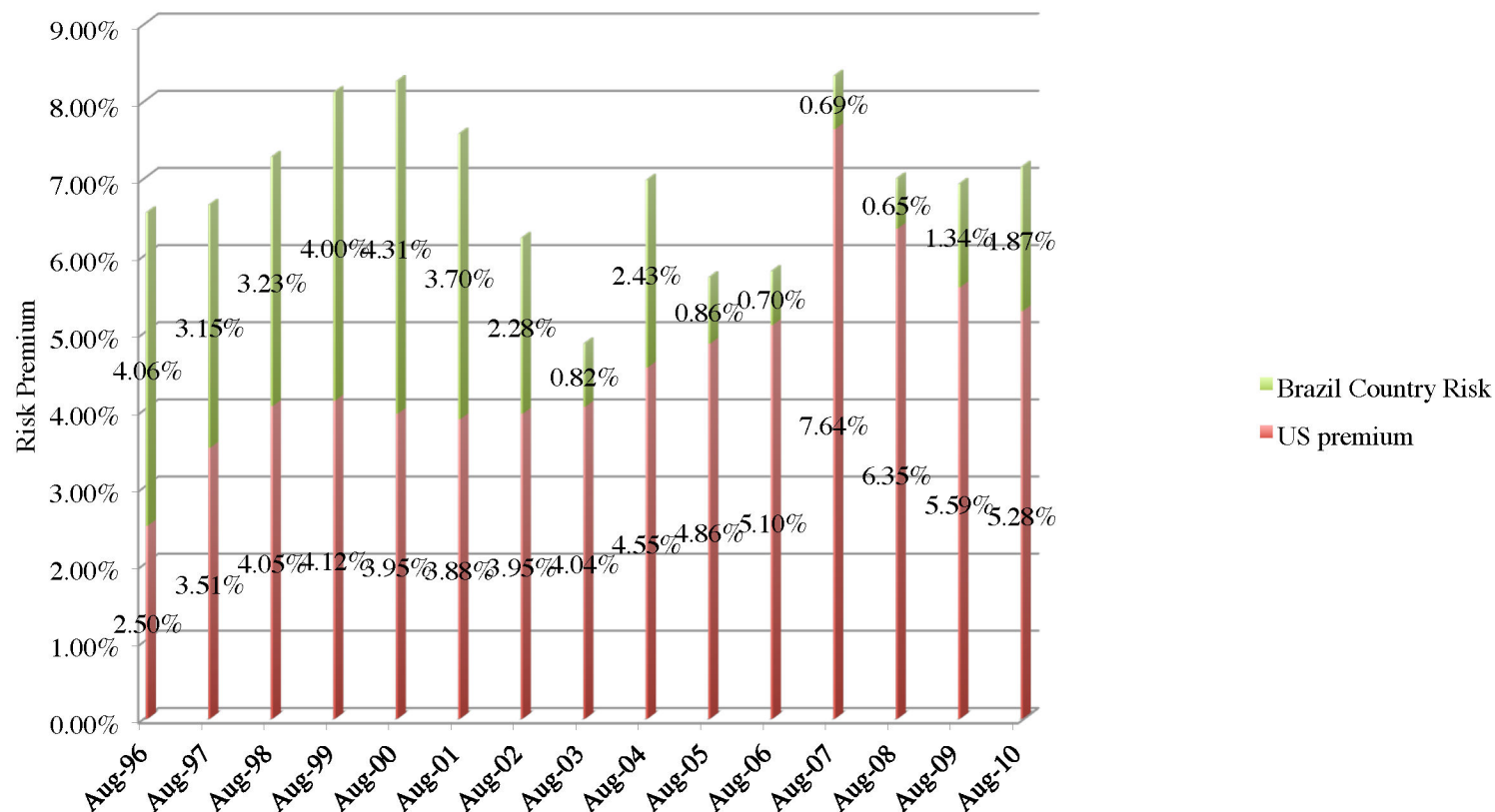
$$15446 = \frac{537.06}{(1+r)} + \frac{612.25}{(1+r)^2} + \frac{697.86}{(1+r)^3} + \frac{795.67}{(1+r)^4} + \frac{907.07}{(1+r)^5} + \frac{907.07(1.0676)}{(r - .0676)(1+r)^5}$$

- Expected return on stocks = 11.18%
- Implied equity risk premium for India = 11.18% - 6.76% = 4.42%

Can country risk premiums change? Brazil CRP & Total ERP from 2000 to 2013

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Figure 15: Implied Equity Risk Premium - Brazil

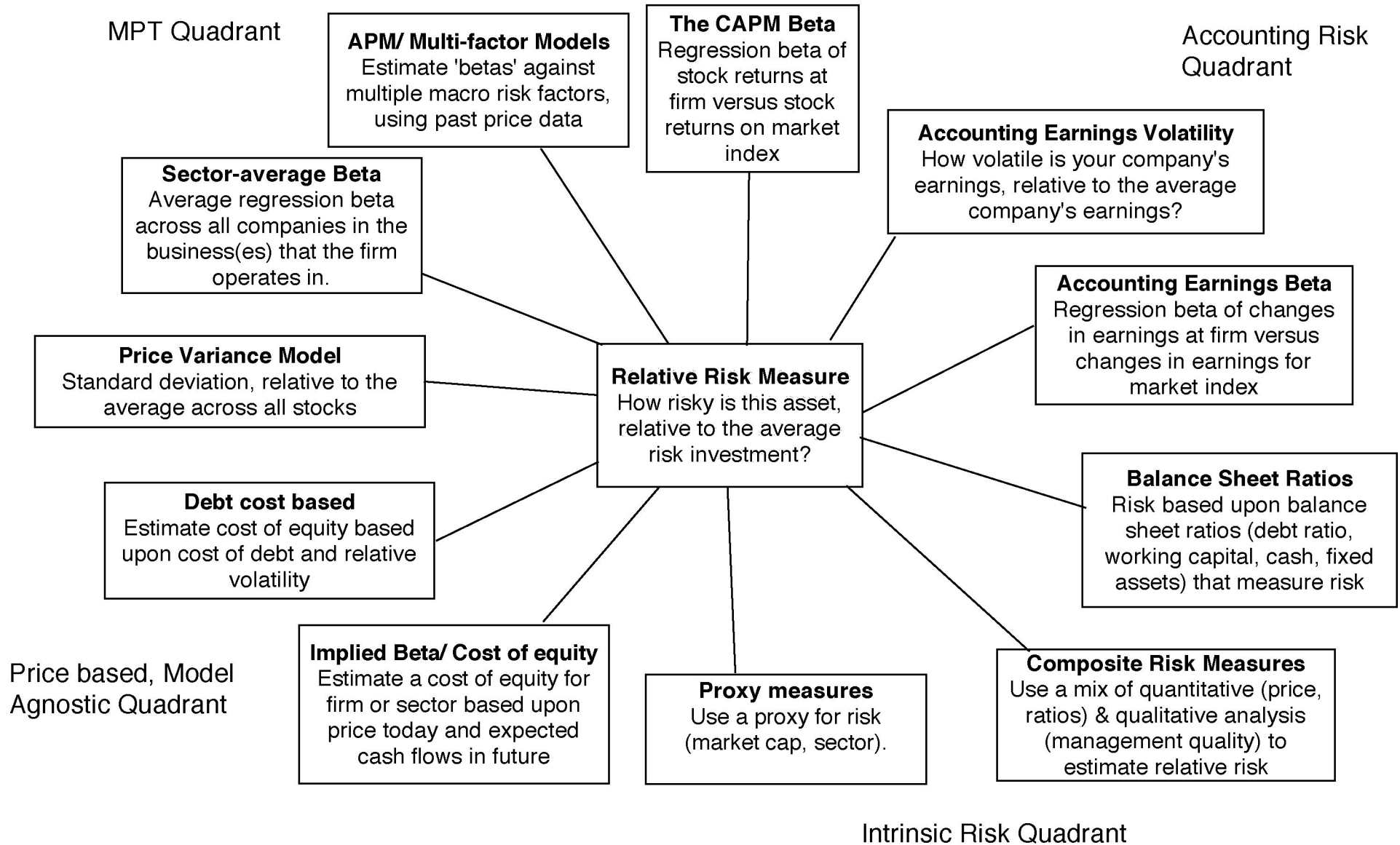


The evolution of Emerging Market Risk

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	<i>PBV Developed</i>	<i>PBV Emerging</i>	<i>ROE Developed</i>	<i>ROE Emerging</i>	<i>US T.Bond rate</i>	<i>Growth rate Developed</i>	<i>Growth rate Emerging</i>	<i>Cost of equity (Developed)</i>	<i>Cost of equity (Emerging)</i>	<i>Differential ERP</i>
2004	2.00	1.19	10.81%	11.65%	4.22%	3.72%	5.22%	7.27%	10.62%	3.36%
2005	2.09	1.27	11.12%	11.93%	4.39%	3.89%	5.39%	7.35%	10.54%	3.19%
2006	2.03	1.44	11.32%	12.18%	4.70%	4.20%	5.70%	7.71%	10.20%	2.49%
2007	1.67	1.67	10.87%	12.88%	4.02%	3.52%	5.02%	7.92%	9.73%	1.81%
2008	0.87	0.83	9.42%	11.12%	2.21%	1.71%	3.21%	10.57%	12.74%	2.17%
2009	1.20	1.34	8.48%	11.02%	3.84%	3.34%	4.84%	7.62%	9.45%	1.83%
2010	1.39	1.43	9.14%	11.22%	3.29%	2.79%	4.29%	7.36%	9.14%	1.78%
2011	1.12	1.08	9.21%	10.04%	1.88%	1.38%	2.88%	8.37%	9.51%	1.14%
2012	1.17	1.18	9.10%	9.33%	1.76%	1.26%	2.76%	7.96%	8.33%	0.37%
Jun-13	1.17	1.17	8.79%	9.37%	2.55%	2.05%	3.55%	7.81%	8.52%	0.71%

Measuring Relative Risk



The CAPM Beta

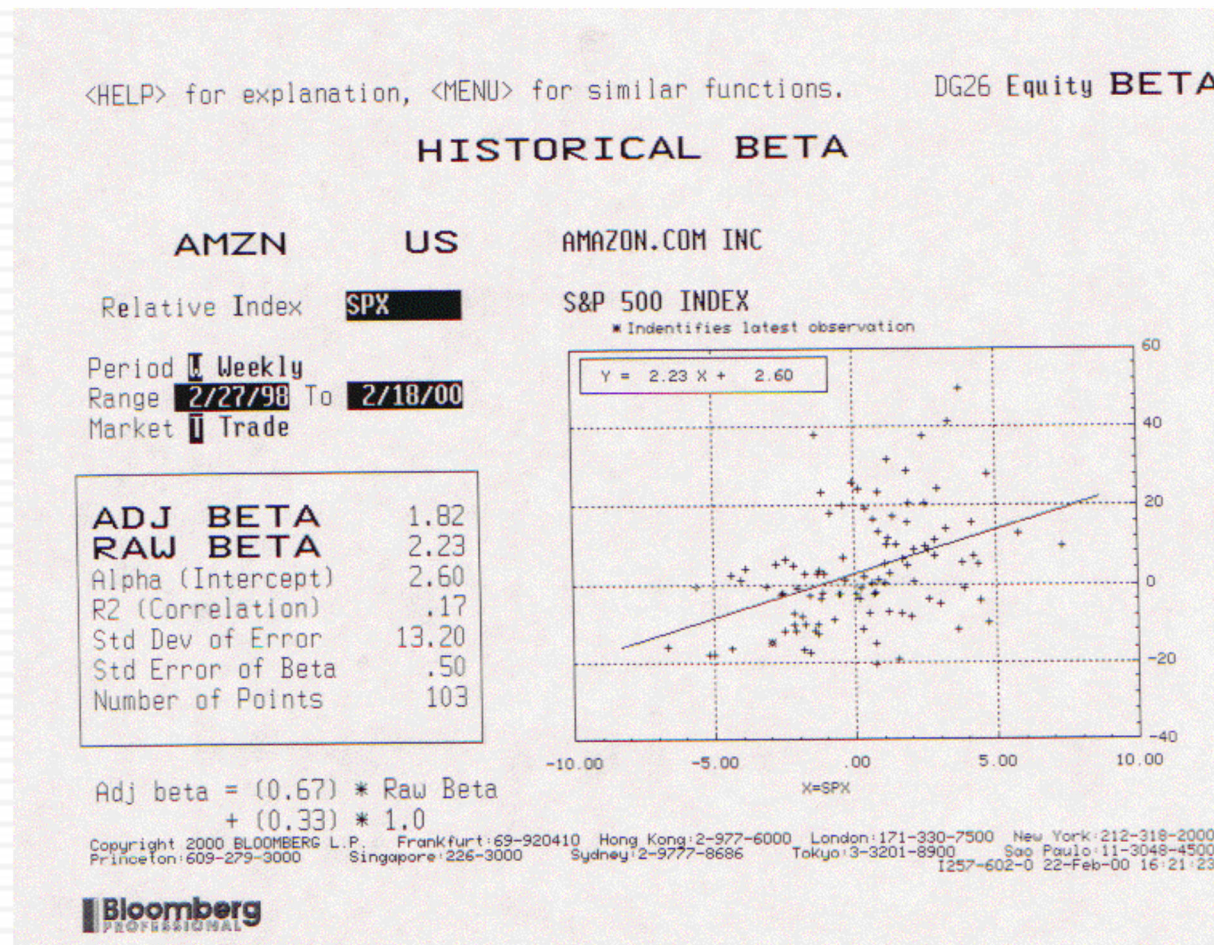
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- The standard procedure for estimating betas is to regress stock returns (R_j) against market returns (R_m) -
$$R_j = a + b R_m$$

where a is the intercept and b is the slope of the regression.
- The slope of the regression corresponds to the beta of the stock, and measures the riskiness of the stock.
- This beta has three problems:
 - It has high standard error
 - It reflects the firm's business mix over the period of the regression, not the current mix
 - It reflects the firm's average financial leverage over the period rather than the current leverage.

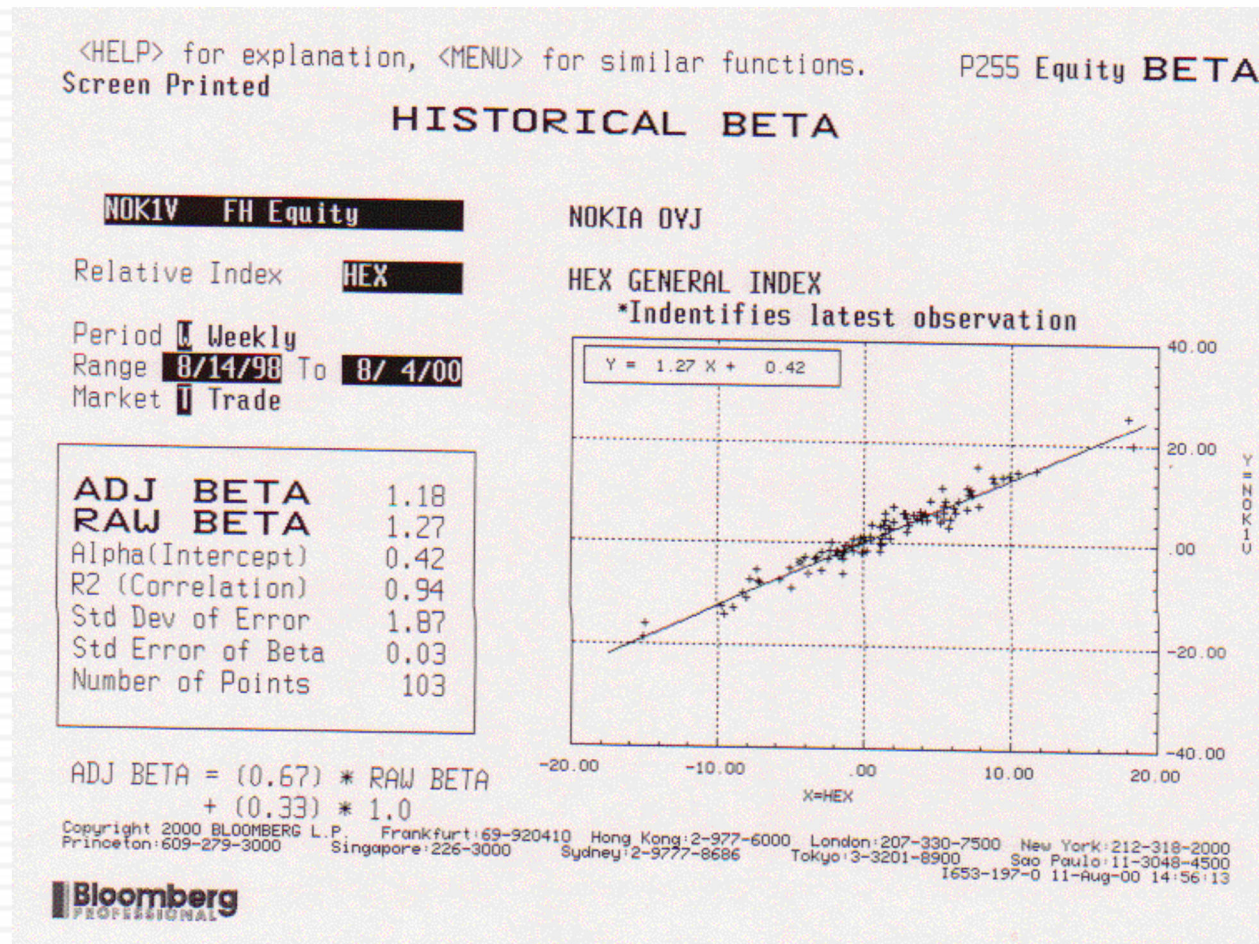
Beta Estimation: The Noise Problem

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Beta Estimation: The Index Effect

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Stock-priced based solutions to the Regression Beta Problem

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- Modify the regression beta by
 - ▣ changing the index used to estimate the beta
 - ▣ adjusting the regression beta estimate, by bringing in information about the fundamentals of the company
- Estimate the beta for the firm using
 - ▣ the standard deviation in stock prices instead of a regression against an index
 - ▣ $\text{Relative risk} = \frac{\text{Standard deviation in stock prices for investment}}{\text{Average standard deviation across all stocks}}$
- Estimate the beta for the firm from the bottom up without employing the regression technique. This will require
 - ▣ understanding the business mix of the firm
 - ▣ estimating the financial leverage of the firm
- Imputed or implied beta (cost of equity) for the sector.

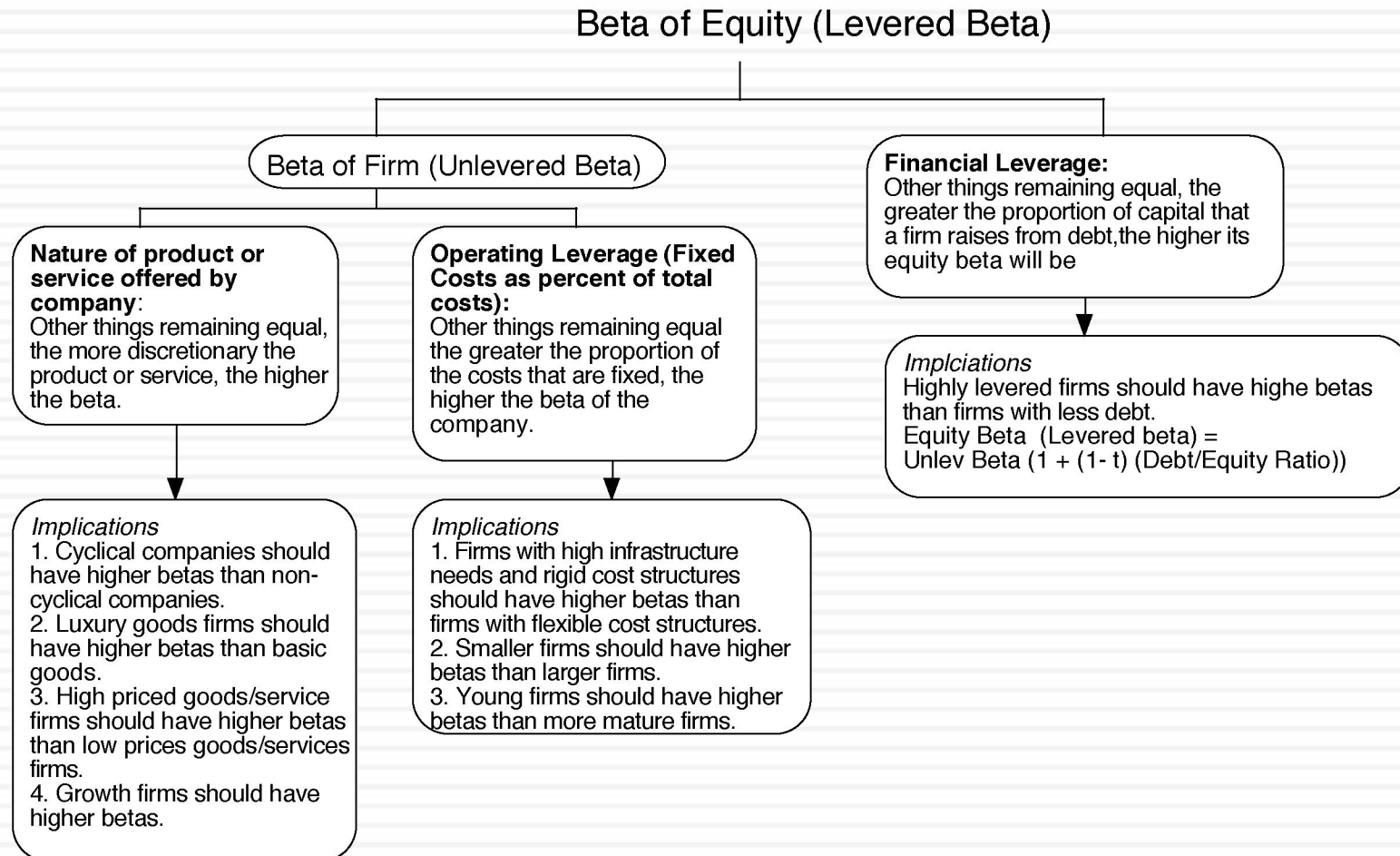
Alternative measures of relative risk for equity

80

- Accounting risk measures: To the extent that you don't trust market-priced based measures of risk, you could compute relative risk measures based on
 - ▣ Accounting earnings volatility: Compute an accounting beta or relative volatility
 - ▣ Balance sheet ratios: You could compute a risk score based upon accounting ratios like debt ratios or cash holdings (akin to default risk scores like the Z score)
- Proxies: In a simpler version of proxy models, you can categorize firms into risk classes based upon size, sectors or other characteristics.
- Qualitative Risk Models: In these models, risk assessments are based at least partially on qualitative factors (quality of management).
- Debt based measures: You can estimate a cost of equity, based upon an observable costs of debt for the company.
 - ▣ $\text{Cost of equity} = \text{Cost of debt} * \text{Scaling factor}$

Determinants of Betas & Relative Risk

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CHAPTER 13

DIVIDEND DISCOUNT MODELS

In the strictest sense, the only cash flow you receive from a firm when you buy publicly traded stock is the dividend. The simplest model for valuing equity is the dividend discount model -- the value of a stock is the present value of expected dividends on it. While many analysts have turned away from the dividend discount model and viewed it as outmoded, much of the intuition that drives discounted cash flow valuation is embedded in the model. In fact, there are specific companies where the dividend discount model remains a useful tool for estimating value.

This chapter explores the general model as well as specific versions of it tailored for different assumptions about future growth. It also examines issues in using the dividend discount model and the results of studies that have looked at its efficacy.

The General Model

When an investor buys stock, she generally expects to get two types of cashflows - dividends during the period she holds the stock and an expected price at the end of the holding period. Since this expected price is itself determined by future dividends, the value of a stock is the present value of dividends through infinity.

$$\text{Value per share of stock} = \sum_{t=1}^{t=\infty} \frac{E(\text{DPS}_t)}{(1 + k_e)^t}$$

where,

DPS_t = Expected dividends per share

k_e = Cost of equity

The rationale for the model lies in the present value rule - the value of any asset is the present value of expected future cash flows discounted at a rate appropriate to the riskiness of the cash flows.

There are two basic inputs to the model - expected dividends and the cost on equity. To obtain the expected dividends, we make assumptions about expected future growth rates in earnings and payout ratios. The required rate of return on a stock is determined by its riskiness, measured differently in different models - the market beta in the CAPM, and the factor betas in the arbitrage and multi-factor models. The model is flexible enough to allow for time-varying discount rates, where the time variation is caused by expected changes in interest rates or risk across time.

Versions of the model

Since projections of dollar dividends cannot be made through infinity, several versions of the dividend discount model have been developed based upon different assumptions about future growth. We will begin with the simplest – a model designed to value stock in a stable-growth firm that pays out what it can afford in dividends and then look at how the model can be adapted to value companies in high growth that may be paying little or no dividends.

I. The Gordon Growth Model

The Gordon growth model can be used to value a firm that is in 'steady state' with dividends growing at a rate that can be sustained forever.

The Model

The Gordon growth model relates the value of a stock to its expected dividends in the next time period, the cost of equity and the expected growth rate in dividends.

$$\text{Value of Stock} = \frac{\text{DPS}_1}{k_e - g}$$

where,

DPS_1 = Expected Dividends one year from now (next period)

k_e = Required rate of return for equity investors

g = Growth rate in dividends forever

What is a stable growth rate?

While the Gordon growth model is a simple and powerful approach to valuing equity, its use is limited to firms that are growing at a stable rate. There are two insights worth keeping in mind when estimating a 'stable' growth rate. First, since the growth rate in the firm's dividends is expected to last forever, the firm's other measures of performance (including earnings) can also be expected to grow at the same rate. To see why, consider the consequences in the long term of a firm whose earnings grow 6% a year forever, while its dividends grow at 8%. Over time, the dividends will exceed earnings. On the other hand, if a firm's earnings grow at a faster rate than dividends in the long term, the payout ratio, in the long term, will converge towards zero, which is also not a steady state. Thus, though the model's requirement is for the expected growth rate in dividends, analysts should be able to substitute in the expected growth rate in earnings and get precisely the same result, if the firm is truly in steady state.

The second issue relates to what growth rate is reasonable as a 'stable' growth rate. As noted in Chapter 12, this growth rate has to be less than or equal to the growth rate of the economy in which the firm operates. This does not, however, imply that analysts will always

agree about what this rate should be even if they agree that a firm is a stable growth firm for three reasons.

- Given the uncertainty associated with estimates of expected inflation and real growth in the economy, there can be differences in the benchmark growth rate used by different analysts, i.e., analysts with higher expectations of inflation in the long term may project a nominal growth rate in the economy that is higher.
- The growth rate of a company may not be greater than that of the economy but it can be less. Firms can become smaller over time relative to the economy.
- There is another instance in which an analyst may stray from a strict limit imposed on the 'stable growth rate'. If a firm is likely to maintain a few years of 'above-stable' growth rates, an approximate value for the firm can be obtained by adding a premium to the stable growth rate, to reflect the above-average growth in the initial years. Even in this case, the flexibility that the analyst has is limited. The sensitivity of the model to growth implies that the stable growth rate cannot be more than 1% or 2% above the growth rate in the economy. If the deviation becomes larger, the analyst will be better served using a two-stage or a three-stage model to capture the 'super-normal' or 'above-average' growth and restricting the Gordon growth model to when the firm becomes truly stable.

Does a stable growth rate have to be constant over time?

The assumption that the growth rate in dividends has to be constant over time is a difficult assumption to meet, especially given the volatility of earnings. If a firm has an average growth rate that is close to a stable growth rate, the model can be used with little real effect on value. Thus, a cyclical firm that can be expected to have year-to-year swings in growth rates, but has an average growth rate that is 5%, can be valued using the Gordon growth model, without a significant loss of generality. There are two reasons for this result. First, since dividends are smoothed even when earnings are volatile, they are less likely to be affected by year-to-year changes in earnings growth. Second, the mathematical effects of using an average growth rate rather than a constant growth rate are small.

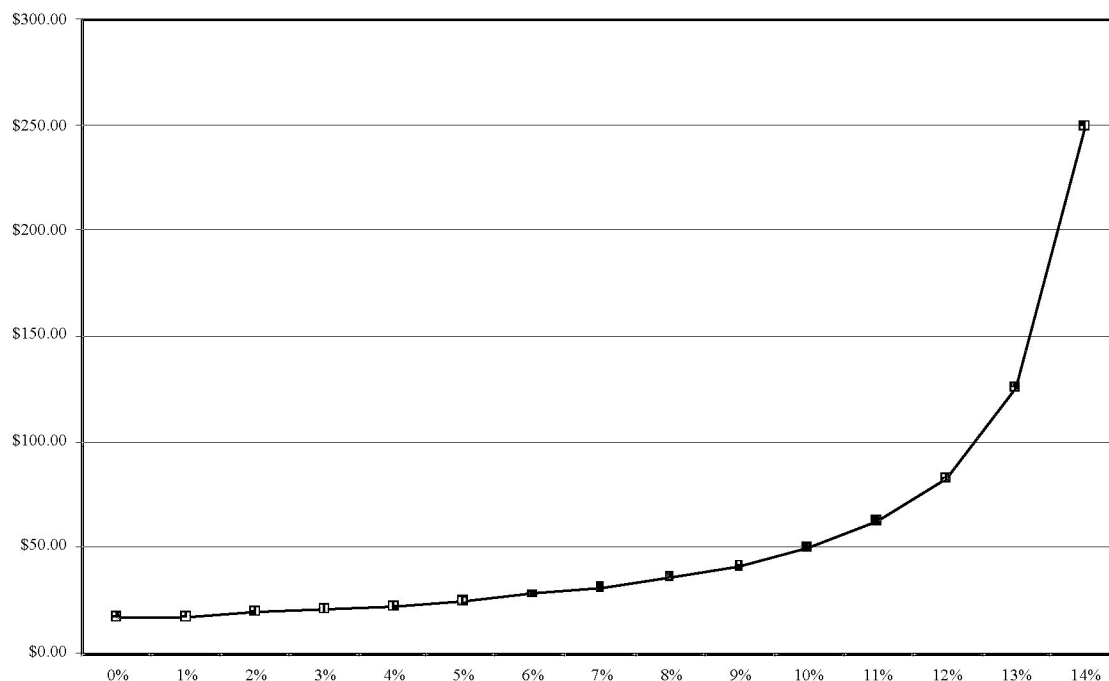
Limitations of the model

The Gordon growth model is a simple and convenient way of valuing stocks but it is extremely sensitive to the inputs for the growth rate. Used incorrectly, it can yield misleading or even absurd results, since, as the growth rate converges on the discount rate, the value goes to infinity. Consider a stock, with an expected dividend per share next period of \$2.50, a cost of equity of 15%, and an expected growth rate of 5% forever. The value of this stock is:

$$\text{Value} = \frac{2.50}{0.15 - 0.05} = \$25.00$$

Note, however, the sensitivity of this value to estimates of the growth rate in Figure 13.1.

Figure 13.1: Value Per Share and Expected Growth Rate



As the growth rate approaches the cost of equity, the value per share approaches infinity. If the growth rate exceeds the cost of equity, the value per share becomes negative.

This issue is tied to the question of what comprises a stable growth rate. If an analyst follows the constraints discussed in the previous chapter in estimating stable growth rates, this will never happen. In this example, for instance, an analyst who uses a 14% growth rate and obtains a \$250 value would have been violating a basic rule on what comprises stable growth.

Works best for:

In summary, the Gordon growth model is best suited for firms growing at a rate comparable to or lower than the nominal growth in the economy and which have well established dividend payout policies that they intend to continue into the future. The dividend payout of the firm has to be consistent with the assumption of stability, since stable

firms generally pay substantial dividends¹. In particular, this model will under estimate the value of the stock in firms that consistently pay out less than they can afford and accumulate cash in the process.



.DDMst.xls: This spreadsheet allows you to value a stable growth firm, with stable firm characteristics (beta and return on equity) and dividends that roughly match cash flows.

Illustration 13.1: Value a regulated firm: Consolidated Edison in May 2001

Consolidated Edison is the electric utility that supplies power to homes and businesses in New York and its environs. It is a monopoly whose prices and profits are regulated by the State of New York.

Rationale for using the model

- The firm is in stable growth; based upon size and the area that it serves. Its rates are also regulated. It is unlikely that the regulators will allow profits to grow at extraordinary rates.
- The firm is in a stable business and regulation is likely to restrict expansion into new businesses.
- The firm is in stable leverage.
- The firm pays out dividends that are roughly equal to FCFE.
 - Average Annual FCFE between 1996 and 2000 = \$551 million
 - Average Annual Dividends between 1996 and 2000 = \$506 million
 - Dividends as % of FCFE = 91.54%

Background Information

Earnings per share in 2000 = \$3.13

Dividend Payout Ratio in 1994 = 69.97%

Dividends per share in 2000 = \$2.19

Return on equity = 11.63%

Estimates

We first estimate the cost of equity, using a bottom-up levered beta for electric utilities of 0.90, a riskfree rate of 5.40% and a market risk premium of 4%.

Con Ed Beta = 0.90

Cost of Equity = 5.4% + 0.90*4% = 9%

We estimate the expected growth rate from fundamentals.

Expected growth rate = (1 - Payout ratio) Return on equity
 = (1 - 0.6997)(0.1163) = 3.49%


¹ The average payout ratio for large stable firms in the United States is about 60%.

Valuation

We now use the Gordon growth model to value the equity per share at Con Ed:

$$\begin{aligned} \text{Value of Equity} &= \frac{\text{Expected dividends next year}}{\text{Cost of equity} - \text{Expected growth rate}} \\ &= \frac{(\$2.19)(1.0349)}{0.09 - 0.0349} = \$41.15 \end{aligned}$$

Con Ed was trading for \$36.59 on the day of this analysis (May 14, 2001). Based upon this valuation, the stock would have been under valued.

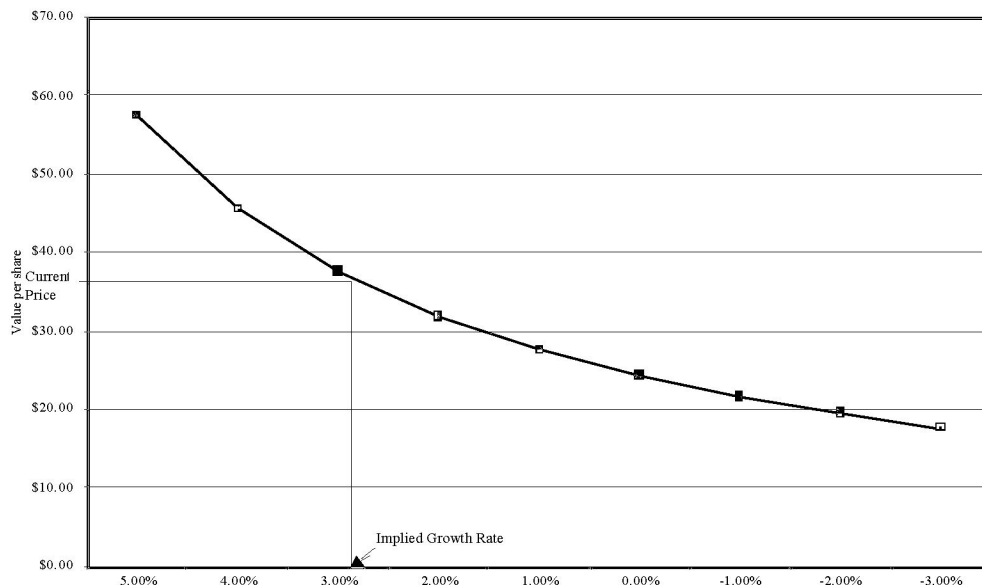
 *.DDMst.xlsx*: This spreadsheet allows you to value a stable growth firm, with stable firm characteristics (beta and return on equity) and dividends that roughly match cash flows.

Implied Growth Rate

Our value for Con Ed is different from the market price and this is likely to be the case with almost any company that you value. There are three possible explanations for this deviation. One is that you are right and the market is wrong. While this may be the correct explanation, you should probably make sure that the other two explanations do not hold – that the market is right and you are wrong or that the difference is too small to draw any conclusions. [

To examine the magnitude of the difference between the market price and your estimate of value, you can hold the other variables constant and change the growth rate in your valuation until the value converges on the price. Figure 13.2 estimates value as a function of the expected growth rate (assuming a beta of 0.90 and current dividends per share of \$2.19).

Figure 13.2: Value per share versus Growth



Solving for the expected growth rate that provides the current price,

$$\$36.59 = \frac{\$2.19(1 + g)}{0.09 - g}$$

The growth rate in earnings and dividends would have to be 2.84% a year to justify the stock price of \$36.59. This growth rate is called an **implied growth rate**. Since we estimate growth from fundamentals, this allows us to estimate an implied return on equity.

$$\text{Implied return on equity} = \frac{\text{Implied growth rate}}{\text{Retention ratio}} = \frac{0.0284}{0.3003} = 9.47\%$$

Illustration 13.2: Value a real estate investment trust: Vornado REIT

Real estate investment trusts were created in the early 1970s by a law that allowed these entities to invest in real estate and pass the income, tax-free, to their investors. In return for the tax benefit, however, REITs are required to return at least 95% of their earnings as dividends. Thus, they provide an interesting case study in dividend discount model valuation. Vornado Realty Trust owns and has investments in real estate in the New York area including Alexander's, the Hotel Pennsylvania and other ventures.

Rationale for using the model

Since the firm is required to pay out 95% of its earnings as dividends, the growth in earnings per share will be modest,² making it a good candidate for the Gordon growth model.

Background Information

In 2000, Vornado paid dividends per share of \$2.12 on earnings per share of \$2.22. The estimated payout ratio is:

$$\text{Expected payout ratio} = \frac{2.12}{2.22} = 95.50\%$$

The firm had a return on equity of 12.29%.

Estimates

We use the average beta for real estate investment trusts of 0.69, a riskfree rate of 5.4% and a risk premium of 4% to estimate a cost of equity:

$$\text{Cost of equity} = 5.4\% + 0.69 (4\%) = 8.16\%$$

The expected growth rate is estimated from the dividend payout ratio and the return on equity:

² Growth in net income may be much higher, since REITs can still issue new equity for investing in new ventures.

Expected growth rate = $(1 - 0.955)(0.1229) = 0.55\%$

Valuation

$$\text{Value per share} = \frac{2.12(1.0055)}{0.0816 - 0.0055} = \$28.03$$

It is particularly important with REITs that we steer away from net income growth, which may be much higher. On May 14, 2001, Vornado Realty was trading at \$36.57, which would make it overvalued.

II. Two-stage Dividend Discount Model

The two-stage growth model allows for two stages of growth - an initial phase where the growth rate is not a stable growth rate and a subsequent steady state where the growth rate is stable and is expected to remain so for the long term. While, in most cases, the growth rate during the initial phase is higher than the stable growth rate, the model can be adapted to value companies that are expected to post low or even negative growth rates for a few years and then revert back to stable growth.

The Model

The model is based upon two stages of growth, an extraordinary growth phase that lasts n years and a stable growth phase that lasts forever afterwards.

Extraordinary growth rate: $g\%$ each year for n years Stable growth: g_n forever

|_____|_____>

Value of the Stock = PV of Dividends during extraordinary phase + PV of terminal price

$$P_0 = \sum_{t=1}^{t=n} \frac{DPS_t}{(1 + k_{e,hg})^t} + \frac{P_n}{(1 + k_{e,hg})^n} \text{ where } P_n = \frac{DPS_{n+1}}{(k_{e,st} - g_n)}$$

where,

DPS_t = Expected dividends per share in year t

k_e = Cost of Equity (hg: High Growth period; st: Stable growth period)

P_n = Price (terminal value) at the end of year n

g = Extraordinary growth rate for the first n years

g_n = Steady state growth rate forever after year n

In the case where the extraordinary growth rate (g) and payout ratio are unchanged for the first n years, this formula can be simplified.

$$P_0 = \frac{DPS_0 * (1+g) * \left(1 - \frac{(1+g)^n}{(1+k_{e,hg})^n}\right)}{k_{e,hg} - g} + \frac{DPS_{n+1}}{(k_{e,st} - g_n)(1+k_{e,hg})^n}$$

where the inputs are as defined above.

Calculating the terminal price

The same constraint that applies to the growth rate for the Gordon Growth Rate model, i.e., that the growth rate in the firm is comparable to the nominal growth rate in the economy, applies for the terminal growth rate (g_n) in this model as well.

In addition, the payout ratio has to be consistent with the estimated growth rate. If the growth rate is expected to drop significantly after the initial growth phase, the payout ratio should be higher in the stable phase than in the growth phase. A stable firm can pay out more of its earnings in dividends than a growing firm. One way of estimating this new payout ratio is to use the fundamental growth model described in Chapter 12.

Expected Growth = Retention ratio * Return on equity

Algebraic manipulation yields the following stable period payout ratio:

$$\text{Stable Payout ratio} = \frac{\text{Stable growth rate}}{\text{Stable period return on equity}}$$

Thus, a firm with a 5% growth rate and a return on equity of 15% will have a stable period payout ratio of 33.33%.

The other characteristics of the firm in the stable period should be consistent with the assumption of stability. For instance, it is reasonable to assume that a high growth firm has a beta of 2.0, but unreasonable to assume that this beta will remain unchanged when the firm becomes stable. In fact, the rule of thumb that we developed in the last chapter – that stable period betas should be between 0.8 and 1.2 – is worth repeating here. Similarly, the return on equity, which can be high during the initial growth phase, should come down to levels commensurate with a stable firm in the stable growth phase. What is a reasonable stable period return on equity? The industry average return on equity and the firm's own stable period cost of equity provide useful information to make this judgment.

Limitations of the model

There are three problems with the two-stage dividend discount model – the first two would apply to any two-stage model and the third is specific to the dividend discount model.

- The first practical problem is in defining the length of the extraordinary growth period. Since the growth rate is expected to decline to a stable level after this period, the value of an investment will increase as this period is made longer. While we did develop criteria

that might be useful in making this judgment in Chapter 12, it is difficult in practice to convert these qualitative considerations into a specific time period.

- The second problem with this model lies in the assumption that the growth rate is high during the initial period and is transformed overnight to a lower stable rate at the end of the period. While these sudden transformations in growth can happen, it is much more realistic to assume that the shift from high growth to stable growth happens gradually over time.
- The focus on dividends in this model can lead to skewed estimates of value for firms that are not paying out what they can afford in dividends. In particular, we will underestimate the value of firms that accumulate cash and pay out too little in dividends.

Works best for:

Since the two-stage dividend discount model is based upon two clearly delineated growth stages, high growth and stable growth, it is best suited for firms which are in high growth and expect to maintain that growth rate for a specific time period, after which the sources of the high growth are expected to disappear. One scenario, for instance, where this may apply is when a company has patent rights to a very profitable product for the next few years and is expected to enjoy super-normal growth during this period. Once the patent expires, it is expected to settle back into stable growth. Another scenario where it may be reasonable to make this assumption about growth is when a firm is in an industry which is enjoying super-normal growth because there are significant barriers to entry (either legal or as a consequence of infra-structure requirements), which can be expected to keep new entrants out for several years.

The assumption that the growth rate drops precipitously from its level in the initial phase to a stable rate also implies that this model is more appropriate for firms with modest growth rates in the initial phase. For instance, it is more reasonable to assume that a firm growing at 12% in the high growth period will see its growth rate drops to 6% afterwards than it is for a firm growing at 40% in the high growth period.

Finally, the model works best for firms that maintain a policy of paying out most of residual cash flows – i.e, cash flows left over after debt payments and reinvestment needs have been met – as dividends.

Illustration 13.3: Valuing a firm with the two-stage dividend discount model: Procter & Gamble

Procter & Gamble (P&G) manufactures and markets consumer products all over the world. Some of its best known brand names include Pampers diapers, Tide detergent, Crest toothpaste and Vicks cough/cold medicines.

A Rationale for using the Model

- *Why two-stage?* While P&G is a firm with strong brand names and an impressive track record on growth, it faces two problems. The first is the saturation of the domestic U.S. market, which represents about half of P&G's revenues. The second is the increased competition from generics across all of its product lines. We will assume that the firm will continue to grow but restrict the growth period to 5 years.
- *Why dividends?* P&G has a reputation for paying high dividends and it has not accumulated large amounts of cash over the last decade.

Background Information

- Earnings per share in 2000 = \$3.00
- Dividends per share in 2000 = \$1.37
- Payout ratio in 2000 = $\frac{1.37}{3.00} = 45.67\%$
- Return on Equity in 2000 = 29.37%

Estimates

We will first estimate the cost of equity for P&G, based upon a bottom-up beta of 0.85 (estimated using the unlevered beta for consumer product firms and P&G's debt to equity ratio), a riskfree rate of 5.4% and a risk premium of 4%.

Cost of equity = 5.4% + 0.85 (4%) = 8.8%

To estimate the expected growth in earnings per share over the five-year high growth period, we use the retention ratio in the most recent financial year (2000) but lower the return on equity to 25% from the current value.

Expected growth rate = Retention ratio * Return on Equity

$$= (1 - 0.4567)(0.25) = 13.58\%$$

In stable growth, we will estimate that the beta for the stock will rise to 1, leading to a cost of equity of 9.40%.

Cost of equity in stable growth = 5.4% + 1 (4%) = 9.40%

The expected growth rate will be assumed to be equal to the growth rate of the economy (5%) and the return on equity will drop to 15%, which is lower than the current industry average (17.4%) but higher than the cost of equity estimated above. The retention ratio in stable growth during the stable growth period is calculated.

Retention ratio in stable growth = $\frac{g}{ROE} = \frac{5\%}{15\%} = 33.33\%$

The payout ratio in stable growth is therefore 66.67%.

Estimating the value:

The first component of value is the present value of the expected dividends during the high growth period. Based upon the current earnings (\$3.00), the expected growth rate (13.58%) and the expected dividend payout ratio (45.67%), the expected dividends can be computed for each year in the high growth period.

Table 13.1: Expected Dividends per share: P&G

<i>Year</i>	<i>EPS</i>	<i>DPS</i>	<i>Present Value</i>
1	\$3.41	\$1.56	\$1.43
2	\$3.87	\$1.77	\$1.49
3	\$4.40	\$2.01	\$1.56
4	\$4.99	\$2.28	\$1.63
5	\$5.67	\$2.59	\$1.70
Sum			\$7.81

The present value is computed using the cost of equity of 8.8% for the high growth period.

Cumulative Present Value of Dividends during high growth (@8.8%) = \$7.81

The present value of the dividends can also be computed in short hand using the following computation:

$$\text{PV of Dividends} = \frac{\$1.37(1.1358) \left(1 - \frac{(1.1358)^5}{(1.088)^5} \right)}{0.088 - 0.1358} = \$7.81$$

The price (terminal value) at the end of the high growth phase (end of year 5) can be estimated using the constant growth model.

$$\text{Terminal price} = \frac{\text{Expected Dividends per share}_{n+1}}{k_{e,st} - g_n}$$

$$\text{Expected Earnings per share}_6 = 3.00 * 1.1358^5 * 1.05 = \$5.96$$

$$\begin{aligned} \text{Expected Dividends per share}_6 &= \text{EPS}_6 * \text{Stable period payout ratio} \\ &= \$5.96 * 0.6667 = \$3.97 \end{aligned}$$

$$\text{Terminal price} = \frac{\text{Dividends}_6}{k_{e,st} - g} = \frac{\$3.97}{0.094 - 0.05} = \$90.23$$

The present value of the terminal price –is:

$$\text{PV of Terminal Price} = \frac{\$90.23}{(1.088)^5} = \$59.18$$

The cumulated present value of dividends and the terminal price can then be calculated.

$$P_0 = \frac{\$1.37(1.1358) \left(1 - \frac{(1.1358)^5}{(1.088)^5} \right)}{0.088 - 0.1358} + \frac{\$90.23}{(1.088)^5} = \$7.81 + \$59.18 = \$66.99$$

P&G was trading at \$63.90 at the time of this analysis on May 14, 2001.



.DDM2st.xls: This spreadsheet allows you to value a firm with a temporary period of high earnings followed by stable growth.

A Trouble Shooting Guide: What is wrong with this valuation? DDM 2 St

If this is your 'problem'

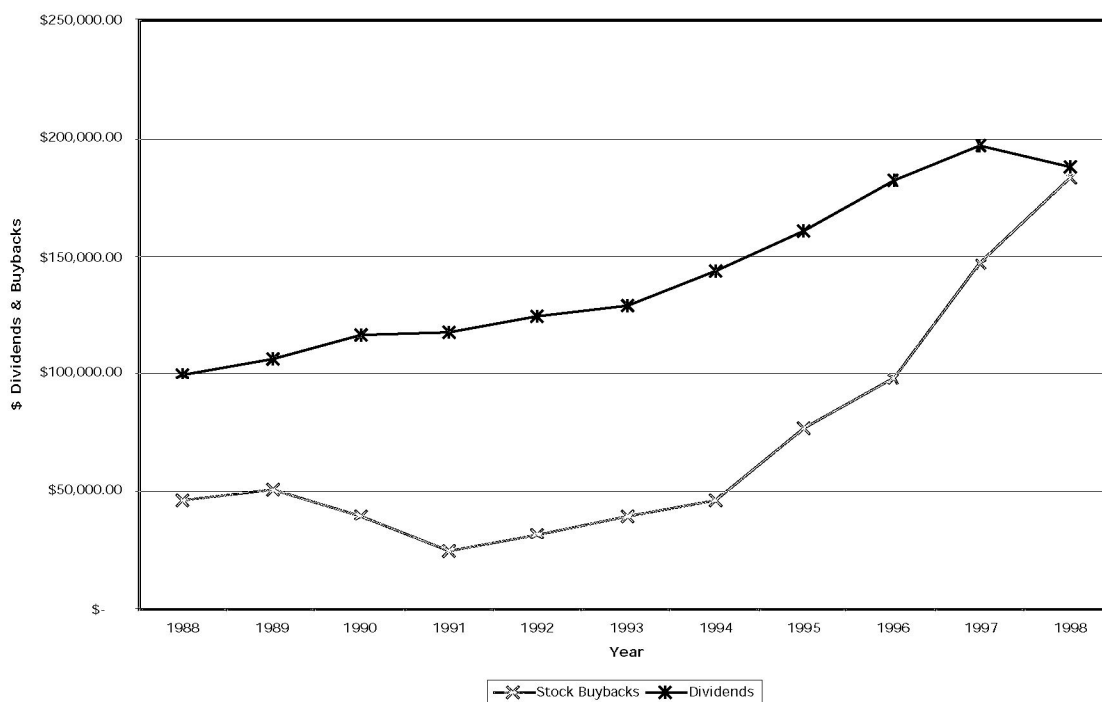
this may be the s

- If you get a extremely low value from the 2-stage DDM, the likely culprits are
 - the stable period payout ratio is too low for a stable firm ($< 40\%$) If using fundame
 - the beta in the stable period is too high for a stable firm If entering direct
 - the use of the two-stage model when the three-stage model is more appropriate Use a beta close
- If you get an extremely high value,
 - the growth rate in the stable growth period is too high for stable firm Use a three-stag
 - the growth rate in the stable growth period is too high for stable firm Use a growth rat

Modifying the model to include stock buybacks

In recent years, firms in the United States have increasingly turned to stock buybacks as a way of returning cash to stockholders. Figure 13.3 presents the cumulative amounts paid out by firms in the form of dividends and stock buybacks from 1960 to 1998.

Figure 13.3: Stock Buybacks and Dividends: Aggregate for US Firms - 1989-98



The trend towards stock buybacks is very strong, especially in the 1990s.

What are the implications for the dividend discount model? Focusing strictly on dividends paid as the only cash returned to stockholders exposes us to the risk that we might be missing significant cash returned to stockholders in the form of stock buybacks. The simplest way to incorporate stock buybacks into a dividend discount model is to add them on to the dividends and compute a modified payout ratio:

$$\text{Modified dividend payout ratio} = \frac{\text{Dividends} + \text{Stock Buybacks}}{\text{Net Income}}$$

While this adjustment is straightforward, the resulting ratio for any one year can be skewed by the fact that stock buybacks, unlike dividends, are not smoothed out. In other words, a firm may buy back \$3 billion in stock in one year and not buy back stock for the next 3 years. Consequently, a much better estimate of the modified payout ratio can be obtained by looking at the average value over a four or five year period. In addition, firms may

sometimes buy back stock as a way of increasing financial leverage. We could adjust for this by netting out new debt issued from the calculation above:

$$\text{Modified dividend payout} = \frac{\text{Dividends} + \text{Stock Buybacks} - \text{Long Term Debt issues}}{\text{Net Income}}$$

Adjusting the payout ratio to include stock buybacks will have ripple effects on the estimated growth and the terminal value. In particular, the modified growth rate in earnings per share can be written as:

$$\text{Modified growth rate} = (1 - \text{Modified payout ratio}) * \text{Return on equity}$$

Even the return on equity can be affected by stock buybacks. Since the book value of equity is reduced by the market value of equity bought back, a firm that buys back stock can reduce its book equity (and increase its return on equity) dramatically. If we use this return on equity as a measure of the marginal return on equity (on new investments), we will overstate the value of a firm. Adding back stock buybacks in recent year to the book equity and re-estimating the return on equity can sometimes yield a more reasonable estimate of the return on equity on investments.

Illustration 13.4: Valuing a firm with modified dividend discount mode: Procter & Gamble

Consider our earlier valuation of Procter and Gamble where we used the current dividends as the basis for our projections. Note that over the last four years, P&G has had significant stock buybacks each period. Table 13.2 summarizes the dividends and buybacks over the period.

Table 13.2: Dividends and Stock Buybacks: P&G

	1997	1998	1999	2000	Total
Net Income	3415	3780	3763	3542	14500
Dividends	1329	1462	1626	1796	6213
Buybacks	2152	391	1881	-1021	3403
Dividends+Buybacks	3481	1853	3507	775	9616
Payout ratio	38.92%	38.68%	43.21%	50.71%	42.85%
Modified payout ratio	101.93%	49.02%	93.20%	21.88%	66.32%
Buybacks	1652	1929	2533	1766	
Net LT Debt issued	-500	1538	652	2787	
Buybacks net of debt	2152	391	1881	-1021	

Over the five-year period, P&G had significant buybacks but it also increased its leverage dramatically in the last three years. Summing up the total cash returned to stockholders over

the last 4 years, we arrive at a modified payout ratio of 66.32%. If we substitute this payout ratio into the valuation in Illustration 13.3, the expected growth rate over the next 5 years drops to 8.42%:

Expected growth rate = (1 - Modified payout ratio) ROE = (1 - 0.6632)(0.25) = 8.42%

We will still assume a five year high growth period and that the parameters in stable growth remain unchanged. The value per share can be estimated.

$$P_0 = \frac{\$3.00(0.6632)(1.0842) \left(1 - \frac{(1.0842)^5}{(1.0880)^5} \right)}{0.0880 - 0.0842} + \frac{\$71.50}{(1.0880)^5} = \$56.75$$

Note that the drop in growth rate in earnings during the high growth period reduces earnings in the terminal year, and the terminal value per share drops to \$71.50.

This value is lower than that obtained in Illustration 13.3 and it reflects our expectation that P&G does not have as many new profitable new investments (earning a return on equity of 25%).

Valuing an entire market using the dividend discount model

All our examples of the dividend discount model so far have involved individual companies, but there is no reason why we cannot apply the same model to value a sector or even the entire market. The market price of the stock would be replaced by the cumulative market value of all of the stocks in the sector or market. The expected dividends would be the cumulated dividends of all these stocks and could be expanded to include stock buybacks by all firms. The expected growth rate would be the growth rate in cumulated earnings of the index. There would be no need for a beta or betas, since you are looking at the entire market (which should have a beta of 1) and you could add the risk premium (or premiums) to the riskfree rate to estimate a cost of equity. You could use a two-stage model, where this growth rate is greater than the growth rate of the economy, but you should be cautious about setting the growth rate too high or the growth period too long because it will be difficult for cumulated earnings growth of all firms in an economy to run ahead of the growth rate in the economy for extended periods.

Consider a simple example. Assume that you have an index trading at 700 and that the average dividend yield of stocks in the index is 5%. Earnings and dividends can be expected to grow at 4% a year forever and the riskless rate is 5.4%. If you use a market risk premium of 4%, the value of the index can be estimated.

Cost of equity = Riskless rate + Risk premium = 5.4% + 4% = 9.4%

Expected dividends next year = (Dividend yield * Value of the index)(1+ expected growth rate) = $(0.05 \times 700) (1.04) = 36.4$

$$\text{Value of the index} = \frac{\text{Expected dividends next year}}{\text{Cost of equity} - \text{Expected growth rate}} = \frac{36.4}{0.094 - 0.04} = 674$$

At its existing level of 700, the market is slightly over priced.

Illustration 13.5: Valuing the S&P 500 using a dividend discount model: January 1, 2001

On January 1, 2001, the S&P 500 index was trading at 1320. The dividend yield on the index was only 1.43%, but including stock buybacks increases the modified dividend yield to 2.50%. Analysts were estimating that the earnings of the stocks in the index would increase 7.5% a year for the next 5 years. Beyond year 5, the expected growth rate is expected to be 5%, the nominal growth rate in the economy. The treasury bond rate was 5.1% and we will use a market risk premium of 4%, leading to a cost of equity of 9.1%:

$$\text{Cost of equity} = 5.1\% + 4\% = 9.1\%$$

The expected dividends (and stock buybacks) on the index for the next 5 years can be estimated from the current dividends and expected growth of 7.50%.

$$\text{Current dividends} = 2.50\% \text{ of } 1320 = 33.00$$

	1	2	3	4	5
Expected Dividends =	\$35.48	\$38.14	\$41.00	\$44.07	\$47.38
Present Value =	\$32.52	\$32.04	\$31.57	\$31.11	\$30.65

The present value is computed by discounting back the dividends at 9.1%. To estimate the terminal value, we estimate dividends in year 6 on the index:

$$\text{Expected dividends in year 6} = \$47.38 (1.05) = \$49.74$$

$$\text{Terminal value of the index} = \frac{\text{Expected Dividends}_6}{r - g} = \frac{\$49.74}{0.091 - 0.05} = \$1213$$

$$\text{Present value of Terminal value} = \frac{\$1213}{1.091^5} = \$785$$

The value of the index can now be computed:

$$\text{Value of index} = \text{Present value of dividends during high growth} + \text{Present value of terminal value} = \$32.52 + \$32.04 + \$31.57 + \$31.11 + \$30.65 + \$785 = \$943$$

Based upon this, we would have concluded that the index was over valued at 1320.

The Value of Growth

Investors pay a price premium when they acquire companies with high growth potential. This premium takes the form of higher price-earnings or price-book value ratios. While no one will contest the proposition that growth is valuable, it is possible to pay too much for growth. In fact, empirical studies that show low price-earnings ratio stocks earning return premiums over high price-earnings ratio stocks in the long term supports the notion that investors overpay for growth. This section uses the two-stage dividend discount model to examine the value of growth and it provides a benchmark that can be used to compare the actual prices paid for growth.

Estimating the value of growth

The value of the equity in any firm can be written in terms of three components:

$$P_0 = \left[\frac{DPS_0 * (1+g) * \left(1 - \frac{(1+g)^n}{(1+k_{e,hg})^n} \right)}{k_{e,hg} - g} + \frac{DPS_{n+1}}{(k_{e,st} - g_n)(1+k_{e,hg})^n} - \frac{DPS_1}{(k_{e,st} - g_n)} \right]$$

|-----|
Extraordinary Growth

$$+ \left[\frac{DPS_1}{(k_{e,st} - g_n)} - \frac{DPS_0}{k_{e,st}} \right] + \frac{DPS_0}{k_{e,st}}$$

|-----| |-----|
Stable Growth Assets in place

where

DPS_t = Expected dividends per share in year t

k_e = Required rate of return

P_n = Price at the end of year n

g = Growth rate during high growth stage

g_n = Growth rate forever after year n

Value of extraordinary growth = Value of the firm with extraordinary growth in first n years - Value of the firm as a stable growth firm³

Value of stable growth = Value of the firm as a stable growth firm - Value of firm with no growth

³ The payout ratio used to calculate the value of the firm as a stable firm can be either the current payout ratio, if it is reasonable, or the new payout ratio calculated using the fundamental growth formula.

Assets in place = Value of firm with no growth

In making these estimates, though, we have to remain consistent. For instance, to value assets in place, you would have to assume that the entire earnings could be paid out in dividends, while the payout ratio used to value stable growth should be a stable period payout ratio.

Illustration 13.6: The Value of Growth: P&G in May 2001

In illustration 13.3, we valued P&G using a 2-stage dividend discount model at \$66.99. We first value the assets in place using current earnings (\$3.00) and assume that all earnings are paid out as dividends. We also use the stable growth cost of equity as the discount rates.

$$\text{Value of the assets in place} = \frac{\text{Current EPS}}{k_{e,st}} = \frac{\$3}{0.094} = \$31.91$$

To estimate the value of stable growth, we assume that the expected growth rate will be 5% and that the payout ratio is the stable period payout ratio of 66.67%:

$$\begin{aligned} \text{Value of stable growth} &= \frac{(\text{Current EPS})(\text{Stable Payout Ratio})(1 + g_n)}{k_{e,st} - g_n} - \$31.91 \\ &= \frac{(\$3.00)(0.6667)(1.05)}{0.094 - 0.05} - \$31.91 = \$15.81 \end{aligned}$$

$$\text{Value of extraordinary growth} = \$66.99 - \$31.91 - \$15.81 = \$19.26$$

The Determinants of the Value of Growth

1. *Growth rate during extraordinary period:* The higher the growth rate in the extraordinary period, the higher the estimated value of growth will be. If the growth rate in the extraordinary growth period had been raised to 20% for the Procter & Gamble valuation, the value of extraordinary growth would have increased from \$19.26 to \$39.45. Conversely, the value of high growth companies can drop precipitously if the expected growth rate is reduced, either because of disappointing earnings news from the firm or as a consequence of external events.
2. *Length of the extraordinary growth period:* The longer the extraordinary growth period, the greater the value of growth will be. At an intuitive level, this is fairly simple to illustrate. The value of \$19.26 obtained for extraordinary growth is predicated on the assumption that high growth will last for five years. If this is revised to last ten years, the value of extraordinary growth will increase to \$43.15.
3. *Profitability of projects:* The profitability of projects determines both the growth rate in the initial phase and the terminal value. As projects become more

profitable, they increase both growth rates and growth period, and the resulting value from extraordinary growth will be greater.

4. *Riskiness of the firm/equity* The riskiness of a firm determines the discount rate at which cashflows in the initial phase are discounted. Since the discount rate increases as risk increases, the present value of the extraordinary growth will decrease.

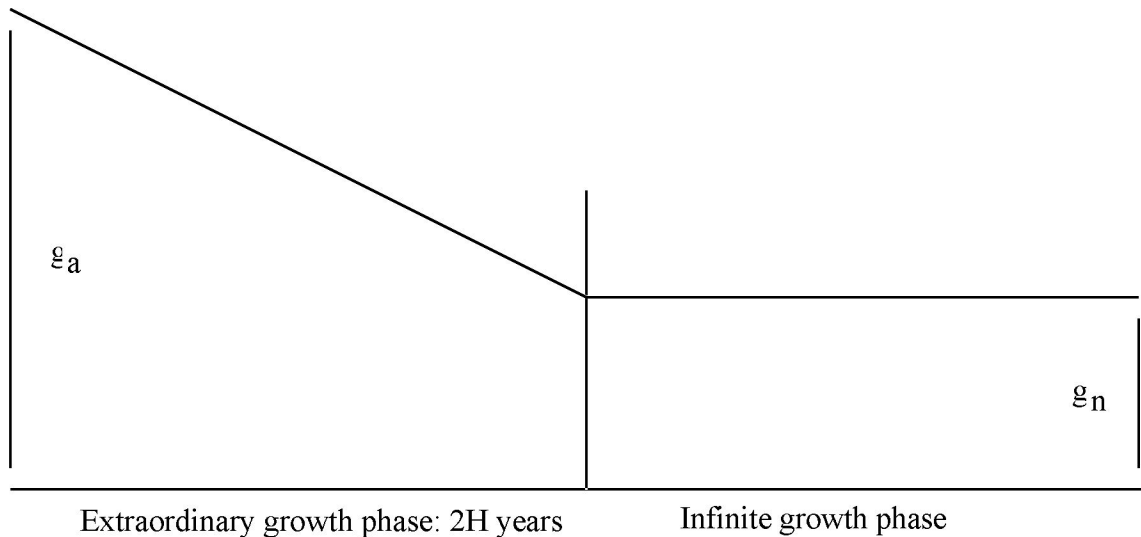
III. The H Model for valuing Growth

The H model is a two-stage model for growth, but unlike the classical two-stage model, the growth rate in the initial growth phase is not constant but declines linearly over time to reach the stable growth rate in steady stage. This model was presented in Fuller and Hsia (1984).

The Model

The model is based upon the assumption that the earnings growth rate starts at a high initial rate (g_a) and declines linearly over the extraordinary growth period (which is assumed to last $2H$ periods) to a stable growth rate (g_n). It also assumes that the dividend payout and cost of equity are constant over time and are not affected by the shifting growth rates. Figure 13.4 graphs the expected growth over time in the H Model.

Figure 13.4: Expected Growth in the H Model



The value of expected dividends in the H Model can be written as:

$$P_0 = \frac{DPS_0 * (1+g_n)}{(k_e - g_n)} + \frac{DPS_0 * H * (g_a - g_n)}{(k_e - g_n)}$$

Stable growth

Extraordinary growth

where,

 P_0 = Value of the firm now per share, DPS_t = DPS in year t k_e = Cost of equity g_a = Growth rate initially g_n = Growth rate at end of $2H$ years, applies forever afterwards*Limitations*

This model avoids the problems associated with the growth rate dropping precipitously from the high growth to the stable growth phase, but it does so at a cost. First, the decline in the growth rate is expected to follow the strict structure laid out in the model -- it drops in linear increments each year based upon the initial growth rate, the stable growth rate and the length of the extraordinary growth period. While small deviations from this assumption do not affect the value significantly, large deviations can cause problems. Second, the assumption that the payout ratio is constant through both phases of growth exposes the analyst to an inconsistency -- as growth rates decline the payout ratio usually increases.

Works best for:

The allowance for a gradual decrease in growth rates over time may make this a useful model for firms which are growing rapidly right now, but where the growth is expected to decline gradually over time as the firms get larger and the differential advantage they have over their competitors declines. The assumption that the payout ratio is constant, however, makes this an inappropriate model to use for any firm that has low or no dividends currently. Thus, the model, by requiring a combination of high growth and high payout, may be quite limited⁴ in its applicability.

Illustration 13.7: Valuing with the H model: Alcatel

Alcatel is a French telecommunications firm, paid dividends per share of 0.72 Ffr on earnings per share of 1.25 Ffr in 2000. The firm's earnings per share had grown at 12% over the prior 5 years but the growth rate is expected to decline linearly over the next 10 years to 5%, while the payout ratio remains unchanged. The beta for the stock is 0.8, the riskfree rate is 5.1% and the market risk premium is 4%.

⁴ Proponents of the model would argue that using a steady state payout ratio for firms which pay little or no dividends is likely to cause only small errors in the valuation.

Cost of equity = $5.1\% + 0.8 \times 4\% = 8.30\%$

The stock can be valued using the H model:

$$\text{Value of stable growth} = \frac{(0.72)(1.05)}{0.083 - 0.05} = \$22.91$$

$$\text{Value of extraordinary growth} = \frac{(0.72)(10/2)(0.12 - 0.05)}{0.083 - 0.05} = 7.64$$

$$\text{Value of stock} = 22.91 + 7.64 = 30.55$$

The stock was trading at 33.40 Ffr in May 2001.

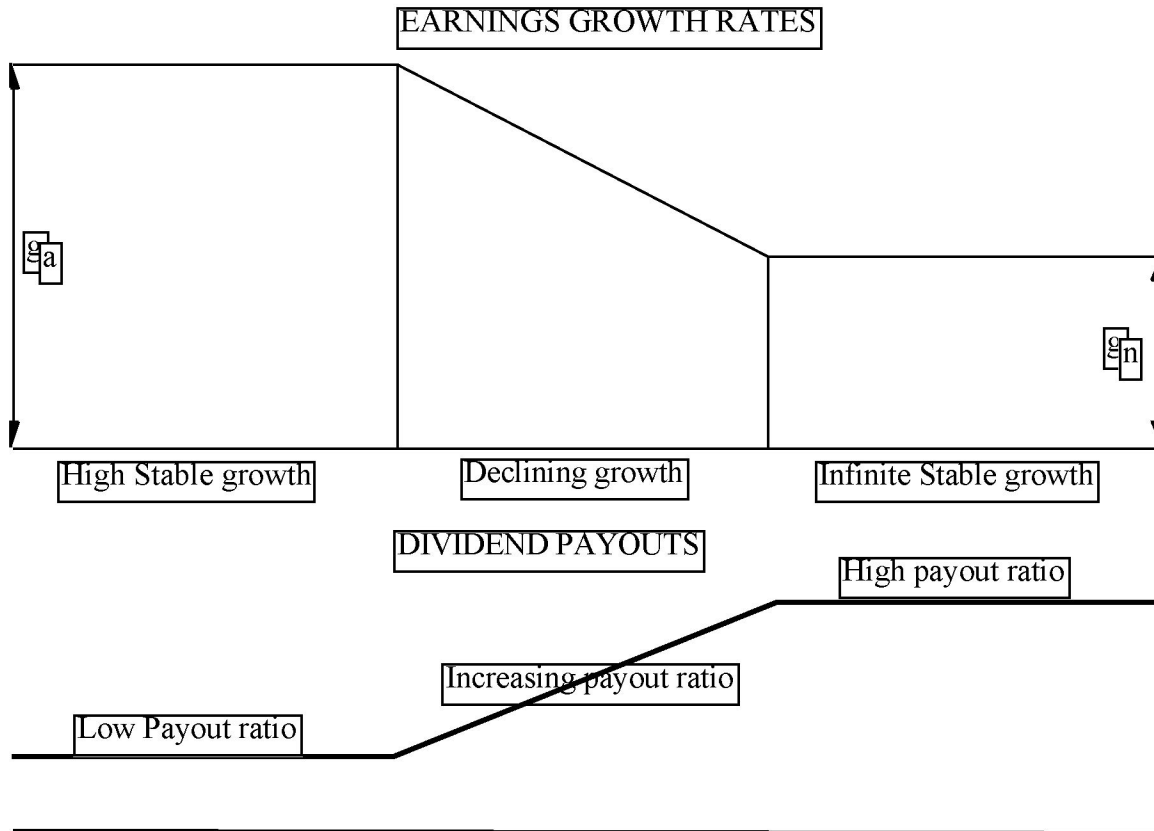
IV. Three-stage Dividend Discount Model

The three-stage dividend discount model combines the features of the two-stage model and the H-model. It allows for an initial period of high growth, a transitional period where growth declines and a final stable growth phase. It is the most general of the models because it does not impose any restrictions on the payout ratio.

The Model

This model assumes an initial period of stable high growth, a second period of declining growth and a third period of stable low growth that lasts forever. Figure 13.5 graphs the expected growth over the three time periods.

Figure 13.5: Expected Growth in the Three-Stage DDM



The value of the stock is then the present value of expected dividends during the high growth and the transitional periods and of the terminal price at the start of the final stable growth phase.

$$P_0 = \underbrace{\sum_{t=1}^{t=n1} \frac{EPS_0 * (1+g_a)^t * \Pi_a}{(1+k_{e,hg})^t}}_{\text{High growth phase}} + \underbrace{\sum_{t=n1+1}^{t=n2} \frac{DPS_t}{(1+k_{e,t})^t}}_{\text{Transition}} + \underbrace{\frac{EPS_{n2} * (1+g_n) * \Pi_n}{(k_{e,st} - g_n)(1+r)^n}}_{\text{Stable growth phase}}$$

where,

EPS_t = Earnings per share in year t

DPS_t = Dividends per share in year t

g_a = Growth rate in high growth phase (lasts $n1$ periods)

g_n = Growth rate in stable phase

Π_a = Payout ratio in high growth phase

Π_n = Payout ratio in stable growth phase

k_e = Cost of equity in high growth (hg), transition (t) and stable growth (st)

Assumptions

This model removes many of the constraints imposed by other versions of the dividend discount model. In return, however, it requires a much larger number of inputs - year-specific payout ratios, growth rates and betas. For firms where there is substantial noise in the estimation process, the errors in these inputs can overwhelm any benefits that accrue from the additional flexibility in the model.

Works best for:

This model's flexibility makes it a useful model for any firm, which in addition to changing growth over time is expected to change on other dimensions as well - in particular, payout policies and risk. It is best suited for firms which are growing at an extraordinary rate now and are expected to maintain this rate for an initial period, after which the differential advantage of the firm is expected to deplete leading to gradual declines in the growth rate to a stable growth rate. Practically speaking, this may be the more appropriate model to use for a firm whose earnings are growing at very high rates⁵, are expected to continue growing at those rates for an initial period, but are expected to start declining gradually towards a stable rate as the firm become larger and loses its competitive advantages.

Illustration 13.8: Valuing with the Three-stage DDM model: Coca Cola

Coca Cola, the owner of the most valuable brand name in the world according to Interbrand, was able to increase its market value ten-fold in the 1980s and 1990s. While growth has leveled off in the last few years, the firm is still expanding both into other products and other markets.

A Rationale for using the Three-Stage Dividend Discount Model

- *Why three-stage?* Coca Cola is still in high growth, but its size and dominant market share will cause growth to slide in the second phase of the high growth period. The high growth period is expected to last 5 years and the transition period is expected to last an additional 5 years.
- *Why dividends?* The firm has had a track record of paying out large dividends to its stockholders, and these dividends tend to mirror free cash flows to equity.
- The financial leverage is stable.

Background Information

- Current Earnings / Dividends
 - Earnings per share in 2000 = \$1.56

⁵ The definition of a 'very high' growth rate is largely subjective. As a rule of thumb, growth rates over 25% would qualify as very high when the stable growth rate is 6-8%.

- Dividends per share in 2000 = \$0.69
- Payout ratio in 2000 = 44.23%
- Return on Equity = 23.37%

Estimate

a. Cost of Equity

We will begin by estimating the cost of equity during the high growth phase, expected. We use a bottom-up levered beta of 0.80 and a riskfree rate of 5.4%. We use a risk premium of 5.6%, significantly higher than the mature market premium of 4%, which we have used in the valuation so far, to reflect Coca Cola's exposure in Latin America, Eastern Europe and Asia. The cost of equity can then be estimated for the high growth period.

$$\text{Cost of equity}_{\text{high growth}} = 5.4\% + 0.8 (5.6\%) = 9.88\%$$

In stable growth, we assume that the beta will remain 0.80, but reduce the risk premium to 5% to reflect the expected maturing of many emerging markets.

$$\text{Cost of equity}_{\text{stable growth}} = 5.4\% + 0.8 (5.0\%) = 9.40\%$$

During the transition period, the cost of equity will linearly decline from 9.88% in year 5 to 9.40% in year 10.

b. Expected Growth and Payout Ratios

The expected growth rate during the high growth phase is estimated using the current return on equity of 23.37% and payout ratio of 44.23%.

$$\text{Expected growth rate} = \text{Retention ratio} * \text{Return on equity} = (1-0.4423)(0.2337) = 13.03\%$$

During the transition phase, the expected growth rate declines linearly from 13.03% to a stable growth rate of 5.5%. To estimate the payout ratio in stable growth, we assume a return on equity of 20% for the firm:

$$\text{Stable period payout ratio} = 1 - \frac{g}{\text{ROE}} = 1 - \frac{5.5\%}{20\%} = 72.5\%$$

During the transition phase, the payout ratio adjusts upwards from 44.23% to 72.5% in linear increments.

Estimating the Value

These inputs are used to estimate expected earnings per share, dividends per share and costs of equity for the high growth, transition and stable periods. The present values are also shown in the last column table 13.3.

Table 13.3: Expected EPS, DPS and Present Value: Coca Cola

<i>Year</i>	<i>Expected Growth</i>	<i>EPS</i>	<i>Payout ratio</i>	<i>DPS</i>	<i>Cost of Equity</i>	<i>Present Value</i>
-------------	------------------------	------------	---------------------	------------	-----------------------	----------------------

High Growth Stage						
1	13.03%	\$1.76	44.23%	\$0.78	9.88%	\$0.71
2	13.03%	\$1.99	44.23%	\$0.88	9.88%	\$0.73
3	13.03%	\$2.25	44.23%	\$1.00	9.88%	\$0.75
4	13.03%	\$2.55	44.23%	\$1.13	9.88%	\$0.77
5	13.03%	\$2.88	44.23%	\$1.27	9.88%	\$0.79
Transition Stage						
6	11.52%	\$3.21	49.88%	\$1.60	9.78%	\$0.91
7	10.02%	\$3.53	55.54%	\$1.96	9.69%	\$1.02
8	8.51%	\$3.83	61.19%	\$2.34	9.59%	\$1.11
9	7.01%	\$4.10	66.85%	\$2.74	9.50%	\$1.18
10	5.50%	\$4.33	72.50%	\$3.14	9.40%	\$1.24

(Note: Since the costs of equity change each year, the present value has to be calculated using the cumulated cost of equity. Thus, in year 7, the present value of dividends is:

$$\text{PV of year 7 dividend} = \frac{\$1.96}{(1.0988)^5 (1.0978) (1.0969)} = \$1.02$$

The terminal price at the end of year 10 can be calculated based upon the earnings per share in year 11, the stable growth rate of 5%, a cost of equity of 9.40% and the payout ratio of 72.5% -

$$\text{Terminal price} = \frac{\$4.33(1.055)(0.725)}{0.094 - 0.055} = \$84.83$$

The components of value are as follows:

Present Value of dividends in high growth phase: \$ 3.76

Present Value of dividends in transition phase: \$ 5.46

Present Value of terminal price at end of transition: \$ 33.50

Value of Coca Cola Stock : \$ 42.72

Coca Cola was trading at \$46.29 in May 21, 2001.



.DDM3st.xls: This spreadsheet allows you to value a firm with a period of high growth followed by a transition period where growth declines to a stable growth rate.

What is wrong with this model? (3 stage DDM)

If this is your problem

this may

- If you are getting too low a value from this model,
 - the stable period payout ratio is too low for a stable firm ($< 40\%$)
If using fundamental data
If entering direct data
Use a beta closer to 1
 - the beta in the stable period is too high for a stable firm
Use a beta closer to 1
- If you get an extremely high value,
 - the growth rate in the stable growth period is too high for stable firm
Use a growth rate closer to 0
 - the period of growth (high + transition) is too high
Use shorter high growth period

Issues in using the Dividend Discount Model

The dividend discount model's primary attraction is its simplicity and its intuitive logic. There are many analysts, however, who view its results with suspicion because of limitations that they perceive it to possess. The model, they claim, is not really useful in valuation, except for a limited number of stable, high-dividend paying stocks. This section examines some of the areas where the dividend discount model is perceived to fall short.

(a) Valuing non-dividend paying or low dividend paying stocks

The conventional wisdom is that the dividend discount model cannot be used to value a stock that pays low or no dividends. It is wrong. If the dividend payout ratio is adjusted to reflect changes in the expected growth rate, a reasonable value can be obtained even for non-dividend paying firms. Thus, a high-growth firm, paying no dividends currently, can still be valued based upon dividends that it is expected to pay out when the growth rate declines. If the payout ratio is not adjusted to reflect changes in the growth rate, however, the dividend discount model will underestimate the value of non-dividend paying or low-dividend paying stocks.

(b) Is the model too conservative in estimating value?

A standard critique of the dividend discount model is that it provides too conservative an estimate of value. This criticism is predicated on the notion that the value is determined by more than the present value of expected dividends. For instance, it is argued that the dividend discount model does not reflect the value of 'unutilized assets'. There is no reason, however, that these unutilized assets cannot be valued separately and added on to the value from the dividend discount model. Some of the assets that are supposedly ignored by the dividend discount model, such as the value of brand names, can be dealt with simply within the context of the model.

A more legitimate criticism of the model is that it does not incorporate other ways of returning cash to stockholders (such as stock buybacks). If you use the modified version of the dividend discount model, this criticism can also be countered.

(c) The contrarian nature of the model

The dividend discount model is also considered by many to be a contrarian model. As the market rises, fewer and fewer stocks, they argue, will be found to be undervalued using the dividend discount model. This is not necessarily true. If the market increase is due to an improvement in economic fundamentals, such as higher expected growth in the economy and/or lower interest rates, there is no reason, a priori, to believe that the values

from the dividend discount model will not increase by an equivalent amount. If the market increase is not due to fundamentals, the dividend discount model values will not follow suit, but that is more a sign of strength than weakness. The model is signaling that the market is overvalued relative to dividends and cashflows and the cautious investor will pay heed.

Tests of the Dividend Discount Model

The ultimate test of a model lies in how well it works at identifying undervalued and overvalued stocks. The dividend discount model has been tested and the results indicate that it does, in the long term, provide for excess returns. It is unclear, however, whether this is because the model is good at finding undervalued stocks or because it proxies for well-known empirical irregularities in returns relating to price-earnings ratios and dividend yields.

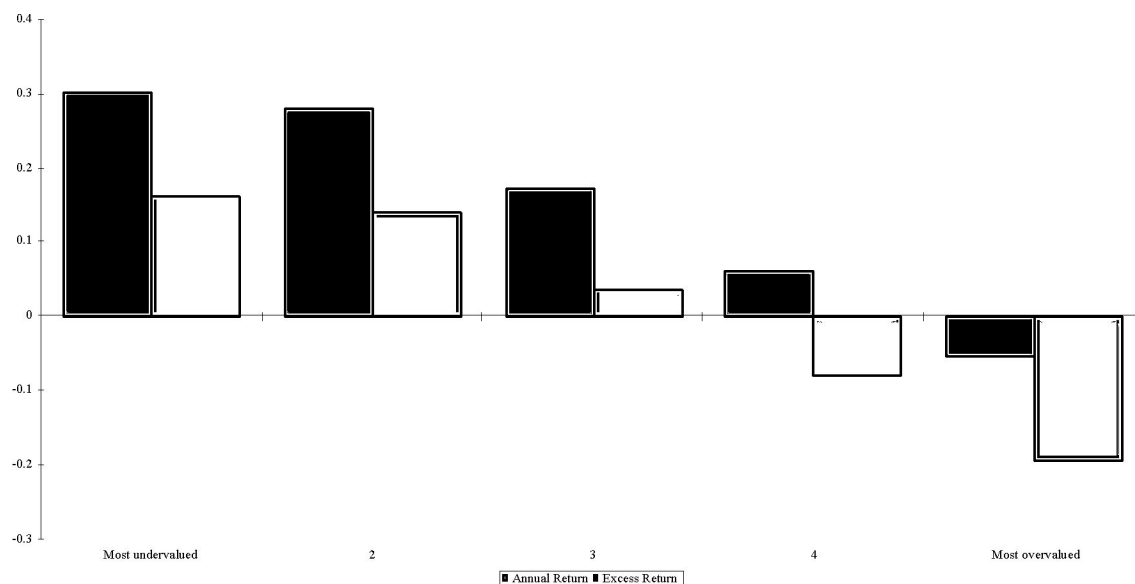
A Simple Test of the Dividend Discount model

A simple study of the dividend discount model was conducted by Sorensen and Williamson, where they valued 150 stocks from the S&P 400 in December 1980, using the dividend discount model. They used the difference between the market price at that time and the model value to form five portfolios based upon the degree of under or over valuation. They made fairly broad assumptions in using the dividend discount model.

- (a) The average of the earnings per share between 1976 and 1980 was used as the current earnings per share.
- (b) The cost of equity was estimated using the CAPM.
- (c) The extraordinary growth period was assumed to be five years for all stocks and the I/B/E/S consensus forecast of earnings growth was used as the growth rate for this period.
- (d) The stable growth rate, after the extraordinary growth period, was assumed to be 8% for all stocks.
- (e) The payout ratio was assumed to be 45% for all stocks.

The returns on these five portfolios were estimated for the following two years (January 1981-January 1983) and excess returns were estimated relative to the S&P 500 Index using the betas estimated at the first stage and the CAPM. Figure 13.6 illustrates the excess returns earned by the portfolio that was undervalued by the dividend discount model relative to both the market and the overvalued portfolio.

Figure 13.6 Performance of the Dividend Discount Model: 1981-83



The undervalued portfolio had a positive excess return of 16% per annum between 1981 and 1983, while the overvalued portfolio had a negative excess return of 15% per annum during the same time period. Other studies which focus only on the dividend discount model come to similar conclusions. In the long term, undervalued (overvalued) stocks from the dividend discount model outperform (under perform) the market index on a risk adjusted basis.

Caveats on the use of the dividend discount model

The dividend discount model provides impressive results in the long term. There are, however, three considerations in generalizing the findings from these studies.

The dividend discount model does not beat the market every year

The dividend discount model outperforms the market over five-year time periods, but there have been individual years where the model has significantly under performed the market. Haugen reports on the results of a fund that used the dividend discount model to analyze 250 large capitalization firms and to classify them into five quintiles from the first quarter of 1979 to the last quarter of 1991. The betas of these quintiles were roughly equal. The valuation was done by six analysts who estimated an extraordinary growth rate for the initial high growth phase, the length of the high growth phase and a transitional phase for each of the firms. The returns on the five portfolios as well as the returns on all 250 stocks and the S&P 500 from 1979 to 1991 are reported in Table 13.4.

Table 13.4: Returns on Quintiles: Dividend Discount Model

Quintile							
	Under Valued	2	3	4	Over Valued	250 Stocks	S&P 500
1979	35.07%	25.92%	18.49%	17.55%	20.06%	23.21%	18.57%
1980	41.21%	29.19%	27.41%	38.43%	26.44%	31.86%	32.55%
1981	12.12%	10.89%	1.25%	-5.59%	-8.51%	28.41%	24.55%
1982	19.12%	12.81%	26.72%	28.41%	35.54%	24.53%	21.61%
1983	34.18%	21.27%	25.00%	24.55%	14.35%	24.10%	22.54%
1984	15.26%	5.50%	6.03%	-4.20%	-7.84%	3.24%	6.12%
1985	38.91%	32.22%	35.83%	29.29%	23.43%	33.80%	31.59%
1986	14.33%	11.87%	19.49%	12.00%	20.82%	15.78%	18.47%
1987	0.42%	4.34%	8.15%	4.64%	-2.41%	2.71%	5.23%
1988	39.61%	31.31%	17.78%	8.18%	6.76%	20.62%	16.48%
1989	26.36%	23.54%	30.76%	32.60%	35.07%	29.33%	31.49%
1990	-17.32%	-8.12%	-5.81%	2.09%	-2.65%	-6.18%	-3.17%
1991	47.68%	26.34%	33.38%	34.91%	31.64%	34.34%	30.57%
1979-91	1253%	657%	772%	605%	434%	722%	654%

The undervalued portfolio earned significantly higher returns than the overvalued portfolio and the S&P 500 for the 1979-91 period, but it under performed the market in five of the twelve years and the overvalued portfolio in four of the twelve years.

Is the model just a proxy for low PE ratios and dividend yields?

The dividend discount model weights expected earnings and dividends in near periods more than earnings and dividends in far periods., It is biased towards finding low price-earnings ratio stocks with high dividend yields to be undervalued and high price-earnings ratio stocks with low or no dividend yields to be overvalued. Studies of market efficiency indicate that low PE ratio stocks have outperformed (in terms of excess returns) high PE ratio stocks over extended time periods. Similar conclusions have been drawn about high-dividend yield stocks relative to low-dividend yield stocks. Thus, the valuation findings of the model are consistent with empirical irregularities observed in the market.

It is unclear how much the model adds in value to investment strategies that use PE ratios or dividend yields to screen stocks. Jacobs and Levy (1988b) indicate that the marginal gain is relatively small.

Attribute

Average Excess Return per Quarter: 1982-87

Dividend Discount Model	0.06% per quarter
Low P/E Ratio	0.92% per quarter
Book/Price Ratio	0.01% per quarter
Cashflow/Price	0.18% per quarter
Sales/Price	0.96% per quarter
Dividend Yield	-0.51% per quarter

This suggests that using low PE ratios to pick stocks adds 0.92% to your quarterly returns, whereas using the dividend discount model adds only a further 0.06% to quarterly returns. If, in fact, the gain from using the dividend discount model is that small, screening stocks on the basis of observables (such as PE ratio or cashflow measures) may provide a much larger benefit in terms of excess returns.

The tax disadvantages from high dividend stocks

Portfolios created with the dividend discount model are generally characterized by high dividend yield, which can create a tax disadvantage if dividends are taxed at a rate greater than capital gains or if there is a substantial tax timing⁶ liability associated with dividends. Since the excess returns uncovered in the studies presented above are pre-tax to the investor, the introduction of personal taxes may significantly reduce or even eliminate these excess returns.

In summary, the dividend discount model's impressive results in studies looking at past data have to be considered with caution. For a tax-exempt investment, with a long time horizon, the dividend discount model is a good tool, though it may not be the only one, to pick stocks. For a taxable investor, the benefits are murkier, since the tax consequences of the strategy have to be considered. For investors with shorter time horizons, the dividend discount model may not deliver on its promised excess returns, because of the year-to-year volatility in its performance.

Conclusion

When you buy stock in a publicly traded firm, the only cash flow you receive directly from this investment are expected dividends. The dividend discount model builds on this simple proposition and argues that the value of a stock then has to be the present value of expected dividends over time. Dividend discount models can range from simple growing perpetuity models such as the Gordon Growth model, where a stock's value is a function of

⁶ Investors do not have a choice of when they receive dividends, whereas they have a choice on the timing of capital gains.

its expected dividends next year, the cost of equity and the stable growth rate, to complex three stage models, where payout ratios and growth rates change over time.

While the dividend discount model is often criticized as being of limited value, it has proven to be surprisingly adaptable and useful in a wide range of circumstances. It may be a conservative model that finds fewer and fewer undervalued firms as market prices rise relative to fundamentals (earnings, dividends, etc.) but that can also be viewed as a strength. Tests of the model also seem to indicate its usefulness in gauging value, though much of its effectiveness may be derived from its finding low PE ratio, high dividend yield stocks to be undervalued.

Problems

1. Respond true or false to the following statements relating to the dividend discount model:
 - A. The dividend discount model cannot be used to value a high growth company that pays no dividends.
 - B. The dividend discount model will undervalue stocks, because it is too conservative.
 - C. The dividend discount model will find more undervalued stocks, when the overall stock market is depressed.
 - D. Stocks that are undervalued using the dividend discount model have generally made significant positive excess returns over long time periods (five years or more).
 - E. Stocks which pay high dividends and have low price-earnings ratios are more likely to come out as undervalued using the dividend discount model.

2. Ameritech Corporation paid dividends per share of \$3.56 in 1992 and dividends are expected to grow 5.5% a year forever. The stock has a beta of 0.90 and the treasury bond rate is 6.25%.
 - a. What is the value per share, using the Gordon Growth Model?
 - b. The stock was trading for \$80 per share. What would the growth rate in dividends have to be to justify this price?

3. Church & Dwight, a large producer of sodium bicarbonate, reported earnings per share of \$1.50 in 1993 and paid dividends per share of \$0.42. In 1993, the firm also reported the following:

Net Income = \$30 million

Interest Expense = \$0.8 million

Book Value of Debt = \$7.6 million

Book Value of Equity = \$160 million

The firm faced a corporate tax rate of 38.5%. (The market value debt to equity ratio is 5%.) The treasury bond rate is 7%.

The firm expected to maintain these financial fundamentals from 1994 to 1998, after which it was expected to become a stable firm with an earnings growth rate of 6%. The firm's financial characteristics were expected to approach industry averages after 1998. The industry averages were as follows:

Return on Capital = 12.5%

Debt/Equity Ratio = 25%

Interest Rate on Debt = 7%

Church and Dwight had a beta of 0.85 in 1993 and the unlevered beta was not expected to change over time.

- a. What is the expected growth rate in earnings, based upon fundamentals, for the high-growth period (1994 to 1998)?
 - b. What is the expected payout ratio after 1998?
 - c. What is the expected beta after 1998?
 - d. What is the expected price at the end of 1998?
 - e. What is the value of the stock, using the two-stage dividend discount model?
 - f. How much of this value can be attributed to extraordinary growth? to stable growth?
4. Oneida Inc, the world's largest producer of stainless steel and silverplated flatware, reported earnings per share of \$0.80 in 1993 and paid dividends per share of \$0.48 in that year. The firm was expected to report earnings growth of 25% in 1994, after which the growth rate was expected to decline linearly over the following six years to 7% in 1999. The stock was expected to have a beta of 0.85. (The treasury bond rate was 6.25%)
- a. Estimate the value of stable growth, using the H Model.
 - b. Estimate the value of extraordinary growth, using the H Model.
 - c. What are the assumptions about dividend payout in the H Model?
5. Medtronic Inc., the world's largest manufacturer of implantable biomedical devices, reported earnings per share in 1993 of \$3.95 and paid dividends per share of \$0.68. Its earnings were expected to grow 16% from 1994 to 1998, but the growth rate was expected to decline each year after that to a stable growth rate of 6% in 2003. The payout ratio was expected to remain unchanged from 1994 to 1998, after which it would increase each year to reach 60% in steady state. The stock was expected to have a beta of 1.25 from 1994 to 1998, after which the beta would decline each year to reach 1.00 by the time the firm becomes stable. (The treasury bond rate was 6.25%)
- a. Assuming that the growth rate declines linearly (and the payout ratio increases linearly) from 1999 to 2003, estimate the dividends per share each year from 1994 to 2003.
 - b. Estimate the expected price at the end of 2003.
 - c. Estimate the value per share, using the three-stage dividend discount model.

Musings on Markets

My not-so-profound thoughts about valuation, corporate finance and the news of the day!

Saturday, April 11, 2015

The Small Cap Premium: Where is the beef?

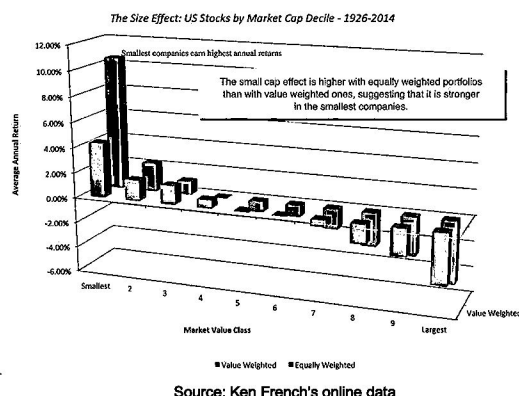
For decades, analysts and investor have bought into the idea of a small cap premium, i.e., that stocks with low market capitalizations can be expected to earn higher returns than stocks with higher market capitalizations. For investors, this has led to the pursuit of small cap stocks and funds for their portfolios, and for analysts, it has translated into the addition of "small cap" premiums of between 3-5% to traditional model-based expected returns, for companies that they classify as small cap. While I understand the origins of the practice, I question the adjustment for three reasons:

1. On closer scrutiny, the historical data, which has been used as the basis of the argument, is yielding more ambiguous results and leading us to question the original judgment that there is a small cap premium.
2. The forward-looking risk premiums, where we look at the market pricing of stocks to get a measure of what investors are demanding as expected returns, are yielding no premiums for small cap stocks.
3. If the justification is intuitive, i.e., that smaller firms are riskier than larger firms, much of that additional risk is either diversifiable, better adjusted for in the expected cash flows (instead of the discount rate) or double counted.

The small cap premium is a testimonial to the power of inertia in corporate finance and valuation, where once a practice becomes established, it becomes difficult to challenge, even if the original reasons for it have long since disappeared.

The Basis

The first studies that uncovered the phenomenon of the small cap premium came out in the 1970s. They broke companies down into deciles, based on market capitalization, and found that companies in the lowest decile earned higher returns, after adjusting for conventional risk measures, than companies in the highest decile. I updated those studies through the end of 2014, and the small cap premium seems intact (at least at first sight). In summary, looking at returns from 1926 to 2014, the smallest cap stocks (in the lowest decile) earned 4.33% more than the market, after adjusting for risk.



This is the strongest (and perhaps) only evidence for a small cap premium and it is reproduced in data services that try to estimate historical risk premiums (Ibbotson, Duff and Phelps etc.). This historical premium has become the foundation for both valuation and investment practice. In valuation, analysts have referenced this table to estimate a small cap premium (4-5%) that they then add to the required return from conventional risk and return models to estimate discount rates. For instance, in the conventional capital asset pricing model, it plays out as follows:

$$\text{Expected Return} = \text{Risk free rate} + \text{Beta} * \text{Equity Risk Premium} + \text{Small Cap Premium}$$

That discount rate is used to estimate the value of future cash flows, and not surprisingly, the use of a small cap premium lowers the value of smaller companies.

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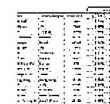
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I get a sense that I am approaching the end of this series of weekly posts, or perhaps I am just hoping that it is true, as the COVI...



A Viral Market Meltdown VI: The Price of Risk
It is a sign of how volatile the last few weeks have been, that a week like the last one, where index levels move only 2-3% a day, high b...



A Viral Market Meltdown V: Back to Basics!
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A Viral Market Meltdown IV: Investing for a post-virus Economy
At the end of each of the weeks leading into the last one, I have done a market update, reflecting the changes that occurred in the week, ...

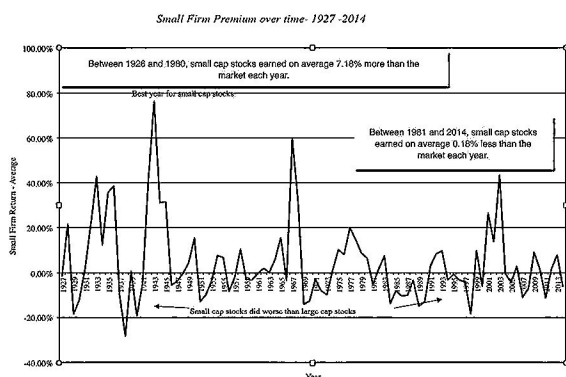
A Viral Market Meltdown III: Pricing or Value? Trading or Investing?

In investing, it has been used as a weapon both for and against active investing. Those who favor active investing have pointed to the small cap premium as a justification for their activity, and during the periods of history when small cap companies outperformed the market, it did make them look like heroes but it quickly gave rise to a counterforce, where performance measurement services (like Morningstar) started incorporating portfolio tilts, comparing small cap funds against small cap indices. Since almost all of the "excess returns" disappeared on this comparison, it was only a matter of time before index funds entered the arena, creating small-cap index funds for investors who wanted to claim the premium, without paying large management fees.

The Problem with the Historical Premium

In the decades since the original small cap premium study, the data on stocks has become richer and deeper, allowing us to take a closer look at the phenomenon. There are some serious questions that can be raised about whether the premium exists and if so, what exactly it is measuring:

1. **Trend lines and Time Periods:** Small cap stocks have earned higher returns than large cap stocks between 1928 and 2014 but the premium has been volatile over history, disappearing for decades and reappearing again. While the premium was strong prior to 1980, it seems to have dissipated since 1981. One reason may be that the small cap premium studies drew attention and investor money to small cap stocks, and in the process led to a repricing of these stocks. Another is that the small cap premium is a side effect of larger macroeconomic variables (inflation, real growth etc.) and that the behavior of those variables has changed since 1980.



Source: Ken French's online data

2. **Microcap, not small cap premium:** Even over the long time period that provides the strongest support for existence of a small cap premium, one study finds that removing stocks with less than \$5 million in market cap causes the small firm effect to vanish. In effect, what you have is microcap premium, isolated in the smallest of stocks, not just small stocks.
3. **Standard Error:** Historical equity returns are noisy and any estimates of risk premium from that data will reflect the noise in the form of large standard errors on estimates. I have made this point about the overall historical equity risk premium but it becomes magnified when you dice and slice historical data into sub-classes. The table below lists standard errors in excess returns by decile class and reinforce the notion that the small cap premium is fragile, barely making the threshold for statistical significance over the entire period.

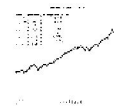
Decile	Average	Standard Error	Maximum	Minimum
Smallest	4.33%	1.96%	76.28%	-28.42%
2	1.63%	1.14%	41.25%	-17.96%
3	1.47%	0.77%	41.98%	-13.54%
4	0.64%	0.55%	15.56%	-7.33%
5	0.05%	0.53%	11.63%	-16.05%
6	-0.01%	0.51%	15.21%	-14.01%
7	-0.51%	0.55%	7.48%	-19.50%
8	-1.50%	0.81%	11.20%	-29.42%
9	-2.13%	1.02%	21.96%	-36.09%
Largest	-3.98%	1.56%	31.29%	-65.57%

Source: Ken French's online data

4. **The January Effect:** One of the most puzzling aspects of the small cap premium is that almost all of it is earned in one month of the year, January, and removing that month makes it disappear. So what? If your argument for the small cap premium is that small cap stocks are riskier, you now have the onus of explaining why that risk shows up only in the first month of every year.



This is the third, and I hope the last, of my viral market updates, reflecting how much change a week can deliver, and last week delivered...



A Viral Market Meltdown: Fear or Fundamentals?
It has become almost a rite of passage for investors, at least since 2008, that they will be tested by a market crisis precipitated some...



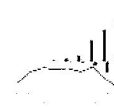
A Do-it-yourself (DIY) Valuation of Tesla: Of Investment Regrets and Disagreements!
I was hoping to move on from Tesla to my data update posts, but my last post on Tesla drew some attention, in good and bad ways, partly be...



An Ode to Luck: Revisiting my Tesla Valuation
When investing, I am often my own biggest adversary, handicapped by the preconceptions and priors that I bring into analysis and decision ...



A Viral Market Meltdown Part II: Clues in the Debris!
Update on 3.9/20: In a sign of how volatile times are, over the weekend, oil prices plummeted to close to \$30, the treasury bond rate to I...



Walmart's India (Flipkart) Gambit: Growth Rebirth or Costly Facelift?
On May 9, 2018, Walmart confirmed officially what had been rumored for weeks, and announced that it would pay \$16 billion to ac...

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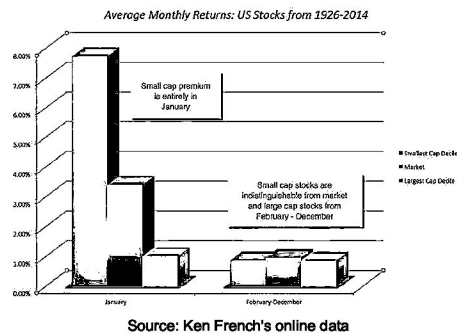
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- 2009 (60)
- 2008 (42)

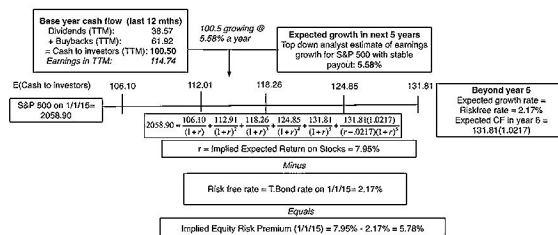
5. Weaker globally: The small cap premium seems to be smaller in non-US markets than in US markets and is non-existent in some. In contrast, the value effect (where low price to book stocks outperform the market) is strong globally.
6. Proxy for other factors: A host of papers argue that the bulk or all of the small size effect can be attributed to a liquidity effect and that putting in a proxy for illiquidity makes the size effect disappear or diminishes it.
7. Works only with market cap: Finally, you can take issue with the use of a market-priced based measure of size in a study of returns. Others have tried other non-price size measures such as income or revenues but there seems to be no size effect in those variables.

A recent working paper by Asness, Frazini, Israel, Moskowitz and Pedersen tries to resurrect the size effect, but accomplishes it only by removing the subset of small companies that they classify as "low quality" or "junk". While the results are interesting and can be used by active small-cap fund managers as a justification for their activity, they are in no way a basis for adding a small cap premium to every small company, and asking analysts to add it on only for small, high quality companies is problematic. In summary, if the only justification that you can offer for the addition of a small cap premium to your discount rate is the historical risk premium, you are on thin ice.

Market-Implied Small Cap Premium

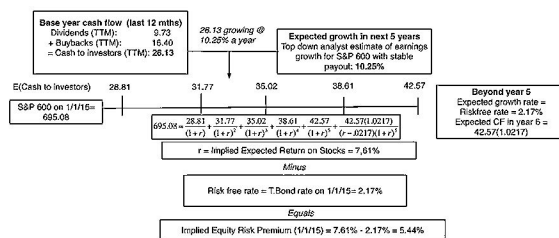
If the historical data ceases to support the use of a historical risk premium, can we then draw on intuition and argue that since small companies tend to be riskier (or we perceive them to be), investors must require higher return when they invest in them? You can, but the onus is then on you to back up that intuition. In fact, you can check to see whether investors are demanding a forward looking "small cap" premium, by looking at how they price small as opposed to large companies, and backing out what investors are demanding as expected returns. Put simply, if small cap stocks are viewed by investors as riskier and that risk is being priced in, you should expect to see, other things remaining equal, higher expected returns on small cap stocks than large cap stocks.

As some of you are aware, I compute a forward-looking equity premium for the S&P 500 at the start of each year, backing out the number from the current level of the index and expected cash flows. On January 1, 2015, this is what I found:



In effect, to the extent that my base year cash flows are reasonable and my expected growth rate reflects market expectations, the expected return on large cap stocks on January 1, 2015 was 7.95% in the US (yielding an overall equity risk premium of 5.78% on that day).

To get a measure of the forward-looking small cap premium, I computed the expected return implied in the S&P 600 Small Cap Index, using the same approach that I used for S&P 500. In spite of using a higher expected earnings growth for small cap stocks, the expected return that I estimate is only 7.61%:



In effect, the market is attaching a smaller expected return for small cap stocks than large ones, stories and intuition notwithstanding.

I am not surprised that the market does not seem to buy into the small cap premiums that academics and practitioners are so attached to. After all, if the proponents of small cap premiums are right, bundling together small companies into a larger company should instantly generate a bonus, since you are replacing the much higher required returns of smaller companies with the lower expected return of a larger one. In fact, small companies should disappear from the market.

The Illiquidity Fig Leaf

Looking at the data, the only argument left, as I see it, for the use of the small cap premium is as a premium for illiquidity, and even on that basis, it fails at one of these four levels:

1. *If illiquidity is your bogey man in valuation, why use market capitalization as a stand-in for it?* Market capitalization and illiquidity don't always go hand in hand, since there are small, liquid companies and large, illiquid ones in the market. Four decades ago, your excuse would have been that the data on illiquidity was either inaccessible or unavailable and that market capitalization was the best proxy you could find for illiquidity. That is no longer the case and there are studies that categorize companies based on measures of illiquidity (bid ask spread, trading volume) and find an "illiquidity premium" for illiquid companies.
2. *If illiquidity is what you are adjusting for in the small cap premium, why is it a constant across companies, buyers and time?* Even if your defense is that the small cap premium is an imperfect (but reasonable) measure of the illiquidity premium, it is unreasonable to expect it to be the same for every company. Thus, even if you are valuing just privately owned businesses (where illiquidity is a clear and present danger), that illiquidity should be greater in some businesses than in others and the illiquidity (or small cap) premium should be larger for the former than the latter. Furthermore, the premium you add to the discount rate should be higher in some periods (during market crises and liquidity crunches) than others and for some buyers (cash poor, impatient) than others (patient, cash rich).
3. *Even if you can argue that illiquidity is your rationale for the small cap premium and that it is the same across companies, why is it not changing over the time horizon of your valuation (and especially in your terminal value)?* In any valuation, you assume through your company's cash flows and growth rates that your company will change over time and it is inconsistent (with your own narrative) to lock in an illiquidity premium into your discount rate that does not change as your company does. Thus, if you are using a 30% expected growth rate on your company, your "small" company is getting bigger (at least according to your estimates) and presumably more liquid over time. Should your illiquidity premium therefore not follow your own reasoning and decrease over time?
4. *If your argument is that size is a good proxy for illiquidity, that all small companies are equally illiquid and that that illiquidity does not change as you make them bigger, why are you reducing your end value by an illiquidity discount?* This question is directed at private company appraisers who routinely use small cap premiums to increase discount rates and also reduce the end (DCF) value by 25% or more, because of illiquidity. You can show me data to back up your discount (I have seen restricted stock and IPO studies) but none of them can justify the double counting of illiquidity in valuation.

Why are we slow to give up on the "small cap" premium?

It is true that the small cap premium is established practice at many appraisal firms, investment banks and companies. Given the shaky base on which it is built and how much that base has been chipped away in the last two decades, you would think that analysts would reconsider their use of small cap premiums, but there are three powerful forces that keep it in play.

1. **Intuition:** Analysts and investors not only start of with the presumption that the discount rates for small companies should be higher than large companies, but also have a "number" in mind. When risk and return models deliver a much lower number, the urge to add to it to make it "more reasonable" is almost unstoppable. Consequently, an analyst who arrives at an 8% cost of equity for a small company feels much more comfortable after adding a 5% small cap premium. It is entirely possible that you are an idiot savant with the uncanny capacity to assess the right discount rate for companies, but if that is the case, why go through this charade of using risk and return models and adding premiums to get to your "intuited" discount rate? For most of us, gut feeling and instinct are not good guides to estimating discount rates and here is why. Not all risk is meant for the discount rate, with some risk (like management skills) being diversifiable (and thus lessened in

portfolios) and other risks (like risk of failure or regulatory approval) better reflected in probabilities an expected cash flow. A discount rate cannot and is not meant to be a receptacle for all your hopes and fears, a number that you can tweak until you get to your comfort zone.

2. Inertia (institutional and individual): The strongest force in corporate finance practice is inertia, where much of what companies, investors and analysts do reflects past practice. The same is true in the use of the small cap premium, where a generation of analysts has been brought up to believe (by valuation handbooks and teaching) that it is the right adjustment to make and now do it by rote. That inertia is reinforced in the legal arena (where many valuations end up, either as part of business or tax disputes) by the legal system's respect for precedence and general practice. You may view this as harsh, but I believe that you will have an easier time defending the use of a bad, widely used practice of long standing in court than you would arguing for an innovative better practice.
3. Bias: My experiences with many analysts who use small cap premiums suggest to me that one motive is to get a "lower" value". Why would they want a lower value? First, in accounting and tax valuation, the client that you are doing the valuation for might be made better off with a lower value than a higher one. Consequently, you will do everything you can to pump up the discount rate with the small cap premium being only one of the many premiums that you use to "build up" your cost of capital. Second, there seems to be a (misplaced) belief that it is better to arrive at too low a value than one that is too high. If you buy into this "conservative" valuation approach, you will view adding a small cap premium as costless, since even it does not exist, all you have done is arrived at "too low" a value. At the risk of bringing up the memories of statistics classes past, there is always a cost. While "over estimating" discount rates reduces type 1 errors (that you will buy an over valued stock), it comes at the expense of type 2 errors (that you will hold off on buying an under valued stock).

A Requiem for the Small Cap Premium?

I have never used a small cap premium, when valuing a company and I don't plan to start now. Needless to say, I am often asked to justify my non-use of a premium and here are my reasons. First, I am not convinced by either the historical data or by current market behavior that a small cap premium exists. Second, I do believe that small cap companies are more exposed to some risks than large cap companies but there are other more effective devices to bring these risks into valuation. If it is that they are capital constrained (i.e., that it is more difficult for small companies to raise new capital), I will limit their reinvestment and expected growth (thus lowering value). If it is that they have a greater chance of failure, I will estimate a probability of failure and reflect that in my expected value (as I do in my standard DCF model). If it is illiquidity that is your concern, it is worth recognizing that one size will not fit all and that the effect on value will vary across investors and across time and will be better captured in a discount on value.

To illustrate how distorted this debate has become, note that those who routinely add small cap premiums to their discount rates are not put to the same test of justifying its use. So, at the risk of opening analysts up to uncomfortable questions, here are some questions that you should pose to anyone who is using a small cap premium (and that includes yourself):

1. *What is your justification for using a small cap premium?* If the defense is pointing to history (or a data table in a service), it is paper thin, since that historical premium defense seems to have more holes in it than Swiss cheese. If it is intuitive, i.e., that small companies are riskier and markets must see them as such, I don't see the basis for the intuition, since the implied costs of equity for small companies are no higher than those of large companies. If the argument is that everyone does it, I am sorry but just because something is established practice does not make it right.
2. *What are the additional risks that you see in small companies that you don't see in large ones?* I am sure that you can come up with a laundry list that is a mile long, but most of the risks on the list either don't belong in the discount rate (either because they are diversifiable or because they are discrete risks) or can be captured through probability estimates. If it is illiquidity that you are concerned about, see the section on illiquidity above for my response.

If you are investors, here are the lessons I draw from looking at the data. If you are following a strategy of buying small cap stocks, expecting to be rewarded with a premium for just doing that, you will be disappointed. Even the most favorable papers on the small cap premium suggest that you have to add refinements, with some suggesting that these refinements should screen out the least liquid, riskiest small cap stocks and others arguing for value characteristics (stable earnings, high returns on equity & capital, solid growth). I do think that there is a glimmer of hope in the recent research that the payoff to looking for under valued stocks may be greater with small companies, partly because they are more likely to be overlooked, but it will take more work on your part and it won't be easy!

Data sets

1. Professor Ken French's data library (on small cap stocks)

Spreadsheets

1. Implied equity risk premium (S&P 500)
2. Implied equity risk premium (S&P Small Cap 600)

Posted by Aswath Damodaran at 1:57 PM



Labels: Cost of equity, Discount Rates, Small Cap Premium, Valuation Practice

21 comments:

Max said...

It's worse when you look at the performance of the most widely used small cap index, the Russell 2000. The R2K has underperformed small caps in general. (Why? Perhaps because membership in a major index grants a stock improved liquidity).

April 11, 2015 at 4:38 PM



Unknown said...

When we talk about January effect, we're in effect talking about a 12 month reversal phenomenon. In other words (dogs of the Dow theory), the losers of the preceding year outperform the winners of the preceding year and quite understandably so, the micro cap firms whose market cap had plummeted in the preceding 12 months would be expected to outperform the broad market. Further small cap premium would be expected to be significantly positive in bull markets and significantly negative in bear markets, in other words small cap effect is a function of investor sentiment (risk-on vs. risk-off sentiment). So splitting the sample period (1926-2014) into three periods of bull markets, bear markets and range bound markets would give us some more insights on the small cap premium. Further there is the migration effect i.e. small caps going onto become mid caps due to stock price surge and mid caps becoming small caps due to stock prices plummeting in the preceding years. In other words when we look at decile 10 of the capitalization strata, the character of the firms in the decile 10 is vastly different every year. Further empirical evidence suggests small cap premium is concentrated in few sectors/industries which emerge out of nowhere and become sunrise industries. In other words analyzing small cap effect is vastly complicated and there are too many forces at work!

April 11, 2015 at 10:12 PM



Aswath Damodaran said...

Yogesh,
It takes this much convoluted back tracking for you to try to explain with small cap stocks, there is no small cap effect. It is not complicated. It is just not there.

April 11, 2015 at 10:16 PM

UniverseofRisks said...

In your calculation of the small cap ERP, shouldn't the terminal growth rate be much higher than the risk free rate. By using the same rate as the S&P 500 you're actually calculating the return on equity of companies that begin as small cap but are treated as large caps after year 5. Wouldn't you want your inputs for calculations beyond year 5 to reflect a constant rate of growth for small caps, which to me is much higher than the risk free rate?

April 12, 2015 at 8:41 AM



Aswath Damodaran said...

UniverseofRisks,
You have a good point, though the growth rate you use can only be marginally higher than the growth rate of the economy, since it is a perpetual number. By the same token, I should probably use a slightly lower than the economy growth rate for larger companies.

April 12, 2015 at 9:43 AM



Unknown said...

Playing with the IFA index calculator it seems like small almost always outperforms large:

<https://www.ifa.com/calculator/?i=sv&g=100000&s=1/1/2000&e=1/31/2014&infl=true&af=true&aorw=false&perc=true>

Also see:

<http://www.marketwatch.com/story/the-one-asset-class-every-investor-needs-2014-06-25>

BR Martin

April 13, 2015 at 9:15 AM

Anonymous said...

What if there were an instance where the implied small cap premium were higher than the large cap? Would it be more precise to use the small implied premium?

April 13, 2015 at 4:54 PM

MD said...

In line with comments from UniverseofRisk, I think the growth assumption, and particularly the assumed growth differential between large and small caps, can easily change your conclusion. Duplicating your math, I can justify a premium of about 1.25% if I assume 10% growth in small caps for 10 years (vice 5), and then capitalize at the risk free rate. This may not justify the 4-5% premium that is ubiquitously applied, but it does highlight that both growth and risk expectations must be considered.

April 15, 2015 at 2:59 PM

David Velasco said...

Having valued hundreds of small and very small businesses over the years I have never been comfortable imposing substantial small cap premiums when developing my discount rates used in DCF analysis. However, valuers do face real differences when valuing "small" companies (i.e., less than \$5 million in annual sales), such as: 1) Investment diversification is rarely achieved where the typical owner owns 100% of the equity; and 2) that ownership comprises a very significant portion of that individual's personal wealth. Given the reality of typically poor diversification in small business valuation, is any premium warranted for the owner's inability to diversify systematic risk?

April 15, 2015 at 5:50 PM

RoE said...

I once had a discussion with you at a CFA Valuation Conference about when cash flow riskiness should be reflected in the cash flow estimate, rather than discount rate, and you talk about that again here. Could you please explain (or point to references) under what circumstances you adjust cash flows and not discount rates? I think its a very important topic. (Maybe a separate blog session on it??)

April 15, 2015 at 8:29 PM



Unknown said...

In a significant level of valuation work, a "micro-cap" premium is being applied as those companies are much smaller than a small cap company. In those cases, is it at least marginally reasonable and justifiable to use a micro-cap premium?

April 16, 2015 at 11:42 AM



Aswath Damodaran said...

MD and UniverseofRisks,
It is entirely possible that giving a longer growth period for small cap stocks or a slightly higher growth rate in stable growth can yield a small cap premium, but the fact that you have work that hard to get any significance is revealing.

David,

The lack of diversification is an entirely different issue and I am not sure why a small cap premium (that comes from publicly traded companies) would yield an answer. I have argued that lack of diversification effectively scales up your exposure to conventional market risk. (I concocted the total beta measure to capture it).

Michael,

I think that the bulk of the premium, if it exists in microcap companies is a reflection of either survival risk or illiquidity and my points about double counting still stand.

April 16, 2015 at 1:39 PM



Rohit said...

Thanks for a great piece, Professor.

"While 'over estimating' discount rates reduces type 1 errors (that you will buy an over valued stock), it comes at the expense of type 2 errors (that you will hold off on buying an under valued stock)."

As a long-only investor, wouldn't it not be okay to commit a few type II errors as a price for incorporating 'margin of safety'?

April 16, 2015 at 5:00 PM



Aswath Damodaran said...

Rohit,

Sure. As long as you don't end up with a lot of cash in your portfolio because you have set your expected return too high (or applied too large a margin of safety in your portfolio). In this market, finding an under valued stock is tough enough.

April 16, 2015 at 6:59 PM

UniverseofRisks said...

My only concern is; intuitively when you're computing the Implied ERP for small cap stocks, you are looking at a cash-flows of a dynamic set of companies. Ones that are small and fast growing now, which will be replaced by newer and younger companies in the future. I think the stable period growth spread over the risk free rate should be significant in this context because you're dealing with a dynamic set of companies that are always young and fast growing

April 17, 2015 at 7:39 AM



SDHakala said...

What you are saying is what I found in the 1990s and have consistently asked others about. In a chapter to an update to a valuation text in 1998, Hakala (me) and Bajaj found no forward-looking small stock premium, found small stock premiums largely disappeared after 1980, and found that the small stock premium was highly correlated with the bid-ask spread (transactions cost) which has gone down steadily over time. Additionally, if you look at actual buy and hold small cap fund returns (like Vanguard's NAESX from 1960 onward) or DFSCX since inception, small caps have only slightly outperformed large caps on a geometric return basis by 1.0% since 1960s and microcaps have only slightly outperformed large caps since 1982 by 1.0%. The NAESX has actually underperformed S&P 500 funds since the end of 1986 on a geometric return basis. Thus, much of the findings appear to be data mining and biases (arithmetic annual returns in Ibbotson and Duff & Phelps; monthly average returns in Fama-French work) and to have largely declined to not being statistically significant post 1980.

April 17, 2015 at 1:15 PM



SDHakala said...

There are a number of problems that have never really been addressed with the return data and methods commonly used to estimate small cap premiums (particularly Duff & Phelps and Fama-French data):

First, the use of arithmetic average returns over relatively short holding periods will mathematically always overstate the correct size premium for a three to five year DCF and for the terminal value discount rate. There have been numerous studies on this in the academic literature that have largely been discounted or ignored (from Blume's work in the 1970s to more recent work by others such as Jacquier's work with Kane and Marcus). Second, the Compustat (and to a much less extent CRSP) data has two biases associated with construction of the combined data: a backfilling bias (as the databases were constructed and filled in over time from the 1970s to current with subsequently successful companies "backfilled" and added to the historical data overtime) and a restatement bias (historic data was restated after mergers or subsequent restatements). There was work done for Compustat on this issue by Northfield around 2000-2001 that found substantially inflated return averages (as much as 6% overstatement of returns in 1980s) using backfilled and restated data as compared with the originally "as reported" data. Compustat now sells a separate set of "corrected, as first reported" financial data that acknowledges this issue. But even the academic studies often do not use this corrected data (because it does not come automatically matched up with the CRSP data, requires a separate purchase, and the data only goes back to 1983). Requiring companies with two or five years of historical trading or financial data is a common "academic" "solution" to the backfilling bias but does not appear to entirely solve the backfilling problem and does not address the significance and importance of restatements in the data (Companies with negative restatements will drop out of the D&P study and companies with positive restatements for mergers and such will suddenly show up in the data, for example.) Despite the academic work going back and forth on this issue, no one has actually studied the true effects of the bias beyond guesses that I know of. Third, microcap stocks are more likely to experience positive jumps in returns due to being acquired/restructured that are not representative of expected returns. These "outliers" have been shown to significantly explain some of the "anomalies" in some of the academic research. At least one could argue this part of the equation has some validity but it is a non-priced excess return on a current basis for most microcaps.

April 17, 2015 at 1:57 PM

Anonymous said...

In estimating implied ERP for small cap you have used dividend discount model (for stable firms), is this model right in the first place for small companies which are typically 1) growing and 2) not stable?

However, I agree with you that small cap premium is actually not warranted since we have to estimate cash flows based on the characteristics of a particular small firm, and discount rate should not capture it.

April 19, 2015 at 8:32 AM

hernando rivas said...

I understand your blog and I agree with your arguments. But is very subjective to adjust the DCF for possible bankruptcy or other adjustments.

Why are better or less subjective those adjustment than use a small cap premium?

Do you have any longer documents about those topic?

My mail is hrivas82@gmail.com

February 16, 2017 at 5:12 PM

Bo said...

If there is no small cap premium, do you see a mid cap premium? Over the long-haul they seem to have had higher returns and are potentially lower risk being larger and more established than their smaller cap counterparts.

April 18, 2017 at 2:48 PM



Sanjay said...

Dear Professor Damodaran,

Thank you for addressing this topic and in advance for consideration of my question. In practice, I have noted that a number of the stocks I am reviewing in the UK that are listed below the FTSE100 have lower equity betas (and asset betas) than direct comparables. How do we adjust for trading volume, at least in the short term, to reflect the lack of liquidity in the specific stocks. Whilst I agree that there is limited logic for a small cap premium, WACCs do appear lower for smaller cap stocks, even when adjusting for the higher marginal after tax cost of debt.

April 25, 2018 at 7:23 AM

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Musings on Markets

My not-so-profound thoughts about valuation, corporate finance and the news of the day!

Saturday, April 11, 2015

The Small Cap Premium: Where is the beef?

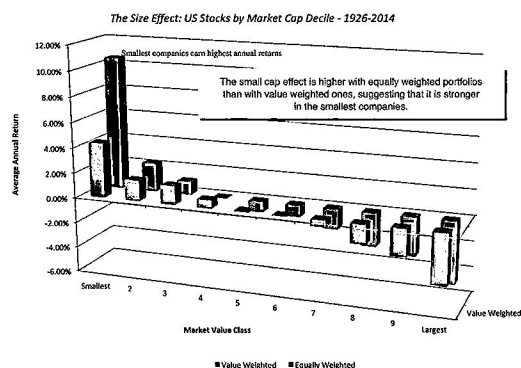
For decades, analysts and investor have bought into the idea of a small cap premium, i.e., that stocks with low market capitalizations can be expected to earn higher returns than stocks with higher market capitalizations. For investors, this has led to the pursuit of small cap stocks and funds for their portfolios, and for analysts, it has translated into the addition of "small cap" premiums of between 3-5% to traditional model-based expected returns, for companies that they classify as small cap. While I understand the origins of the practice, I question the adjustment for three reasons:

1. On closer scrutiny, the historical data, which has been used as the basis of the argument, is yielding more ambiguous results and leading us to question the original judgment that there is a small cap premium.
2. The forward-looking risk premiums, where we look at the market pricing of stocks to get a measure of what investors are demanding as expected returns, are yielding no premiums for small cap stocks.
3. If the justification is intuitive, i.e., that smaller firms are riskier than larger firms, much of that additional risk is either diversifiable, better adjusted for in the expected cash flows (instead of the discount rate) or double counted.

The small cap premium is a testimonial to the power of inertia in corporate finance and valuation, where once a practice becomes established, it becomes difficult to challenge, even if the original reasons for it have long since disappeared.

The Basis

The first studies that uncovered the phenomenon of the small cap premium came out in the 1970s. They broke companies down into deciles, based on market capitalization, and found that companies in the lowest decile earned higher returns, after adjusting for conventional risk measures, than companies in the highest decile. I updated those studies through the end of 2014, and the small cap premium seems intact (at least at first sight). In summary, looking at returns from 1926 to 2014, the smallest cap stocks (in the lowest decile) earned 4.33% more than the market, after adjusting for risk.



Source: Ken French's online data

This is the strongest (and perhaps) only evidence for a small cap premium and it is reproduced in data services that try to estimate historical risk premiums (Ibbotson, Duff and Phelps etc.). This historical premium has become the foundation for both valuation and investment practice. In valuation, analysts have referenced this table to estimate a small cap premium (4-5%) that they then add to the required return from conventional risk and return models to estimate discount rates. For instance, in the conventional capital asset pricing model, it plays out as follows:

$$\text{Expected Return} = \text{Risk free rate} + \text{Beta} * \text{Equity Risk Premium} + \text{Small Cap Premium}$$

That discount rate is used to estimate the value of future cash flows, and not surprisingly, the use of a small cap premium lowers the value of smaller companies.

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Aswath Damodaran

I am a Professor of Finance at the Stern School of Business at NYU. I teach

classes in corporate finance and valuation, primarily to MBAs, but generally to anyone who will listen.

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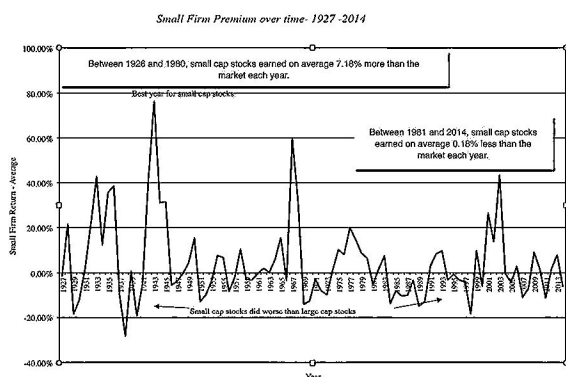
A Viral Market Meltdown III: Pricing or Value? Trading or Investing?

In investing, it has been used as a weapon both for and against active investing. Those who favor active investing have pointed to the small cap premium as a justification for their activity, and during the periods of history when small cap companies outperformed the market, it did make them look like heroes but it quickly gave rise to a counterforce, where performance measurement services (like Morningstar) started incorporating portfolio tilts, comparing small cap funds against small cap indices. Since almost all of the "excess returns" disappeared on this comparison, it was only a matter of time before index funds entered the arena, creating small-cap index funds for investors who wanted to claim the premium, without paying large management fees.

The Problem with the Historical Premium

In the decades since the original small cap premium study, the data on stocks has become richer and deeper, allowing us to take a closer look at the phenomenon. There are some serious questions that can be raised about whether the premium exists and if so, what exactly it is measuring:

1. **Trend lines and Time Periods:** Small cap stocks have earned higher returns than large cap stocks between 1928 and 2014 but the premium has been volatile over history, disappearing for decades and reappearing again. While the premium was strong prior to 1980, it seems to have dissipated since 1981. One reason may be that the small cap premium studies drew attention and investor money to small cap stocks, and in the process led to a repricing of these stocks. Another is that the small cap premium is a side effect of larger macroeconomic variables (inflation, real growth etc.) and that the behavior of those variables has changed since 1980.



Source: Ken French's online data

2. **Microcap, not small cap premium:** Even over the long time period that provides the strongest support for existence of a small cap premium, one study finds that removing stocks with less than \$5 million in market cap causes the small firm effect to vanish. In effect, what you have is microcap premium, isolated in the smallest of stocks, not just small stocks.
3. **Standard Error:** Historical equity returns are noisy and any estimates of risk premium from that data will reflect the noise in the form of large standard errors on estimates. I have made this point about the overall historical equity risk premium but it becomes magnified when you dice and slice historical data into sub-classes. The table below lists standard errors in excess returns by decile class and reinforce the notion that the small cap premium is fragile, barely making the threshold for statistical significance over the entire period.

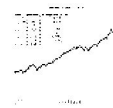
Decile	Average	Standard Error	Maximum	Minimum
Smallest	4.33%	1.96%	76.28%	-28.42%
2	1.63%	1.14%	41.25%	-17.96%
3	1.47%	0.77%	41.98%	-13.54%
4	0.64%	0.55%	15.56%	-7.33%
5	0.05%	0.53%	11.63%	-16.05%
6	-0.01%	0.51%	15.21%	-14.01%
7	-0.51%	0.55%	7.48%	-19.50%
8	-1.50%	0.81%	11.20%	-29.42%
9	-2.13%	1.02%	21.96%	-36.09%
Largest	-3.98%	1.56%	31.29%	-65.57%

Source: Ken French's online data

4. **The January Effect:** One of the most puzzling aspects of the small cap premium is that almost all of it is earned in one month of the year, January, and removing that month makes it disappear. So what? If your argument for the small cap premium is that small cap stocks are riskier, you now have the onus of explaining why that risk shows up only in the first month of every year.



This is the third, and I hope the last, of my viral market updates, reflecting how much change a week can deliver, and last week delivered...



A Viral Market Meltdown: Fear or Fundamentals?
It has become almost a rite of passage for investors, at least since 2008, that they will be tested by a market crisis precipitated some...



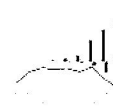
A Do-it-yourself (DIY) Valuation of Tesla: Of Investment Regrets and Disagreements!
I was hoping to move on from Tesla to my data update posts, but my last post on Tesla drew some attention, in good and bad ways, partly be...



An Ode to Luck: Revisiting my Tesla Valuation
When investing, I am often my own biggest adversary, handicapped by the preconceptions and priors that I bring into analysis and decision ...



A Viral Market Meltdown Part II: Clues in the Debris!
Update on 3.9/20: In a sign of how volatile times are, over the weekend, oil prices plummeted to close to \$30, the treasury bond rate to I...



Walmart's India (Flipkart) Gambit: Growth Rebirth or Costly Facelift?
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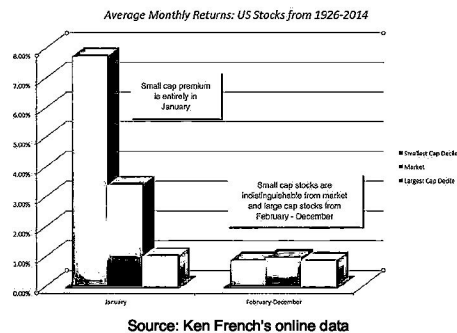
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► 2014 (44)

► 2013 (36)

► 2012 (49)

► 2011 (55)

► 2010 (45)

► 2009 (60)

► 2008 (42)

5. Weaker globally: The small cap premium seems to be smaller in non-US markets than in US markets and is non-existent in some. In contrast, the value effect (where low price to book stocks outperform the market) is strong globally.

6. Proxy for other factors: A host of papers argue that the bulk or all of the small size effect can be attributed to a liquidity effect and that putting in a proxy for illiquidity makes the size effect disappear or diminishes it.

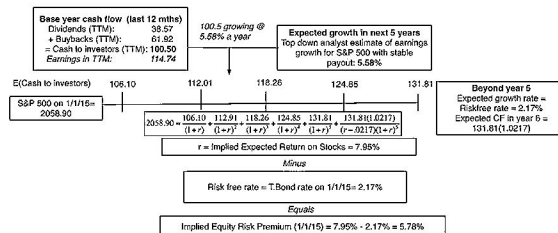
7. Works only with market cap: Finally, you can take issue with the use of a market-priced based measure of size in a study of returns. Others have tried other non-price size measures such as income or revenues but there seems to be no size effect in those variables.

A recent working paper by Asness, Frazini, Israel, Moskowitz and Pedersen tries to resurrect the size effect, but accomplishes it only by removing the subset of small companies that they classify as "low quality" or "junk". While the results are interesting and can be used by active small-cap fund managers as a justification for their activity, they are in no way a basis for adding a small cap premium to every small company, and asking analysts to add it on only for small, high quality companies is problematic. In summary, if the only justification that you can offer for the addition of a small cap premium to your discount rate is the historical risk premium, you are on thin ice.

Market-Implied Small Cap Premium

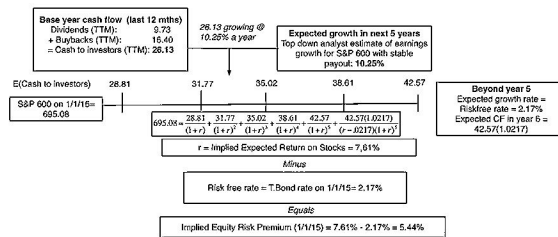
If the historical data ceases to support the use of a historical risk premium, can we then draw on intuition and argue that since small companies tend to be riskier (or we perceive them to be), investors must require higher return when they invest in them? You can, but the onus is then on you to back up that intuition. In fact, you can check to see whether investors are demanding a forward looking "small cap" premium, by looking at how they price small as opposed to large companies, and backing out what investors are demanding as expected returns. Put simply, if small cap stocks are viewed by investors as riskier and that risk is being priced in, you should expect to see, other things remaining equal, higher expected returns on small cap stocks than large cap stocks.

As some of you are aware, I compute a forward-looking equity premium for the S&P 500 at the start of each year, backing out the number from the current level of the index and expected cash flows. On January 1, 2015, this is what I found:



In effect, to the extent that my base year cash flows are reasonable and my expected growth rate reflects market expectations, the expected return on large cap stocks on January 1, 2015 was 7.95% in the US (yielding an overall equity risk premium of 5.78% on that day).

To get a measure of the forward-looking small cap premium, I computed the expected return implied in the S&P 600 Small Cap Index, using the same approach that I used for S&P 500. In spite of using a higher expected earnings growth for small cap stocks, the expected return that I estimate is only 7.61%:



In effect, the market is attaching a smaller expected return for small cap stocks than large ones, stories and intuition notwithstanding.

I am not surprised that the market does not seem to buy into the small cap premiums that academics and practitioners are so attached to. After all, if the proponents of small cap premiums are right, bundling together small companies into a larger company should instantly generate a bonus, since you are replacing the much higher required returns of smaller companies with the lower expected return of a larger one. In fact, small companies should disappear from the market.

The Illiquidity Fig Leaf

Looking at the data, the only argument left, as I see it, for the use of the small cap premium is as a premium for illiquidity, and even on that basis, it fails at one of these four levels:

1. *If illiquidity is your bogey man in valuation, why use market capitalization as a stand-in for it?* Market capitalization and illiquidity don't always go hand in hand, since there are small, liquid companies and large, illiquid ones in the market. Four decades ago, your excuse would have been that the data on illiquidity was either inaccessible or unavailable and that market capitalization was the best proxy you could find for illiquidity. That is no longer the case and there are studies that categorize companies based on measures of illiquidity (bid ask spread, trading volume) and find an "illiquidity premium" for illiquid companies.
2. *If illiquidity is what you are adjusting for in the small cap premium, why is it a constant across companies, buyers and time?* Even if your defense is that the small cap premium is an imperfect (but reasonable) measure of the illiquidity premium, it is unreasonable to expect it to be the same for every company. Thus, even if you are valuing just privately owned businesses (where illiquidity is a clear and present danger), that illiquidity should be greater in some businesses than in others and the illiquidity (or small cap) premium should be larger for the former than the latter. Furthermore, the premium you add to the discount rate should be higher in some periods (during market crises and liquidity crunches) than others and for some buyers (cash poor, impatient) than others (patient, cash rich).
3. *Even if you can argue that illiquidity is your rationale for the small cap premium and that it is the same across companies, why is it not changing over the time horizon of your valuation (and especially in your terminal value)?* In any valuation, you assume through your company's cash flows and growth rates that your company will change over time and it is inconsistent (with your own narrative) to lock in an illiquidity premium into your discount rate that does not change as your company does. Thus, if you are using a 30% expected growth rate on your company, your "small" company is getting bigger (at least according to your estimates) and presumably more liquid over time. Should your illiquidity premium therefore not follow your own reasoning and decrease over time?
4. *If your argument is that size is a good proxy for illiquidity, that all small companies are equally illiquid and that that illiquidity does not change as you make them bigger, why are you reducing your end value by an illiquidity discount?* This question is directed at private company appraisers who routinely use small cap premiums to increase discount rates and also reduce the end (DCF) value by 25% or more, because of illiquidity. You can show me data to back up your discount (I have seen restricted stock and IPO studies) but none of them can justify the double counting of illiquidity in valuation.

Why are we slow to give up on the “small cap” premium?

It is true that the small cap premium is established practice at many appraisal firms, investment banks and companies. Given the shaky base on which it is built and how much that base has been chipped away in the last two decades, you would think that analysts would reconsider their use of small cap premiums, but there are three powerful forces that keep it in play.

1. **Intuition:** Analysts and investors not only start off with the presumption that the discount rates for small companies should be higher than large companies, but also have a "number" in mind. When risk and return models deliver a much lower number, the urge to add to it to make it "more reasonable" is almost unstoppable. Consequently, an analyst who arrives at an 8% cost of equity for a small company feels much more comfortable after adding a 5% small cap premium. It is entirely possible that you are an idiot savant with the uncanny capacity to assess the right discount rate for companies, but if that is the case, why go through this charade of using risk and return models and adding premiums to get to your "intuited" discount rate? For most of us, gut feeling and instinct are not good guides to estimating discount rates and here is why. Not all risk is meant for the discount rate, with some risk (like management skills) being diversifiable (and thus lessened in

portfolios) and other risks (like risk of failure or regulatory approval) better reflected in probabilities an expected cash flow. A discount rate cannot and is not meant to be a receptacle for all your hopes and fears, a number that you can tweak until you get to your comfort zone.

2. Inertia (institutional and individual): The strongest force in corporate finance practice is inertia, where much of what companies, investors and analysts do reflects past practice. The same is true in the use of the small cap premium, where a generation of analysts has been brought up to believe (by valuation handbooks and teaching) that it is the right adjustment to make and now do it by rote. That inertia is reinforced in the legal arena (where many valuations end up, either as part of business or tax disputes) by the legal system's respect for precedence and general practice. You may view this as harsh, but I believe that you will have an easier time defending the use of a bad, widely used practice of long standing in court than you would arguing for an innovative better practice.
3. Bias: My experiences with many analysts who use small cap premiums suggest to me that one motive is to get a "lower" value". Why would they want a lower value? First, in accounting and tax valuation, the client that you are doing the valuation for might be made better off with a lower value than a higher one. Consequently, you will do everything you can to pump up the discount rate with the small cap premium being only one of the many premiums that you use to "build up" your cost of capital. Second, there seems to be a (misplaced) belief that it is better to arrive at too low a value than one that is too high. If you buy into this "conservative" valuation approach, you will view adding a small cap premium as costless, since even it does not exist, all you have done is arrived at "too low" a value. At the risk of bringing up the memories of statistics classes past, there is always a cost. While "over estimating" discount rates reduces type 1 errors (that you will buy an over valued stock), it comes at the expense of type 2 errors (that you will hold off on buying an under valued stock).

A Requiem for the Small Cap Premium?

I have never used a small cap premium, when valuing a company and I don't plan to start now. Needless to say, I am often asked to justify my non-use of a premium and here are my reasons. First, I am not convinced by either the historical data or by current market behavior that a small cap premium exists. Second, I do believe that small cap companies are more exposed to some risks than large cap companies but there are other more effective devices to bring these risks into valuation. If it is that they are capital constrained (i.e., that it is more difficult for small companies to raise new capital), I will limit their reinvestment and expected growth (thus lowering value). If it is that they have a greater chance of failure, I will estimate a probability of failure and reflect that in my expected value (as I do in my standard DCF model). If it is illiquidity that is your concern, it is worth recognizing that one size will not fit all and that the effect on value will vary across investors and across time and will be better captured in a discount on value.

To illustrate how distorted this debate has become, note that those who routinely add small cap premiums to their discount rates are not put to the same test of justifying its use. So, at the risk of opening analysts up to uncomfortable questions, here are some questions that you should pose to anyone who is using a small cap premium (and that includes yourself):

1. *What is your justification for using a small cap premium?* If the defense is pointing to history (or a data table in a service), it is paper thin, since that historical premium defense seems to have more holes in it than Swiss cheese. If it is intuitive, i.e., that small companies are riskier and markets must see them as such, I don't see the basis for the intuition, since the implied costs of equity for small companies are no higher than those of large companies. If the argument is that everyone does it, I am sorry but just because something is established practice does not make it right.
2. *What are the additional risks that you see in small companies that you don't see in large ones?* I am sure that you can come up with a laundry list that is a mile long, but most of the risks on the list either don't belong in the discount rate (either because they are diversifiable or because they are discrete risks) or can be captured through probability estimates. If it is illiquidity that you are concerned about, see the section on illiquidity above for my response.

If you are investors, here are the lessons I draw from looking at the data. If you are following a strategy of buying small cap stocks, expecting to be rewarded with a premium for just doing that, you will be disappointed. Even the most favorable papers on the small cap premium suggest that you have to add refinements, with some suggesting that these refinements should screen out the least liquid, riskiest small cap stocks and others arguing for value characteristics (stable earnings, high returns on equity & capital, solid growth). I do think that there is a glimmer of hope in the recent research that the payoff to looking for under valued stocks may be greater with small companies, partly because they are more likely to be overlooked, but it will take more work on your part and it won't be easy!

Data sets

1. Professor Ken French's data library (on small cap stocks)

Spreadsheets

1. Implied equity risk premium (S&P 500)
2. Implied equity risk premium (S&P Small Cap 600)

Posted by Aswath Damodaran at 1:57 PM



Labels: Cost of equity, Discount Rates, Small Cap Premium, Valuation Practice

21 comments:

Max said...

It's worse when you look at the performance of the most widely used small cap index, the Russell 2000. The R2K has underperformed small caps in general. (Why? Perhaps because membership in a major index grants a stock improved liquidity).

April 11, 2015 at 4:38 PM



Unknown said...

When we talk about January effect, we're in effect talking about a 12 month reversal phenomenon. In other words (dogs of the Dow theory), the losers of the preceding year outperform the winners of the preceding year and quite understandably so, the micro cap firms whose market cap had plummeted in the preceding 12 months would be expected to outperform the broad market. Further small cap premium would be expected to be significantly positive in bull markets and significantly negative in bear markets, in other words small cap effect is a function of investor sentiment (risk-on vs. risk-off sentiment). So splitting the sample period (1926-2014) into three periods of bull markets, bear markets and range bound markets would give us some more insights on the small cap premium. Further there is the migration effect i.e. small caps going onto become mid caps due to stock price surge and mid caps becoming small caps due to stock prices plummeting in the preceding years. In other words when we look at decile 10 of the capitalization strata, the character of the firms in the decile 10 is vastly different every year. Further empirical evidence suggests small cap premium is concentrated in few sectors/industries which emerge out of nowhere and become sunrise industries. In other words analyzing small cap effect is vastly complicated and there are too many forces at work!

April 11, 2015 at 10:12 PM



Aswath Damodaran said...

Yogesh,
It takes this much convoluted back tracking for you to try to explain with small cap stocks, there is no small cap effect. It is not complicated. It is just not there.

April 11, 2015 at 10:16 PM

UniverseofRisks said...

In your calculation of the small cap ERP, shouldn't the terminal growth rate be much higher than the risk free rate. By using the same rate as the S&P 500 you're actually calculating the return on equity of companies that begin as small cap but are treated as large caps after year 5. Wouldn't you want your inputs for calculations beyond year 5 to reflect a constant rate of growth for small caps, which to me is much higher than the risk free rate?

April 12, 2015 at 8:41 AM



Aswath Damodaran said...

UniverseofRisks,
You have a good point, though the growth rate you use can only be marginally higher than the growth rate of the economy, since it is a perpetual number. By the same token, I should probably use a slightly lower than the economy growth rate for larger companies.

April 12, 2015 at 9:43 AM



Unknown said...

Playing with the IFA index calculator it seems like small almost always outperforms large:

<https://www.ifa.com/calculator/?i=sv&g=100000&s=1/1/2000&e=1/31/2014&infl=true&af=true&aorw=false&perc=true>

Also see:

<http://www.marketwatch.com/story/the-one-asset-class-every-investor-needs-2014-06-25>

BR Martin

April 13, 2015 at 9:15 AM

Anonymous said...

What if there were an instance where the implied small cap premium were higher than the large cap? Would it be more precise to use the small implied premium?

April 13, 2015 at 4:54 PM

MD said...

In line with comments from UniverseofRisk, I think the growth assumption, and particularly the assumed growth differential between large and small caps, can easily change your conclusion. Duplicating your math, I can justify a premium of about 1.25% if I assume 10% growth in small caps for 10 years (vice 5), and then capitalize at the risk free rate. This may not justify the 4-5% premium that is ubiquitously applied, but it does highlight that both growth and risk expectations must be considered.

April 15, 2015 at 2:59 PM

David Velasco said...

Having valued hundreds of small and very small businesses over the years I have never been comfortable imposing substantial small cap premiums when developing my discount rates used in DCF analysis. However, valuers do face real differences when valuing "small" companies (i.e., less than \$5 million in annual sales), such as: 1) Investment diversification is rarely achieved where the typical owner owns 100% of the equity; and 2) that ownership comprises a very significant portion of that individual's personal wealth. Given the reality of typically poor diversification in small business valuation, is any premium warranted for the owner's inability to diversify systematic risk?

April 15, 2015 at 5:50 PM

RoE said...

I once had a discussion with you at a CFA Valuation Conference about when cash flow riskiness should be reflected in the cash flow estimate, rather than discount rate, and you talk about that again here. Could you please explain (or point to references) under what circumstances you adjust cash flows and not discount rates? I think its a very important topic. (Maybe a separate blog session on it??)

April 15, 2015 at 8:29 PM



Unknown said...

In a significant level of valuation work, a "micro-cap" premium is being applied as those companies are much smaller than a small cap company. In those cases, is it at least marginally reasonable and justifiable to use a micro-cap premium?

April 16, 2015 at 11:42 AM



Aswath Damodaran said...

MD and UniverseofRisks,

It is entirely possible that giving a longer growth period for small cap stocks or a slightly higher growth rate in stable growth can yield a small cap premium, but the fact that you have work that hard to get any significance is revealing.

David,

The lack of diversification is an entirely different issue and I am not sure why a small cap premium (that comes from publicly traded companies) would yield an answer. I have argued that lack of diversification effectively scales up your exposure to conventional market risk. (I concocted the total beta measure to capture it).

Michael,

I think that the bulk of the premium, if it exists in microcap companies is a reflection of either survival risk or illiquidity and my points about double counting still stand.

April 16, 2015 at 1:39 PM



Rohit said...

Thanks for a great piece, Professor.

"While 'over estimating' discount rates reduces type 1 errors (that you will buy an over valued stock), it comes at the expense of type 2 errors (that you will hold off on buying an under valued stock)."

As a long-only investor, wouldn't it not be okay to commit a few type II errors as a price for incorporating 'margin of safety'?

April 16, 2015 at 5:00 PM



Aswath Damodaran said...

Rohit,

Sure. As long as you don't end up with a lot of cash in your portfolio because you have set your expected return too high (or applied too large a margin of safety in your portfolio). In this market, finding an under valued stock is tough enough.

April 16, 2015 at 6:59 PM

UniverseofRisks said...

My only concern is; intuitively when you're computing the Implied ERP for small cap stocks, you are looking at a cash-flows of a dynamic set of companies. Ones that are small and fast growing now, which will be replaced by newer and younger companies in the future. I think the stable period growth spread over the risk free rate should be significant in this context because you're dealing with a dynamic set of companies that are always young and fast growing

April 17, 2015 at 7:39 AM



SDHakala said...

What you are saying is what I found in the 1990s and have consistently asked others about. In a chapter to an update to a valuation text in 1998, Hakala (me) and Bajaj found no forward-looking small stock premium, found small stock premiums largely disappeared after 1980, and found that the small stock premium was highly correlated with the bid-ask spread (transactions cost) which has gone down steadily over time. Additionally, if you look at actual buy and hold small cap fund returns (like Vanguard's NAESX from 1960 onward) or DFSCX since inception, small caps have only slightly outperformed large caps on a geometric return basis by 1.0% since 1960s and microcaps have only slightly outperformed large caps since 1982 by 1.0%. The NAESX has actually underperformed S&P 500 funds since the end of 1986 on a geometric return basis. Thus, much of the findings appear to be data mining and biases (arithmetic annual returns in Ibbotson and Duff & Phelps; monthly average returns in Fama-French work) and to have largely declined to not being statistically significant post 1980.

April 17, 2015 at 1:15 PM



SDHakala said...

There are a number of problems that have never really been addressed with the return data and methods commonly used to estimate small cap premiums (particularly Duff & Phelps and Fama-French data):

First, the use of arithmetic average returns over relatively short holding periods will mathematically always overstate the correct size premium for a three to five year DCF and for the terminal value discount rate. There have been numerous studies on this in the academic literature that have largely been discounted or ignored (from Blume's work in the 1970s to more recent work by others such as Jacquier's work with Kane and Marcus). Second, the Compustat (and to a much less extent CRSP) data has two biases associated with construction of the combined data: a backfilling bias (as the databases were constructed and filled in over time from the 1970s to current with subsequently successful companies "backfilled" and added to the historical data overtime) and a restatement bias (historic data was restated after mergers or subsequent restatements). There was work done for Compustat on this issue by Northfield around 2000-2001 that found substantially inflated return averages (as much as 6% overstatement of returns in 1980s) using backfilled and restated data as compared with the originally "as reported" data. Compustat now sells a separate set of "corrected, as first reported" financial data that acknowledges this issue. But even the academic studies often do not use this corrected data (because it does not come automatically matched up with the CRSP data, requires a separate purchase, and the data only goes back to 1983). Requiring companies with two or five years of historical trading or financial data is a common "academic" "solution" to the backfilling bias but does not appear to entirely solve the backfilling problem and does not address the significance and importance of restatements in the data (Companies with negative restatements will drop out of the D&P study and companies with positive restatements for mergers and such will suddenly show up in the data, for example.) Despite the academic work going back and forth on this issue, no one has actually studied the true effects of the bias beyond guesses that I know of. Third, microcap stocks are more likely to experience positive jumps in returns due to being acquired/restructured that are not representative of expected returns. These "outliers" have been shown to significantly explain some of the "anomalies" in some of the academic research. At least one could argue this part of the equation has some validity but it is a non-priced excess return on a current basis for most microcaps.

April 17, 2015 at 1:57 PM

Anonymous said...

In estimating implied ERP for small cap you have used dividend discount model (for stable firms), is this model right in the first place for small companies which are typically 1) growing and 2) not stable?

However, I agree with you that small cap premium is actually not warranted since we have to estimate cash flows based on the characteristics of a particular small firm, and discount rate should not capture it.

April 19, 2015 at 8:32 AM

hernando rivas said...

I understand your blog and I agree with your arguments. But is very subjective to adjust the DCF for possible bankruptcy or other adjustments.

Why are better or less subjective those adjustment than use a small cap premium?

Do you have any longer documents about those topic?

My mail is hrivas82@gmail.com

February 16, 2017 at 5:12 PM

Bo said...

If there is no small cap premium, do you see a mid cap premium? Over the long-haul they seem to have had higher returns and are potentially lower risk being larger and more established than their smaller cap counterparts.

April 18, 2017 at 2:48 PM



Sanjay said...

Dear Professor Damodaran,

Thank you for addressing this topic and in advance for consideration of my question. In practice, I have noted that a number of the stocks I am reviewing in the UK that are listed below the FTSE100 have lower equity betas (and asset betas) than direct comparables. How do we adjust for trading volume, at least in the short term, to reflect the lack of liquidity in the specific stocks. Whilst I agree that there is limited logic for a small cap premium, WACCs do appear lower for smaller cap stocks, even when adjusting for the higher marginal after tax cost of debt.

April 25, 2018 at 7:23 AM

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Musings on Markets

My not-so-profound thoughts about valuation, corporate finance and the news of the day!

Saturday, August 5, 2023

The Price of Risk: With Equity Risk Premiums, Caveat Emptor!

If you have been reading my posts, you know that I have an obsession with equity risk premiums, which I believe lie at the center of almost every substantive debate in markets and investing. As part of that obsession, since September 2008, I have estimated an equity risk premium for the S&P 500 at the start of each month, and not only used that premium, when valuing companies during that month, but shared my estimate on my webpage and on social media. In my last post, on country risk premiums, I used the equity risk premium of 5.00% that I estimated for the US at the start of July 2023, for the S&P 500. That said, I don't blame you, if are confused not only about how I estimate this premium, but what it measures. In fact, an article in MarketWatch earlier this year referred to the equity risk premium as an esoteric concept, a phrasing that suggested that it had little relevance to the average investor. Adding to the confusion are the proliferation of very different numbers that you may have seen attached to the current equity risk premium, each usually quoting an expert in the field, but providing little context. Just in the last few weeks, I have seen a Wall Street Journal article put the equity risk premium at 1.1%, a Reuters report put it at 2.2%, and a bearish (and widely followed) money manager estimate the equity risk premium to be negative. How, you may ask, can equity risk premiums be that divergent, and does that imply that anything goes? In this post, I will not try to argue that my estimate is better than others, since that would be hubris, but instead focus on explaining why these ERP differences exist, and let you make your own judgment on which one you should use in your investing decisions.

ERP: Definition and Determinants

The place to start this discussion is with an explanation of what an equity risk premium is, the determinants of that number and why it matters for investors. I will try to steer away from models and economic jargon in this section, simply because they do little to advance understanding and much to muddy the waters.

What is it?

Investors are risk averse, at least in the aggregate, and while that risk aversion can wax and wane, they need at least the expectation of a higher return to be induced to invest in riskier investments. In short, the expected return on a risky investment can be constructed as the sum of the returns you can expect on a guaranteed investment, i.e., a riskfree rate, and a risk premium, which will scale up as risk increases.

$$\text{Expected Return} = \text{Risk free Rate} + \text{Risk Premium}$$

Note that this proposition holds even if you believe that there is nothing out there that is truly risk free, which is the case when you worry about governments defaulting, though it does imply that you have cleaning up to do to get to a riskfree rate. Note also that expectations do not always pan out, and the actual returns on a risky investment can be much lower than the risk free rate, and sometimes sharply negative.

The risk premium that you demand has different names in different markets. In the corporate bond market, it is a *default spread*, an augmentation to the interest rate that you demand on a bond with more default risk. In the real estate market, it is embedded in a *capitalization rate*, an expected return used by real estate investors to convert the income on a real estate property into a value for that property. In the equity market, it is the *equity risk premium*, the price of risk for investing in equities as a class.

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I am a Professor of Finance at the Stern School of Business at NYU. I teach classes in corporate finance and valuation, primarily to MBAs, but generally to anyone who will listen.

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