- The historical average equity risk premium, measured relative to 10-year government bonds as the risk premium investors might objectively have expected on their equity investments, is about 2.4 percent, half what most investors believe.
- The "normal" risk premium might well be a notch lower than 2.4 percent because the 2.4 percent objective expectation preceded *actual* excess returns for stocks relative to bonds that were nearly 100 bps higher, at 3.3 percent a year.
- The current risk premium is approximately zero, and a sensible expectation for the future real return for both stocks and bonds is 2–4 percent, far lower than the actuarial assumptions on which most investors are basing their planning and spending.[33]
- On the hopeful side, because the "normal" level of the risk premium is modest (2.4 percent or quite possibly less), current market valuations need not return to levels that can deliver the 5 percent risk premium (excess return) that the Ibbotson data would suggest. If reversion to the mean occurs, then to restore a 2 percent risk premium, the difference between 2 percent and zero still requires a near halving of stock valuations or a 2 percent drop in real bond yields (or some combination of the two). Either scenario is a less daunting picture than would be required to facilitate a reversion to a 5 percent risk premium.
- Another possibility is that the modest difference between a 2.4 percent normal risk premium and the negative risk premiums that have prevailed in recent quarters permitted the recent bubble. Reversion to the mean might not ever happen, in which case, we should see stocks sputter along delivering bondlike returns, but at a higher risk than bonds, for a long time to come.

The consensus that a normal risk premium is about 5 percent was shaped by deeply rooted naiveté in the investment community, where most participants have a career span reaching no farther back than the monumental 25-year bull market of 1975–1999. This kind of mind-set is a mirror image of the attitudes of the chronically bearish veterans of the 1930s. Today, investors are loathe to recall that the real total returns on stocks were negative for most 10-year spans during the two decades from 1963 to 1983 or that the excess return of stocks relative to long bonds was negative as recently as the 10 years ended August 1993.[34]

When reminded of such experiences, today's investors tend to retreat behind the mantra "things will be different this time." No one can kneel before the notion of the long run and at the same time deny that such circumstances will occur in the decades ahead. Indeed, such crises are more likely than most of us would like to believe. Investors greedy enough or naive enough to expect a 5 percent risk premium and to substantially overweight equities accordingly may well be doomed to deep disappointments in the future as the realized risk premium falls far below this inflated expectation.

What if we are wrong about today's low equity risk premium? Maybe real yields on bonds are lower than they seem. This chance is a frail reed to rely on for support. At this writing, at the end of 2001, an investor can buy TIPS, which provide government-guaranteed yields of about 3.4 percent, but inflation-indexed bond yields are a relatively recent phenomenon in the United States. So, we could not estimate historical real yields for prior years directly, only through a model such as the one described here. If we compare our model for real stock returns, at 2.4 percent in mid-2001, with a TIPS yield of 3.4 percent, we get an estimate for the equity risk premium of –100 bps.

Perhaps real earnings and dividend growth will exceed economic growth in the years ahead, or perhaps economic growth will sharply exceed the historical 1.6 percent real per capita GDP growth rate. These scenarios are certainly possible, but they represent the dreams of the "new paradigm" advocates. The scenarios are unlikely. Even if they prove correct, it will likely be in the context of unprecedented entrepreneurial capitalism, unprecedented new enterprise creation, and hence, unprecedented dilution of shareholders in existing enterprises.

The recurring pattern of history is that exceptionally poor or exceptionally rapid economic growth is never sustained for long. The best performance that dividend growth has ever managed, relative to real per capita GDP, is a scant 10 bp outperformance. This rate, the best 40-year real dividend growth *ever seen*, fell far short of real GDP growth: Real dividend growth was some 2 percent a year below real GDP growth during those same 40-year spans. So, history does not support those who hope that dividend growth will exceed GDP growth. This evidence is not encouraging for those who wish to see a 1.4 percent dividend yield somehow transformed into a 5 percent (or higher) real stock return.

The negative risk premium that precipitated the writing of "The Death of the Risk Premium" (Arnott and Ryan) in early 2000 was not without precedent, although most of the precedents, until recently, are found in the 19th century. In 1984 and again just before the 1987 market crash, real bond yields rose materially above the estimated real return on stocks. How well did this development predict subsequent relative returns? Stated more provocatively, why didn't our model work? Why didn't bonds beat stocks in the past decade? After all, with the 1984 peak in real bond returns and again shortly before the 1987 crash, the risk premium dipped even lower than the levels seen at the market peak in early 2000. Yet, stocks subsequently outpaced bonds. For an answer, recall that the context was a more than doubling of stock valuations, whether measured in price-to-book ratios, price-to-dividend ratios, or P/E multiples. If valuation multiples had held constant, the bonds would have prevailed.[35]

Appendix A. Estimating the Constituents of Return

An analysis of historical data is only as good as the data themselves. Accordingly, we availed ourselves of multiple data sources whenever possible. We were encouraged by the fact that the discrepancies between the various sources led to compounded rates of return that were no more than 0.2 percent different from one another.

Long Government Bond Yields, *BY*(*t*). Our data sources are as follows: for January 1800 to May 2001, 10-year government bond yields from Global Financial Data of the National Bureau of Economic Research (NBER) (data were annual until 1843 and were interpolated for monthly estimates); for June 2001 to December 2001, Bloomberg; and for January 1926 to December 2000, Ibbotson Associates, long-term government bond yields and returns. In cases of differences, we (1) averaged the yield data and (2) recomputed monthly total returns based on an assumed 10-year maturity standard.

Inflation, *INF*(*t*). We used two sources of inflation and U.S. Consumer Price Index data. For January 1801 to May 2001, NBER (annual until 1950; interpolated for monthly estimates); for June 2001 to December 2001, Bloomberg; and for January 1926 to December 2000, Ibbotson Associates. In cases of differences, we averaged the available data. Ibbotson data were given primary (two-thirds) weighting for 1926–1950 because the NBER data are annual through 1950.

Gross Domestic Product, *GDP*(*t*). For January 1800 to September 2001, NBER GNP data annually through 1920, interpolated July-to-July; for 1921–2001, quarterly GDP data; and for December 2001, *Wall Street Journal* consensus estimates.

Dividend Yield in Month *t*, *DY*(*t*), and Return on Stocks in Month *t*, *RS*(*t*). For January 1802 to December 1925, G. William Schwert (1990); for February 1871 to March 2001, Robert Shiller (2000); for January 1926 to December 2000, Ibbotson Associates (2001); and for April 2001 to December 2001, Bloomberg. In cases of differences, we averaged the available data. In Shiller's data, monthly dividend and earnings data are computed from the S&P four-quarter data for the quarter since 1926, with linear interpolation to monthly figures. Dividend and earnings data before 1926 are from Cowles (1939), interpolated from annual data.

Notes

1. The "bible" for the return assumptions that drive our industry is the work of Ibbotson Associates, building on the pioneering work of Ibbotson and Sinquefield (1976a, 1976b). The most recent update of the annual Ibbotson Associates data (2001) shows returns for U.S. stocks, bonds, bills, and inflation of, respectively, 11.0 percent, 5.3 percent, 3.8 percent, and 3.1 percent. These figures imply a real return for stocks of 7.9 percent and a risk premium over bonds of 5.7 percent (570 bps), both measured over a 75-year span. These data shape the expectations of the actuarial community, much of the consulting community, and many fund sponsors.
2. Fischer Black was fond of pointing out that examining the same history again and again with one new year added each passing year is an insidious form of data mining (see, for example, Black 1976). The past looks best when nonrecurring developments and valuation-level changes have distorted the results; extrapolating the past tacitly implies a belief that these nonrecurring developments can recur and that the changes in valuation levels will continue.
3. We strongly suggest that the investment community draw a distinction between past excess returns (observed returns from the past) and expected risk premiums (expected return differences in the future) to avoid continued confusion and to reduce the dangerous temptation to merely extrapolate past excess returns in shaping expectations for the risk premium. This habit is an important source of confusion that, quite literally, (mis)shapes decisions about the management of trillions in assets worldwide. We propose that the investment community begin applying the label "risk premium" *only* to expected future return differences and apply the label "excess returns" to observed historical return differences.
4. To see the effect of compounding at this rate, consider that if our ancestors could have earned a mere 1.6 percent real return on a $1 investment from the birth of Christ in roughly 4 B.C. to today, we would today have enough to buy more than the entire world economy. Similarly, the island of Manhattan was ostensibly purchased for $24 of goods, approximately the same as an ounce of gold when the dollar was first issued. This modest sum invested to earn a mere 5 percent real return would have grown to more than $20 billion in the 370 years since the transaction. At an 8 percent real return, as stocks earned from 1926 to 2000 in the Ibbotson data, this $24 investment would now suffice to buy more than the entire world economy.

5. No rational investor buys if he or she expects less than 1 percent real growth a year in capital, but objective analysis will demonstrate that this return is what stocks have actually delivered, plus their dividend yield, plus or minus any profits or losses from changes in yields. As Asness pointed out in "Bubble Logic" (2000), few buyers of Cisco would have *expected* a 1 percent internal rate of return at the peak, although the stock was priced to deliver just that, even if the overly optimistic consensus earnings and growth forecasts at the time were used. These buyers were focused on the view that the stock would produce handsome gains, as it had in the past, rather than on pursuing an objective evaluation, by using IRR or similar objective valuation tools, of expected returns. Such a focus plants the seeds of major disappointment.
6. The Welch study investigated an expected *arithmetic* risk premium for stocks relative to *cash*, not bonds. The difference between arithmetic and geometric returns is often illustrated by someone earning 50 percent in one year and –50 percent in the next. The arithmetic average is zero, but the person is down 25 percent (or 13.4 percent a year). Most practitioners think in terms of compounded geometric returns; in this example, practitioners would focus on the 13 percent a year loss, not on the zero arithmetic mean. If stocks have 16 percent average annual volatility (the average since World War II), the result is that the arithmetic mean is 130 bps higher than the geometric mean return (the difference is approximately half the variance, or 16 percent × 16 percent/2). Such a difference might be considered a "penalty for risk." If we add a 70 bp real cash yield (the historical average) plus a 720 bp risk premium minus a 130 bp penalty for risk, we find 6.6 percent to be the implied consensus of the economists for the geometric real stock return.
7. Such a return could easily fall to 0–2 percent net of taxes, especially in light of government's taxes on the inflation component of returns.
8. Smith's work even won a favorable review from John Maynard Keynes (for Keynes' approach, see his 1936 classic).
9. TIPS is the acronym for Treasury Inflation-Protected Securities, which have been replaced by Treasury Inflation-Indexed Securities.
10. In fairness, growth is now an explicit part of the picture. Dividend payout ratios are substantially lower than in the early 1920s and the 19th century as a result, at least in part, of corporate desires to finance growth. That said, our own evidence would suggest that internal reinvestment is not necessarily successful: High payout ratios precede higher growth than do low payout ratios.
11. We are indebted to G. William Schwert and Jeremy Siegel for some of the raw data for this analysis (see also Schwert 1990 and Siegel 1998). Although multiple sources exist for data after 1926 and a handful of sources provide data beginning in 1855 or 1870, Professor Schwert was very helpful in assembling these difficult early data. Professor Siegel provided earnings data back to 1870. We have not found a source for earnings data before 1870.
12. The U.S. Bureau of Labor Statistics maintains GDP data from 1921 to date; the earlier data are for GNP (gross national product). Because the two were essentially the same thing until international commerce became the substantial share of the economy that it is today, we used the GNP data from the Bureau of Labor Statistics for the 19th century and the first 20 years of the 20th century.
13. We stripped out reinvestment in the measure of real dividend growth shown in Figure 3 because investors are already receiving the dividend. To include dividends in the real dividend growth would double-count these dividends. What should be of interest to us is the internal growth in dividends stemming from reinvestment of the retained earnings.
14. We multiplied the real dividends by 10 to bring the line visually closer to the others; the result is that on those few occasions when the price line and dividend line touch, the dividend yield is 10 percent.
15. The fact that growth in real dividends and earnings is closer to per capita GDP growth than it is to overall GDP growth is intuitively appealing on one fundamental basis: Real per capita GDP growth measures the growth in productivity. It is sensible to expect real income, real per share earnings, and real per share dividends to grow with productivity rather than to mirror overall GDP growth.
16. This history holds a cautionary tale with regard to today's stock option practices.
17. This fall in dividends of existing enterprises is not surprising when one considers that the companies that existed in 1802 probably encompass, at most, 1 percent of the economy of 2001. The world has so changed that, at least from the perspective of the dominant stocks, today's economy would be unrecognizable in 1802.
18. Another way to think about this idea is to recognize the distinction between a market portfolio and a market index. The market portfolio shows earnings and dividend growth that are wholly consistent with growth in the overall economy (Bernstein 2001a). But if one were to unitize that market portfolio, the unit values would not grow as fast as the total capitalization and the earnings and dividends per unit (per "share" of the index) would not keep pace with the growth in the aggregate dollar earnings and dividends of the companies that compose the market portfolio. (When one stock is dropped and another added to a market index, typically the added stock is larger in capitalization than the deletion, which increases the divisor for constructing the index.) Precisely the same thing would happen in the management of an actual index fund. When a stock was replaced, the proceeds from the deleted stock would rarely suffice to fund the purchase of the added stock. So, all stocks would be trimmed slightly to fund that purchase; this consequence is implied by the change in the divisor for an index. It is this mechanism that drives the difference between the growth of the aggregate dollar earnings and dividends for the market portfolio, which will keep pace with GDP growth over time, and the growth of the "per share" earnings and dividends for the market *index* that creates the dilution we attribute to entrepreneurial capitalism. After all, entrepreneurial capitalism creates the companies that we must add to the market portfolio, thus changing our divisor and driving a wedge between the growth in market earnings or dividends and the growth in earnings and dividends per share in a market index.
19. See Bernstein (2001b). Over the past 131 years, the correlation between payout ratios and subsequent 10-year growth in real earnings has been 0.39; over the past 50 years, this correlation has soared to 0.66. Apparently, the larger the fraction of earnings paid out as dividends, the faster earnings subsequently grow, which is directly contrary to the Miller–Modigliani maxim (see Miller and Modigliani 1961 and Modigliani and Miller 1958).
20. To produce a 3.4 percent real return from stocks, matching the yield on TIPS, real growth in dividends needs to be 1.9 percent (twice the long-term historical real growth rate) while valuation levels remain where they are. Less than twice the historical growth in real dividends, or a return to the 3–6 percent yields of the past, will not get us there.
21. We have made the simplifying assumption that "long term" is a 10-year horizon. Redefining the long-term returns over a 5-year or 20-year horizon produces similar results.
22. Because this adjusted dividend is always at or above the true dividend, we have introduced a positive error into the average dividend yield. We offset this error by subtracting the 40-year average difference between the adjusted dividend and the true dividend. In this way, $EDY(t)$ is not overstated, on average, over time.

23. Of course, stock buybacks increase the share of the economy held by existing shareholders.
24. Arnott and Asness (2002) have shown that since 1945, the payout ratio has had a 77 percent correlation with subsequent real earnings growth. That is, higher retained earnings have historically led to slower, not faster, earnings growth.
25. Throughout this article, when we refer to a 10-year average or a 40-year average, we have used the available data if fewer years of data were available. For instance, for 1820, we used the 20-year GDP growth rate because 40 years of data were unavailable. We followed a convention of requiring at least 25 percent of the intended data; so, if the analysis was based on a 40-year average, we tolerated a 10-year average if necessary. To do otherwise would have forced us to begin our analysis in about 1840 and lose decades of interesting results. Because data before 1800 are very shaky and we required at least 10 years of data, our analysis begins, for the most part, in 1810.
26. We cannot know the 10-year returns from starting dates after 1991, so 192 years of expected return data lead to 182 years of correlation with subsequent 10-year actual returns.
27. Another way to deal with serially correlated data is to test correlations of differenced data. When we carried out such tests, we found that over the full span, the R^2 actually *rose* to 0.446 from the 0.214 shown in Panel A of Table 1; moreover, since 1945, the differenced results showed a still impressive 46 percent correlation. These results are available from the authors on request.
28. In an *ex ante* regression, the model is respecified for each monthly forecast with the use of all previously available data only.
29. We made the simplifying assumption that "long term" is a 10-year horizon. Redefining the long-term returns over a 5-year or 20-year horizon produced similar results.
30. Even when we considered successive differences to eliminate the huge serial correlation of real bond yields and 10-year real bond returns, the result from 1945 to date (available from the authors) was identical to the result for the raw data—a correlation of 0.63.
31. For investors accustomed to the notion that stock returns are uncertain and bond returns are assured over the life of the bond, this result will come as a surprise. But conventional bonds do *not* assure real returns; their expected real returns, therefore, should be highly uncertain. Stocks do, in a fashion, pass inflation through to the shareholder. So, nominal returns for stocks may be volatile and uncertain, but expected real stock returns are much more tightly defined than expected real bond returns.
32. Differencing caused the correlation for the full 182-year span to fall from 0.66 to 0.61 and, for the span following World War II, caused it to fall from 0.79 to 0.48.
33. For the taxable investor, the picture is worse, of course. In the United States, investors are even taxed on the inflation component of returns. From valuation levels that are well above historical norms, a negative real after-tax return is not at all improbable.
34. The excess return of stocks over bonds was negative also in the decades ended September 1991, November 1990, most 10-year spans ending August 1977 to June 1979, and the spans ending September 1974 to January 1975.
35. Consider the 10 years starting just before the stock market crash in September 1987. This span began with double-digit bond yields. The bond yield of 9.8 percent minus a regression-based inflation expectation of 3.6 percent led to an expected real bond return of 6.2 percent. The stock yield of 2.9 percent plus expected real per capita GDP growth of 1.6 percent minus an expected dividend shortfall relative to per capita GDP of 0.4 percent led to an expected real stock return of 4.0 percent. The risk premium was –2.0 percent. But stocks beat bonds by 4.9 percent a year over the next 10 years ending September 1997. What happened? The dividend yield plunged to 1.7 percent. This plunge in yields contributed 5.8 percent a year to stock returns; in the absence of this revaluation, stocks would have underperformed bonds by –0.9 percent. So, the –2.0 percent forecast was not bad; dividends rose a notch faster than normal, and more importantly, the price that the market was willing to pay for each dollar of dividends nearly doubled.

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Stocks versus Bonds: Explaining the Equity Risk Premium

Clifford S. Asness

From the 19th century through the mid-20th century, the dividend yield (dividends/price) and earnings yield (earnings/price) on stocks generally exceeded the yield on long-term U.S. government bonds, usually by a substantial margin. Since the mid-20th century, however, the situation has radically changed. In addressing this situation, I argue that the difference between stock yields and bond yields is driven by the long-run difference in volatility between stocks and bonds. This model fits 1871–1998 data extremely well. Moreover, it explains the currently low stock market dividend and earnings yields. Many authors have found that although both stock yields forecast stock returns, they generally have more forecasting power for long horizons. I found, using data up to May 1998, that the portion of dividend and earnings yields explained by the model presented here has predictive power only over the long term whereas the portion not explained by the model has power largely over the short term.

The dividend yield on the S&P 500 Index has long been examined as a measure of stock market value. For instance, the well-known Gordon growth model expresses a stock price (or a stock market's price) as the discounted value of a perpetually growing dividend stream:

$$P = \frac{D}{R-G}. \tag{1}$$

where

P = price
D = dividends in Year 0
R = expected return
G = annual growth rate of dividends in perpetuity

Now, solving this equation for the expected return on stocks produces

$$R = \frac{D}{P} + G. \tag{2}$$

Thus, if growth is constant, changes in dividends to price, D/P, are exactly changes in expected (or required) return. Empirically, studies by Fama and French (1988, 1989), Campbell and Shiller (1998), and others, have found that the dividend yield on the market portfolio of stocks has forecasting power for aggregate stock market returns and that this power increases as forecasting horizon lengthens.

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The market earnings yield or earnings to price, E/P (the inverse of the commonly tracked P/E), represents how much investors are willing to pay for a given dollar of earnings. E/P and D/P are linked by the payout ratio, dividends to earnings, which represents how much of current earnings are being passed directly to shareholders through dividends. Studies by Sorenson and Arnott (1988), Cole, Helwege, and Laster (1996), Lander, Orphanides, and Douvogiannis (1997), Campbell and Shiller (1998), and others, have found that the market E/P has power to forecast the aggregate market return.

Under certain assumptions, a bond's yield-to-maturity, Y, will equal the nominal holding-period return on the bond.[1] Like the equity yields examined here, the inverse of the bond yield can be thought of as a price paid for the bond's cash flows (coupon payments and repayment of principal). When the yield is low (high), the price paid for the bond's cash flow is high (low). Bernstein (1997), Ilmanen (1995), Bogle (1995), and others, have shown that bond yield levels (unadjusted or adjusted for the level of inflation or short-term interest rates) have power to predict future bond returns.

This article examines the relationship between stock and bond yields and, by extension, the relationship between stock and bond market returns (the difference between stock and bond expected returns is commonly called the equity risk premium). I hypothesize that the relative yield stocks must provide versus bonds today is driven by the experience of each generation of investors with each asset class.

The article also addresses the observation of many authors, economists, and market strategists that today's dividend and earnings yields on stocks are, by historical standards, shockingly low. I find they are not.

Finally, I report the results of decomposing stock yields into a fitted portion (i.e., stock yields explained by the model presented here) and a residual portion (i.e., stock yields not explained by the model).

Historical Yields on Stocks and Bonds

As far as yields are concerned, 1927–1998 tells a tale of two periods—as **Figure 1** clearly shows. Figure 1 plots the dividend yield for the S&P 500 and the yield to maturity for a 10-year U.S. T-bond from January 1927 through May 1998.[2] Prior to the mid-1950s, the stock market's yield was consistently above the bond market's yield. Anecdotally, investors of this era believed that stocks should yield more than bonds because stocks are riskier investments. Since 1958, the stock yield has been below the bond yield, usually substantially below. As of the latest data in Figure 1 (May 1998), the stock market yield was at an all-time low of 1.5 percent whereas the bond market yield was at 5.5 percent, not at all a corresponding low point. This observation has led many analysts to assert that the role of dividends has changed and that dividend yields in the late 1990s are not comparable to those of the past. Although this assertion may have some merit, I will argue that it is largely unnecessary to explain today's low D/P.

As did dividend yields, the stock market's earnings yields systematically exceeded bond yields early in the sample period, but as **Figure 2** shows, since the late-1960s, earnings yields have been comparable to bond yields and clearly strongly related (as are dividend yields, albeit from a lower level).[3] **Table 1** presents monthly correlation coefficients for various periods between the levels of D/P and *Y* and E/P and *Y*. The numbers in Table 1 clearly bear out what is seen in Figures 1 and 2. For the entire period, D/P and *Y* were negatively correlated because of their reversals; E/P was essentially uncorrelated with *Y*. For the later period, however, stock and bond yields show the strong positive relationship many economists and market strategists have noted.

Thus, we are left with several puzzles:

- Why did the stock market strongly outyield bonds for so long only to now consistently underyield bonds?
- Why did stock and bond yields move relatively independently, or even perversely, in the overall 1927–98 period but move strongly together in the later 40 years of this period?
- Perhaps most important, why are today's stock market yields so low and what does that fact mean for the future?

The rest of this article tries to answer these questions.

Figure 1. S&P 500 Dividend Yield and T-Bond Yield to Maturity, January 1927–May 1998

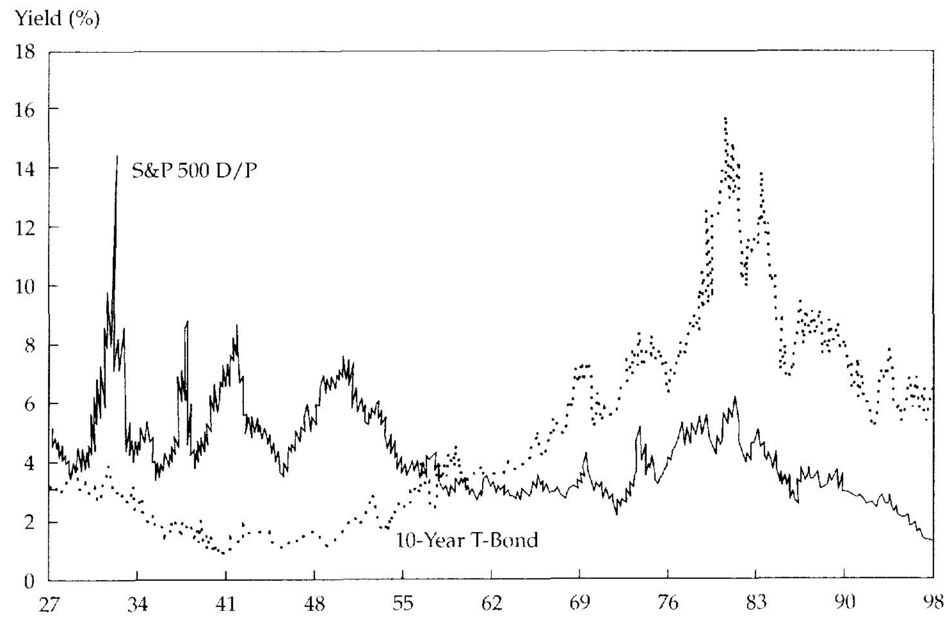


Figure 2. S&P 500 Earnings Yield and T-Bond Yield to Maturity, January 1927–May 1998

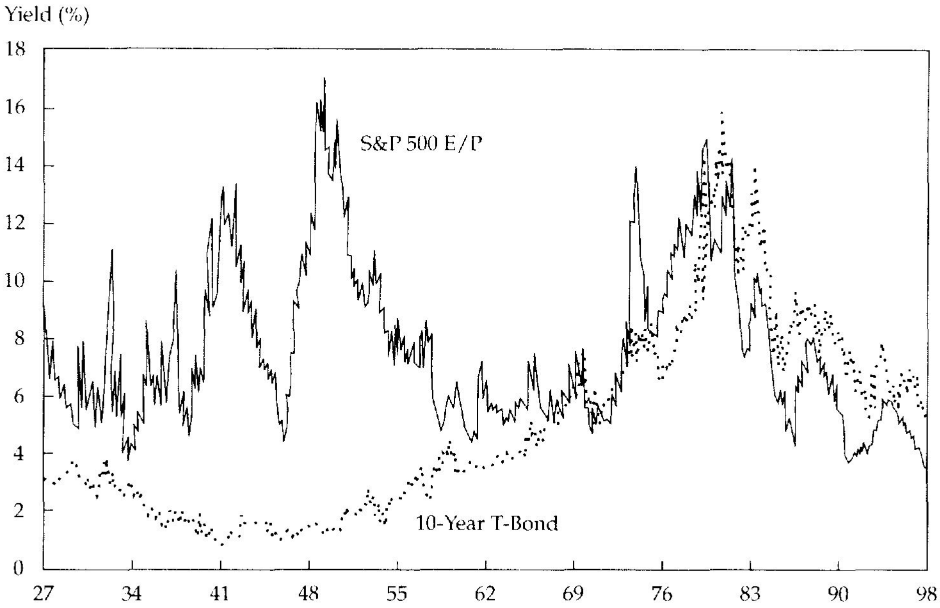


Table 1. Monthly Correlation Coefficients, Various Periods

Period	Correlation of D/P and Y	Correlation of E/P and Y
Full (January 1927–May 1998)	-0.28	+0.08
Early (January 1927–December 1959)	-0.23	-0.49
Late (January 1960–May 1998)	+0.71	+0.69

Model for Stock Market Yields

Researchers have shown a strong link between aggregate dividend and earnings yields and expected stock market returns, especially for long horizons. When stock market yields are high (low), expected future stock returns are high (low). This predictability has two possible explanations that are at least partly consistent with efficient markets (there are many *inefficient*-market explanations). One, investors' taste for risk varies. When investors are relatively less risk averse, they demand less in the way of an expected return premium to bear stock market risk. Fama and French (1988, 1989), among others, explored this hypothesis. Two, the perceived level of risk can change even if investors' taste for risk is constant.

I explore the hypothesis that the perceived level of risk can change (although the two hypotheses are not mutually exclusive). Note that investor perception of long-term risk need not be accurate for this hypothesis to be true. If investor perception of risk is accurate, then the evidence presented here may be consistent with an efficient market. If investor perception of risk is inaccurate but explains the pricing of stocks versus bonds, then the hypothesis may be deemed accurate but still pose a dilemma for fans of efficient markets.

Consider a simple model in which the required long-term returns on aggregate stocks and bonds vary through time. Expected stock returns, $E(\text{Stocks})$, are assumed to be proportional to dividend yields, whereas expected bond returns, $E(\text{Bonds})$, are assumed to move one-for-one with current bond yields; that is,

$$E(\text{Stocks})_t = a + b(\text{D/P}_t) + \varepsilon_{Stocks,t}, \qquad (3)$$

$$E(\text{Bonds})_t = Y_t + \varepsilon_{Bonds}, \qquad (4)$$

(where a is the intercept, b is the slope, D/P_t is dividend yield at time t, and ε is an error term). The hypothesis is that b is positive, so expected stock returns vary positively with current stock dividend yields, and that the ε terms are identically and independently distributed error terms representing the portion of expected returns not captured by the model.[4]

Now, I assume that expected stock and bond returns are linked through the long-run stock and bond volatility experienced by investors. So,

$$E(\text{Stocks})_t - E(\text{Bonds})_t = c + d\sigma(\text{Stocks})_t + e\sigma(\text{Bonds})_t. \quad (5)$$

The hypothesis is that d is positive whereas e is negative. That is, I assume that the expected (or required) return differential between stocks and bonds is a positive linear function of a weighted difference of their volatilities.[5] Although Equations 3, 4, and 5 do not represent a formal asset-pricing model, they do capture the spirit of allowing expected returns to vary through time as a function of volatility. Moreover, they yield empirically testable implications.[6]

Rearranging these equations (and aggregating coefficients) produces the following model:

$$D/P = \gamma_0 + \gamma_1 Y + \gamma_2 \sigma(\text{Stocks}) + \gamma_3 \sigma(\text{Bonds}) + \varepsilon_{D/P,t}. \quad (6)$$

Now, the hypothesis is that γ_1 is positive, γ_2 is positive, and γ_3 is negative. This model, and the precisely corresponding model for E/P, is tested in the following section.[7] Other authors (e.g., Merton 1980; French, Schwert, and Stambaugh 1987) have tested the link between expected stock returns and volatility by examining the relationship between realized stock returns and *ex ante* measures of volatility.[8] However, as these authors noted, realized stock returns are a noisy proxy for expected stock returns. I believe that linking Equations 3, 4, and 5 and focusing on the long term will reveal a clearer relationship between stock market volatility and expected stock market returns as represented by stock market yield (D/P or E/P).[9]

Preliminary Evidence

To investigate Equation 6, I defined a generation as 20 years and used a simple rolling 20-year annualized monthly return volatility for σ(Stocks) and σ(Bonds).[10] The underlying argument is that each generation's perception of the relative risk of stocks and bonds is shaped by the volatility it has experienced. For instance, Campbell and Shiller (1998) mentioned (but did not necessarily advocate) the argument that Baby Boomers are more risk tolerant "perhaps because they do not remember the extreme economic conditions of the 1930s." Another example is Glassman and Hassett (1999), who argued in *Dow 36,000* that remembrances of the Great Depression have led investors to require too high an equity risk premium.

A 20-year period captures the long-term generational phenomenon that I hypothesized.[11] The hypothesis is inherently behavioral because it states that the long-term, slowly changing relationship between stock and bond yields is driven by the long-term volatility of stocks and bonds experienced by the bulk of current investors. Although I believe a 20-year period is intuitively reasonable, given the hypothesis, I am encouraged by the fact that the results that follow are robust to alternative specifications of long-term volatility (i.e., from 10-year to 30-year trailing volatility) and still showed up significantly when windows as short as 5 years were used.

The regressions in this section are simple linear regressions that do not account for some significant econometric problems; for example, the following regressions have highly autocorrelated independent variables, dependent variables, and residuals. But the goal of these regressions is to initially establish the existence of an economically significant relationship. Because statistical inference is problematic, I do not focus on (but do report) the t-statistics. The focus is on the economic significance of the estimated coefficients and R^2 figures. (Subsequent sections explore the issue of statistical significance and report robustness checks.)

Because I required 20 years to estimate volatility and the monthly data began in 1926, I estimated Equation 6 by using monthly data from January 1946 through May 1998. Before examining this equation in full, I first examine the regression of D/P on bond yields only and D/P on the rolling volatility of stock and bond markets only for the 1946–98 period (the first data points are dividend and bond yields in January 1946 and stock and bond volatility estimated from January 1926 through December 1945; the t-statistics are in parentheses under the equations. The results are as follows:

$$\underset{}{D/P} = \underset{(40.72)}{4.10\%} - \underset{(-2.26)}{0.03Y} \quad (7)$$

(with an adjusted R^2 of 0.7 percent) and

$$D/P = \underset{(11.87)}{2.02\%} + \underset{(18.96)}{0.14\sigma(\text{Stocks})} - \underset{(-5.24)}{0.07\sigma(\text{Bonds})} \quad (8)$$

(with an adjusted R^2 of 43.0 percent).[12]

Equation 7 shows that D/P and Y have a mildly negative relationship for 1946–1998, similar to what I found for the entire 1926–98 period (Table 1). Equation 8 shows that a significant amount of the variance of D/P (note the adjusted R^2) is explained by stock and bond volatility, with D/P rising with stock market volatility and falling with bond market volatility. This relationship is economically significant. An increase in stock market volatility from 15 percent to 20 percent, all else being equal, raises the required dividend yield on stocks by 70 basis points (bps). Now, note the estimate for Equation 6:

$$D/P = \underset{(-0.05)}{0.00\%} + \underset{(28.77)}{0.35Y} + \underset{(39.51)}{0.23\,\sigma(\text{Stocks})} - \underset{(-25.69)}{0.31\sigma(\text{Bonds})} \quad (9)$$

(with an adjusted R^2 of 75.4 percent).

This result supports the hypothesis. The dividend yield is mildly negatively related to the bond yield when measured alone (Equation 7), but this negative relationship is a highly misleading indicator of how stock and bond yields covary. When I adjusted for different levels of volatility, I found stock and bond yields to be strongly positively related. My interpretation of this regression is that stock and bond market yields are strongly positively related and the difference between stock and bond yields is a direct positive function of the weighted difference between stock and bond volatility. Intuitively, the more volatile stocks have been versus bonds, the higher the yield premium (or smaller a yield deficit) stocks must offer. In any case, when volatility is held constant, stock yields do rise and fall with bond yields.

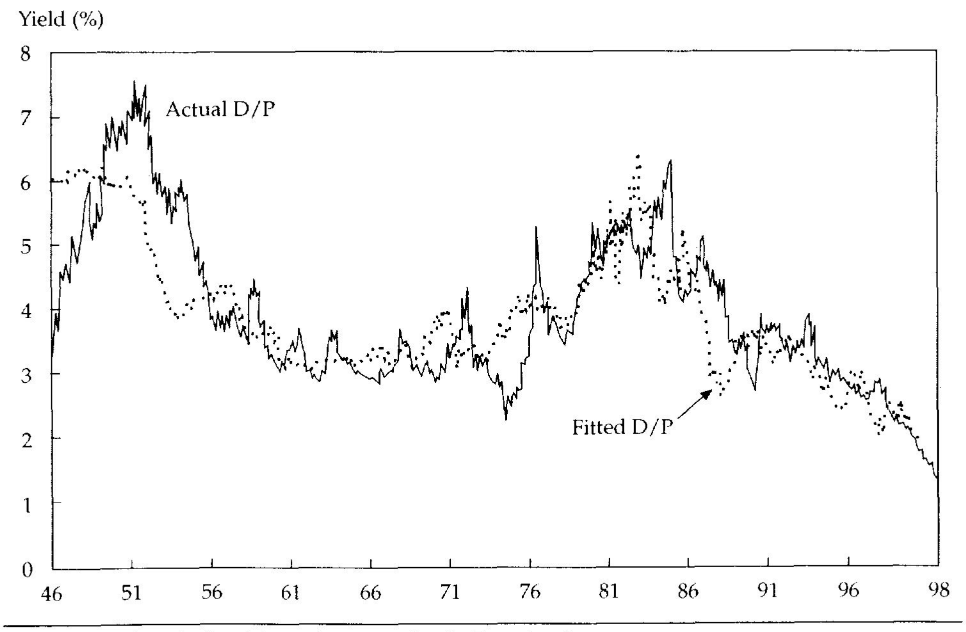
Again, these results are economically significant. For example, a 100 bp rise in bond yields translates to a 35 bp rise in the required stock market dividend yield, whereas a rise in stock market volatility from 15 percent to 20 percent leads to a rise of 115 bps in the required stock market dividend yield.

The fact that stock and bond yields are univariately unrelated (or even negatively related) over long periods (Table 1) is a result of changes in relative stock and bond volatility that obscure the strong positive relationship between stock and bond yields. The reason stock and bond yields are univariately positively related over shorter periods (e.g., 1960–1998) is because of the stable relationship between stock and bond volatility over short periods. In other words, a missing-variable problem is not much of a problem if the missing variable was not changing greatly during the period being examined (such as in 1960–1998). The problem is potentially destructive, however, if the missing variable varied significantly during the period (such as in 1927–1998).

Figure 3 presents the actual market D/P and the in-sample D/P fitted from the regression in Equation 9. **Figure 4** presents the residual from this regression (actual D/P minus fitted D/P). For today's reader, perhaps the most interesting part of Figures 3 and 4 is the latest results. The actual D/P at the end of May 1998 (the last data point) is 1.5 percent, a historic low. The forecasted D/P is also at a historic low, however—2.1 percent—which is a forecasting error of only 60 bps.

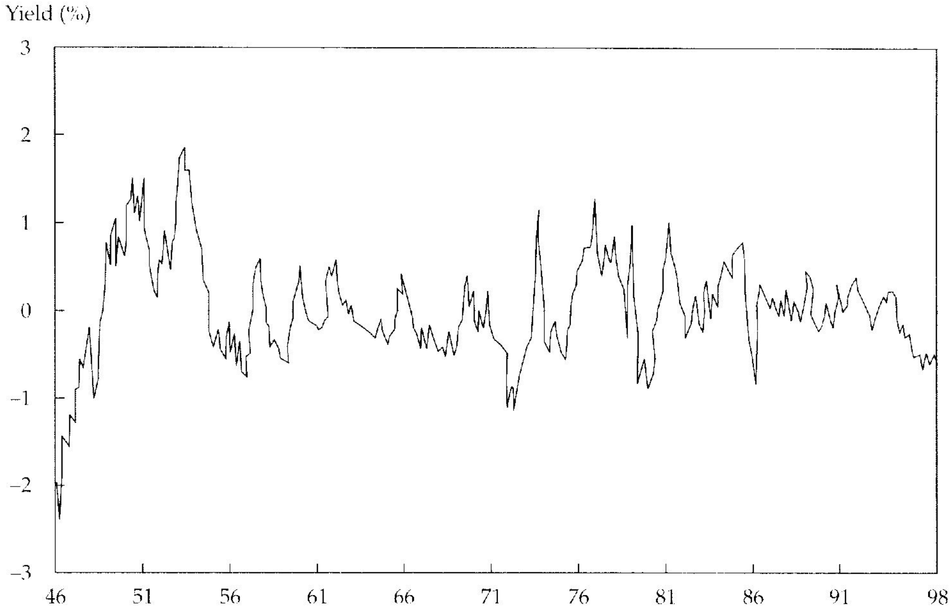
Simply examining the D/P series leads to a belief that recent D/Ps are shockingly low. These regressions suggest a different interpretation: Given the recent low bond yields and a low realized differential in volatility between stocks and bonds, I would forecast an all-time historically low D/P for stocks as of May 1998. The fact that the model does not forecast the actual low in dividend yield is not statistically anomalous (May's forecast error is about 1 standard deviation below zero) and may be a result of the stories other authors have cited to explain today's low D/P (e.g., stock buy-backs

Figure 3. Actual S&P 500 Dividend Yield and In-Sample Dividend Yield, January 1946–May 1998



Note: In-sample D/P fitted from the regression in Equation 9.

Figure 4. Regression Residual: Actual D/P minus Fitted D/P, January 1946–May 1998



replacing dividends). But these stories might not be at all necessary. For example, the story of stock buybacks replacing dividends has been around since at least the late 1980s (Bagwell and Shoven 1989), yet the average in-sample forecasting error of my model for D/P for 1990–1998 is only –9 bps. Apparently, nothing more than Equation 9 is needed to explain recent low dividend yields.

Running a similar regression for E/P, I obtained the following result:

$$E/P = -1.39\% + 0.96Y + 0.49\sigma(\text{Stocks}) - 0.76\sigma(\text{Bonds}) \quad (10)$$
$$(-3.70) \quad (27.33) \quad (29.58) \quad (-21.56)$$

(with an adjusted R^2 of 64.8 percent). The model explains about as much of the variance for earnings yield as dividend yield. As of the end of May 1998, the E/P for the S&P 500 was 3.6 percent, corresponding to a P/E of 27.8. The forecasted E/P from the Equation 10 regression is 3.4 percent, or a forecasted P/E ratio of 29.1. Unlike the case for D/P, I am not (even to a small degree) failing to explain the recent high P/Es on stocks; rather, one would have to explain the opposite, because according to the model, the May 1998 P/E of 27.8 is slightly *lower* than it should be.

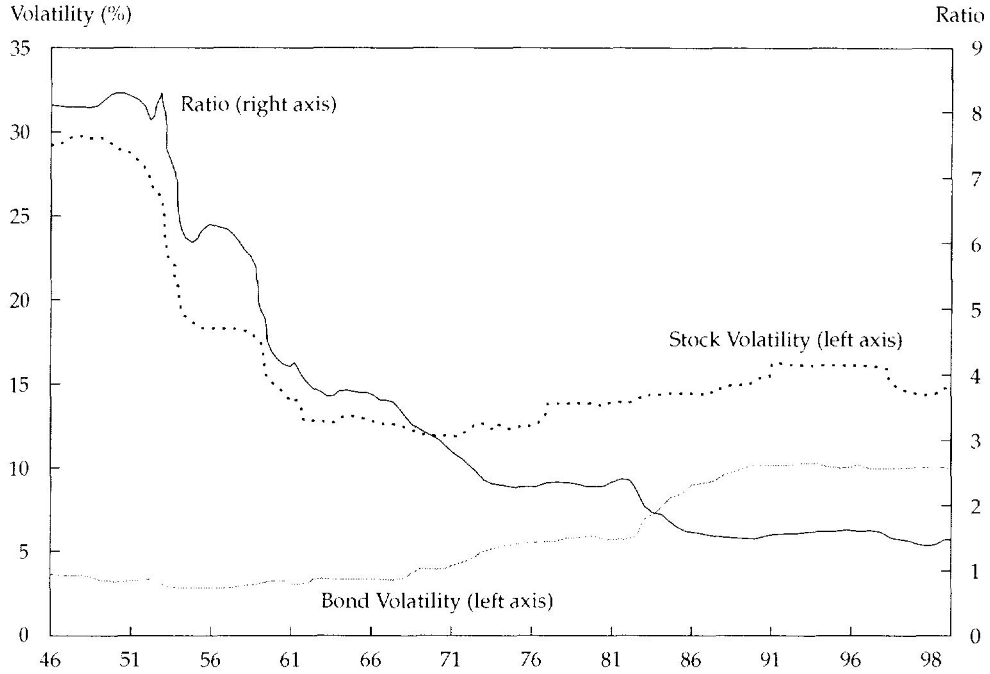
Again, these results are economically significant: The required earnings yield was moving virtually one-for-one with 10-year T-bond yields and increasing 245 bps for each 5 percent rise in stock market volatility (all else being equal). Examining Figure 2 and Table 1 shows that E/P and *Y* were strongly positively correlated only for the later period of the sample (in the earlier period, they were actually negatively correlated, and for the whole period, they were close to uncorrelated). When changing stock- and bond-market volatility is accounted for in Equation 10, however, the strong positive relationship between E/P and *Y* is extended to the full period.

Critique and Further Evidence

The regression results presented in the previous section fit intuition and the hypothesis as formalized in Equation 6, but they are certainly open to criticism. They are in-sample regression results and are thus particularly open to charges of data mining. They are level-on-level regressions, which renders the *t*-statistics invalid and makes the high R^2 figures potentially spurious.[13] Worse, they are level-on-level regressions that use 20-year rolling data and a highly autocorrelated dependent variable.[14] Because the inference is suspect, stock and bond volatility may have followed a pattern that explained a secular-level change in dividend and earnings yield merely by chance.

To examine this possibility, **Figure 5** shows the rolling 20-year volatilities of the stock and bond markets used in the preceding regressions and the ratio of stock to bond volatility. Aside from the very early and very late years of the period, the ratio of rolling 20-year stock volatility to bond volatility was dropping nearly monotonically from 1946 through mid-1998. Thus, a hypothesis that fits the regression results and Figure 5 is that stock yields and bond yields are positively related but, exogenous to this relationship, the level of stock yields has been declining over time.

Figure 5. Rolling 20-Year Volatilities of Stock and Bond Markets and Ratio of Stock to Bond Volatility, January 1946–May 1998



The issue is one of causality. Was the drop in the level of stock yields versus that of bond yields occurring because of changes in their relative experienced volatilities (as I hypothesize), or were other factors causing this drop through time and thus producing spurious regression results? A 50-year regression that uses 20-year rolling data makes answering this question difficult. So, the next subsections attempt to explore this critique.

Performance of the Model versus a Time Trend. If the drop in stock yields versus bond yields is coincidentally, not causally, related to volatility, then a time trend might do as well as volatility in the regression tests. For ease of comparison, recall the results for D/P regressed on bond yields and stock and bond volatility; Equation 9 was

$$D/P = 0.00\% + 0.35Y + 0.23\,\sigma(\text{Stocks}) - 0.31\sigma(\text{Bonds}),$$
$$(-0.05)\quad(28.77)\quad(39.51)\qquad(-25.69)$$

and the adjusted R^2 was 75.4 percent. The next equations report similar regressions in which, instead of stock and bond volatility, either a linear or loglinear time trend was used:

$$D/P = 6.18\% + 0.25(Y) - 0.00(\text{Linear trend}) \qquad (11)$$
$$(51.44)\quad(14.69)\quad(-21.97)$$

(with an adjusted R^2 of 43.8 percent) and

$$D/P = 27.97\% + 0.33(Y) - 0.04(\text{Loglinear trend}) \qquad (12)$$
$$(32.32)\quad(19.88)\quad(-27.61)$$

(with an adjusted R^2 of 55.1 percent).

The time-trend variables capture much of the effect being studied. That is, the relationship between D/P and Y goes from weakly negative (Equation 7) to strongly positive in the presence of the trend variable—meaning that the expected difference between stock and bond yields was declining through time and, after accounting for this trend, stock and bond yields were positively related. The volatility-based regression, however, is clearly the strongest: The adjusted R^2 is higher, and the coefficient on bond yields is larger and more significant.

Next, the loglinear time trend is added to Equation 9 to see how the volatility variables fare in head-on competition:

$$D/P = -10.00\% + 0.35Y + 0.28\sigma(\text{Stocks}) \qquad (9a)$$
$$(-3.98)\quad(28.50)\quad(19.63)$$
$$-\,0.46\sigma(\text{Bonds}) + 0.02(\text{Loglinear trend})$$
$$(-11.87)\qquad(3.99)$$

(with an adjusted R^2 of 76.0 percent).

Clearly, the volatility variables drive out the time trend (analogous results held for the linear time trend) to the point at which the trend's coefficient is slightly positive (the wrong sign). Although the nearly monotonic fall in bond versus stock volatility makes it hard to distinguish between causality and coincidence for the 1946–98 period, the superiority of the volatility-based model over a time trend gives comfort. Analogous results favoring the volatility model were found for E/P.

Rolling Regression Forecasts. I formed rolling out-of-sample forecasts of D/P starting with January 1966. (I began in 1966 because I needed the 20 years from 1926 to 1946 to estimate volatility and the 20 years from 1946 to 1966 to formulate the first predictive regression.) The regressions used an "expanding window" that always started in January 1946 and went up to the month before the forecast.

For comparison purposes, I formed these forecasts based on five models. Model 1 attempted to forecast D/P by using only the average D/P (so the forecast of D/P on January 1966 was the average D/P from January 1946 through December 1965). Model 2 attempted to forecast D/P by using a rolling regression on bond yields only. Model 3 used a rolling version of the complete model from Equation 9 (a regression on bond yields, stock volatility, and bond volatility). Model 4 and Model 5 corresponded to rolling versions of, respectively, the linear trend model in Equation 11 and the log-linear trend model in Equation 12. **Table 2** presents the results of these out-of-sample forecasts.

The volatility-based Model 3 was nearly unbiased over the 1966–98 period, had the lowest absolute bias of any of the five models, and had the lowest standard deviation of forecast error. The out-of-sample rolling regressions thus support the superiority of the volatility model, although again, the time-trend models are somewhat effective when compared with the more naive Models 1 and 2.

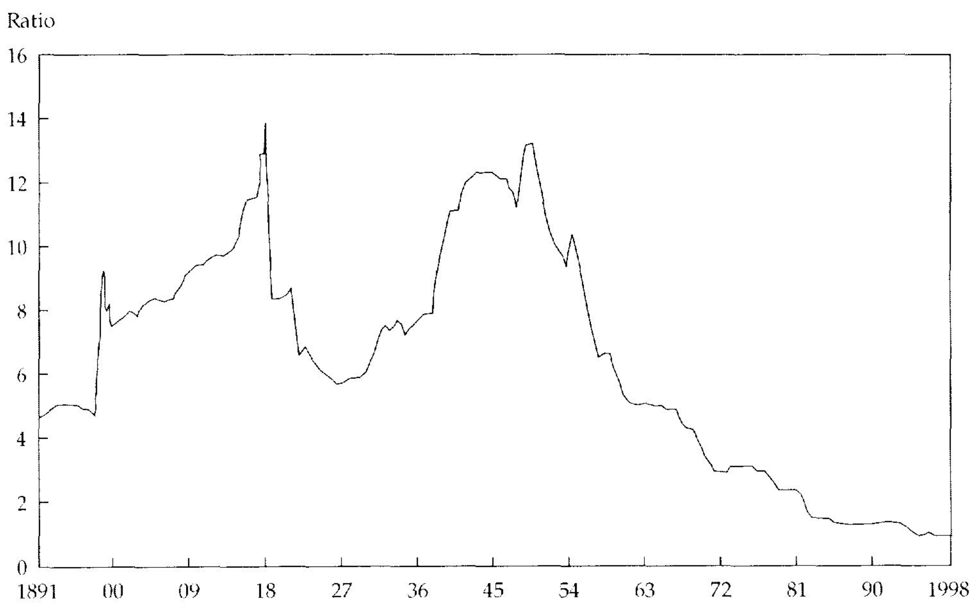
Earlier Data. The best response to many statistical problems is extensive out-of-sample testing—that is, tests with data for a previously unexamined period. All of the tests so far used monthly data for the commonly studied period commencing in 1926. For the tests reported in this section, I used earlier data. Although perhaps not as reliable as the modern data, annual data on the aggregate stock and bond markets are available for as early as 1871.[15]

In addition to simply using new data points, examining the older information provides an advantage that is specific to this study. In **Figure 6**, the new data are used to plot the ratio of rolling 20-year stock market volatility to rolling 20-year bond market volatility over the entire 1891–1998 period.[16]

Table 2. Out-of-Sample Forecasts, January 1966–May 1998

Model	Average Forecasting Error	σ(Forecasting error)
1. Using average D/P	–0.56%	0.97%
2. Using regression on Y	0.29	1.38
3. Using the full model	0.14	0.50
4. Using linear time trend	0.71	0.66
5. Using loglinear time trend	0.54	0.62

Figure 6. Ratio of Rolling 20-Year Stock Market Volatility to Bond Market Volatility, January 1891–May 1998



Recall that one problem with testing the hypothesis for 1946–1998 was that the volatility ratio declined nearly monotonically. Figure 6 shows that the new data preserve this property for this same time period but that the 1891–1945 period reflects no monotonic trend. Thus, if the model works for 1891–1945, or 1891–1998, a spurious time trend is not driving the results. I found that dividend yields also trended down strongly over the 1946–98 period but appear much more stationary when viewed over the entire 1891–1998 period (this figure is available upon request).

As a data check, before examining the pre-1946 data, I reexamined the 1946–98 period with the new annual data set. The following are *annual* regressions for the already-studied 1946–98 period:

$$D/P = 4.12\% - 0.04Y \quad (13)$$
$$(10.78) \quad (-0.65)$$

(with an adjusted R^2 of −1.1 percent);

$$D/P = -1.15\% + 0.29Y + 0.24\sigma(\text{Stocks}) - 0.16\sigma(\text{Bonds}) \quad (14)$$
$$(-1.64) \quad (6.07) \quad (8.03) \quad (-4.88)$$

(with an adjusted R^2 of 66.0 percent;

$$E/P = 6.98\% + 0.13Y \quad (15)$$
$$(7.57) \quad (0.95)$$

(with an adjusted R^2 of −1.8 percent);

$$E/P = -3.12\% + 0.85Y + 0.46\sigma(\text{Stocks}) - 0.40\sigma(\text{Bonds}) \quad (16)$$
$$(-1.64) \quad (6.07) \quad (8.03) \quad (-4.88)$$

(with an adjusted R^2 of 48.9 percent).

Although not precisely the same as the monthly regressions presented earlier, the annual regressions on the new data set are similar enough to be encouraging.

Now, consider the results for these same regressions for the earlier 1891–1945 data:

$$D/P = 2.60\% + 0.77Y \quad (17)$$
$$(2.70) \quad (2.72)$$

(with an adjusted R^2 of 10.6 percent);

$$D/P = -1.65\% + 1.36Y + 0.19\sigma(\text{Stocks}) - 0.53\sigma(\text{Bonds}) \quad (18)$$
$$(-1.18) \quad (5.00) \quad (4.75) \quad (-2.10)$$

(with an adjusted R^2 of 35.7 percent);

$$E/P = 4.20\% + 1.06Y \quad (19)$$
$$(2.20) \quad (1.90)$$

(with an adjusted R^2 of 4.6 percent);

$$E/P = 2.90\% + 1.68Y + 0.25\sigma(\text{Stocks}) - 2.23\sigma(\text{Bonds}) \quad (20)$$
$$(1.05) \quad (3.13) \quad (3.15) \quad (-4.50)$$

(with an adjusted R^2 of 31.5 percent).

These regressions provide bad news and good news. The bad news is that some of the regression coefficients are very different for the 1891–1945 period from what they were for the 1946–98 period. Apparently, the (admittedly simple) model is not completely stable over time. Given changes in the world economy from 1871 to 1998, to think that the coefficients would be completely stable is perhaps wildly optimistic.[17] The good news is that, although over the 1891–1945 period the stock market's D/P and E/P were univariately weakly positively related to Y (see Equations 17 and 19), this relationship became much more strongly positive when I allowed for changing relative stock and bond market volatilities (as in the completely separate 1946–98 period). This relationship was, as my hypothesis forecasted, a strong positive function of the previous 20 years' relative stock versus bond volatility.

Finally, I present the regressions for D/P for the full 1891–1998 period. For comparison, I also present full-period tests of the time-trend variables (the E/P results were highly analogous for all regressions):[18]

$$D/P = 5.20\% - 0.14Y \quad (21)$$
$$(17.79) \quad (-2.53)$$

(with an adjusted R^2 of 4.8 percent);

$$D/P = 5.90\% + 0.03Y - 0.00\text{Linear trend} \quad (22)$$
$$(17.32) \quad (0.42) \quad (-3.54)$$

(with an adjusted R^2 of 14.1 percent);

$$D/P = 7.75\% - 0.06Y - 0.07\text{Loglinear trend} \quad (23)$$
$$(6.09) \quad (-0.91) \quad (-2.06)$$

(with an adjusted R^2 of 7.6 percent);

$$D/P = 1.98\% + 0.26Y + 0.14\sigma(\text{Stocks}) - 0.29\sigma(\text{Bonds}) \quad (24)$$
$$(2.96) \quad (3.52) \quad (4.95) \quad (-5.65)$$

(with an adjusted R^2 of 35.5 percent).

The earlier data and the full-period data strongly support the central tenet of the hypothesis: Without adjusting for volatility and with or without a time trend (Equations 21–23), either a negative or flat relationship appears between D/P and bond yields over the entire period. After adjustment for relative stock and bond volatility, this relationship is strongly positive (Equation 24). Unlike the 1946–98 results, these results are clearly present in the absence of a significant trend in the ratio of stock to bond market volatility and despite any changes in the world economy from 1871 to 1998. In fact, unlike the volatility-based model, the time trends utterly fail to resurrect the positive relationship between stock and bond yields over the full period. When I used the data for 1946–1998, I introduced the issue of distinguishing whether the volatility-based model was spuriously supported because the changes in relative volatility approximated a time trend. The earlier and full-period evidence powerfully indicates that it is the time trend whose efficacy is spurious for 1946–1998, not the volatility-based model.

Full-Period Scatter Plots. As a final and perhaps most compelling test, I examined nonoverlapping 20-year periods from 1878 until 1998. I report the results for the resulting six observations in **Figure 7.** Figure 7 plots the ratio of annualized monthly stock market volatility over corresponding monthly bond volatility for the 20 years ending before the labeled year against the excess of stock market earnings yields over bond yields for the year in question. I chose earnings yields for this investigation because the evidence is that they are directly close to being comparable to bond market yields whereas dividend yields move as a dampened function of bond yields (that is, the coefficient on Y in Equation 10 is nearly 1.0, which makes the simple difference relevant to examine).

Figure 7 clearly supports the model: The greater stock volatility is versus bond volatility, the higher E/P must be versus Y. In contrast to the earlier regression tests, which were admittedly an econometric nightmare, nonoverlapping observations were used for Figure 7, and the autocorrelation of both the dependent and independent series was close to zero.[19] Thus, any need for econometric corrections (e.g., first differencing) was avoided.

The problem now is that I have only six observations, so the tests might lack power, but this is not the case. The t-statistic of the regression line is +7.64, and the adjusted R^2 is 92.0 percent. With six observations, a t-statistic must exceed +2.45 to be significant at a p value of 2.5 percent in a one-tailed test. Clearly, the t-statistic for this test is well past this level of significance.

As a robustness check, I recreated Figure 7 but starting 10 years later (resulting in only five observations over this period). The results are in **Figure 8.** This figure is even more striking than Figure 7 (the t-statistic in Figure 8 is +12.46, and the adjusted R^2 is 97.5 percent). Note from Figure 6 (the graph of the rolling volatility ratio) that two peaks are visible in the ratio of stock to bond volatility. These peaks roughly correspond to the right side of, respectively, Figures 7 and 8. In both cases, the model fits these extreme observations exceptionally well (that is, the largest volatility ratio corresponded to the largest end-of-period gap of stock earnings yield over bond yield). Also note that these two periods (the 20 years ending in 1918 and the 20 years ending in 1948) share no overlapping observations, yet the model fits both perfectly.

Figure 7. Ratio of Annualized Monthly Stock Market Volatility to Corresponding Monthly Bond Volatility versus Excess of Stock Market Earnings Yield over Bond Yield, 1871–May 1998

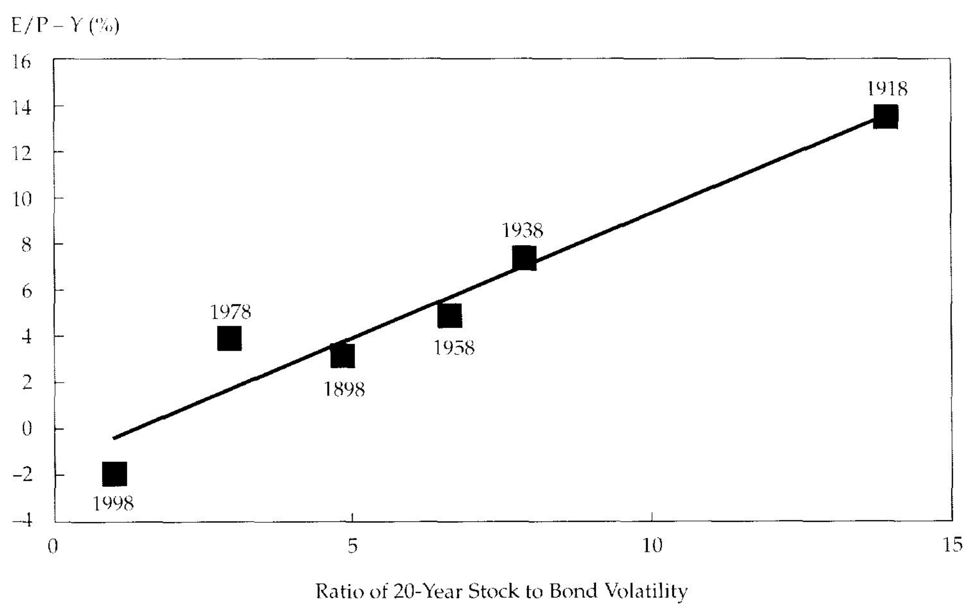
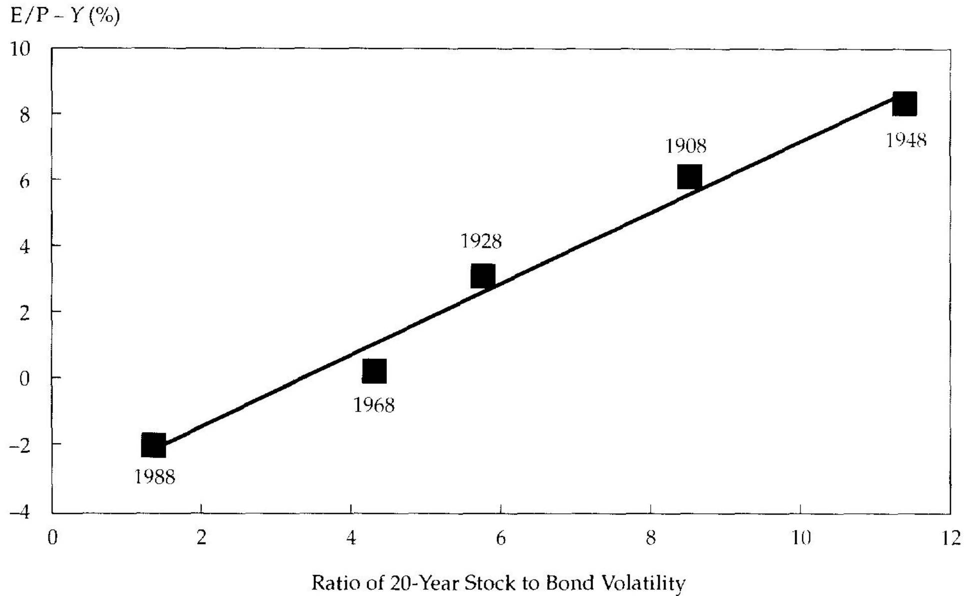


Figure 8. Ratio of Annualized Monthly Stock Market Volatility to Corresponding Monthly Bond Volatility versus Excess of Stock Market Earnings Yield over Bond Yield, 1881–May 1998



Finally, for completeness, I present in **Table 3** the adjusted R^2 and t-statistics for each of eight possible regressions on nonoverlapping periods for which I have six 20-year data points (each row in Table 3 presents the results of a regression that differs by one year in its starting and ending point from the prior/next row). Only one of these eight regressions produced results well below traditional levels of significance, and even in this case, the sign is correct.[20]

We believe these nonoverlapping tests are compelling evidence, irrespective of the econometric problems with our earlier tests, that following long-term periods of high (low) stock market volatility relative to bond market volatility, the required yield on stocks is relatively high (low) versus bonds.

Table 3. Statistics for Eight Regressions

Period	Adjusted R^2	t-Statistic
1891–1991	88.5%	+6.28
1892–1992	73.7	+3.87
1893–1993	81.0	+4.72
1894–1994	45.6	+2.28
1895–1995	9.9	+1.25
1896–1996	91.5	+7.42
1897–1997	78.3	+4.36
1898–1998	92.0	+7.64
Mean	70.1	+4.73
Median	79.7	+4.54

Note: Each row presents the results of a regression that differs by one year in its starting and ending point from the prior/next row.

Market Predictability

Researchers have found that variables D/P and E/P have power to forecast aggregate stock market returns. Moreover, this power appears to increase as time horizon lengthens (e.g., Fama and French 1988, 1989). I tested this finding for 1946–1998 using predictive regressions of excess monthly and annualized 5- and 10-year compound S&P 500 returns on aggregate D/P (t-statistics on all multiperiod regressions were adjusted for overlapping observations and heteroscedasticity). Here are the findings:

$$\text{S\&P monthly return} = \underset{(-1.03)}{-0.56\%} + \underset{(2.38)}{0.32\text{D/P}} \quad (25)$$

(with an adjusted R^2 of 0.7 percent);

$$\text{S\&P 5-year return} = \underset{(-0.88)}{-4.13\%} + \underset{(4.77)}{4.09\text{D/P}} \quad (26)$$

(with an adjusted R^2 of 56.1 percent);

$$\text{S\&P 10-year return} = \underset{(-0.38)}{-1.443\%} + \underset{(4.34)}{3.22\text{D/P}} \quad (27)$$

(with an adjusted R^2 of 58.7percent).

Equations 25–27 verify the findings of other authors that D/P has weak, but statistically significant, power for forecasting monthly returns and strong statistically significant power for forecasting longer-horizon returns.

Now, a new predictive variable, D/P(Error), is introduced. It is the in-sample residual term from the regression of D/P on Y, σ(Stocks), and σ(Bonds) for the 1946–98 period (Equation 9). It represents the

D/P on the S&P 500 in excess or deficit of what I would have predicted had I been using this model to forecast D/P (i.e., the unexplained portion). The results of the same regression tests as done for Equations 25–27 on this new variable are as follows (all results of this section were analogous when tested on E/P):

$$\text{S\&P monthly return} = \underset{(4.29)}{0.67\%} + \underset{(6.74)}{1.75}\text{D/P(Error)} \quad (28)$$

(with an adjusted R^2 of 6.6 percent);

$$\text{S\&P 5-year return} = \underset{(6.50)}{12.60\%} + \underset{(3.00)}{4.65}\text{D/P(Error)} \quad (29)$$

(with an adjusted R^2 of 21.2 percent);

$$\text{S\&P 10-year return} = \underset{(5.64)}{12.08\%} + \underset{(1.35)}{2.01}\text{D/P(Error)} \quad (30)$$

(with an adjusted R^2 of 7.1 percent).

Comparing the results for D/P(Error) with D/P shows that D/P(Error) has far more predictive power than D/P at short (monthly) horizons but far less power at longer horizons.[21] The power of D/P(Error) to forecast short-horizon returns can be interpreted as picking up time-varying risk aversion or, alternatively, as market mispricing (I leave this decision to future work). In either case, when D/P(Error) is high, stocks are selling for lower prices than is usual in the same interest rate and volatility environment and those low prices indicate higher short-horizon expected returns (and vice versa).

Finally, I formed D/P(Fit) as the fitted values from regression Equation 9. D/P(Fit) can be interpreted as the normal dividend yield as forecasted by the model considering the level of bond yields and stock and bond market volatility. By construction, the following relationship holds:

$$\text{D/P} = \text{D/P(Fit)} + \text{D/P(Error)}. \quad (31)$$

By regressing stock returns on both D/P(Fit) and D/P(Error), I decomposed the forecasting power of D/P into a portion coming from fitted D/P and a portion coming from residual D/P. The following regressions were carried out for 1946–98 data:[22]

$$\text{S\&P monthly return} = \underset{(2.07)}{1.25\%} - \underset{(-0.99)}{0.15}\text{D/P(Fit)} + \underset{(6.74)}{1.75}\text{D/P(Error)} \quad (32)$$

(with an adjusted R^2 of 6.6 percent);

$$\text{S\&P 5-year return} = \underset{(-0.56)}{-2.80\%} + \underset{(3.93)}{3.77}\text{D/P(Fit)} + \underset{(4.97)}{4.96}\text{D/P(Error)} \quad (33)$$

(with an adjusted R^2 of 57.1 percent);

$$\text{S\&P 10-year return} = \underset{(-0.76)}{-3.00\%} + \underset{(4.81)}{3.61}\text{D/P(Fit)} + \underset{(2.00)}{2.29}\text{D/P(Error)} \quad (34)$$

(with an adjusted R^2 of 61.1 percent).

Clearly, the power of D/P for predicting short-run (monthly) S&P 500 returns is driven by D/P(Error). As horizon lengthens, D/P(Fit) becomes more and more important, and at the 10-year horizon, D/P(Fit) is considerably more important.

To examine even longer forecast horizons and over longer periods, I again used annual data back to 1871 and formed D/P(Fit) and D/P(Error) from Equation 24. Recall that the first 20 years are needed to estimate volatility, so the following regressions are for 1891–1998 (all returns are annualized compound returns):

$$\text{S\&P annual return} = \underset{(1.89)}{18.1\%} - \underset{(-0.71)}{1.46}\text{D/P(Fit)} + \underset{(1.91)}{2.89}\text{D/P(Error)} \quad (35)$$

(with an adjusted R^2 of 2.0 percent);

$$\text{S\&P 5-year return} = \underset{(0.59)}{5.32\%} + \underset{(0.51)}{0.99}\text{D/P(Fit)} + \underset{(3.67)}{2.32}\text{D/P(Error)} \quad (36)$$

(with an adjusted R^2 of 12.2 percent);

$$\text{S\&P 10-year return} = \underset{(-0.21)}{-1.78\%} + \underset{(1.43)}{2.43}\text{D/P(Fit)} + \underset{(1.89)}{0.81}\text{D/P(Error)} \quad (37)$$

(with an adjusted R^2 of 12.4 percent);

$$\text{S\&P 15-year return} = \underset{(-3.91)}{-10.89\%} + \underset{(9.72)}{4.24}\text{D/P(Fit)} + \underset{(0.43)}{0.18}\text{D/P(Error)} \quad (38)$$

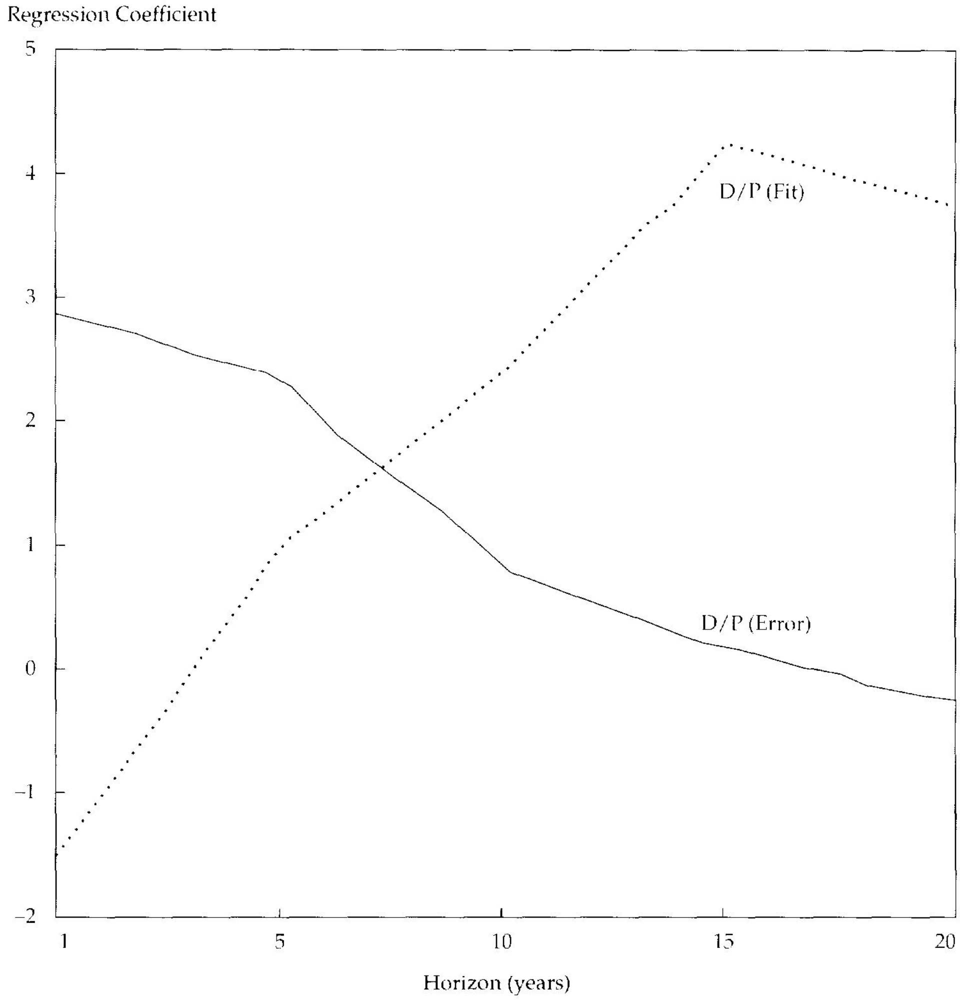
(with an adjusted R^2 of 33.7 percent);

$$\text{S\&P 20-year return} = \underset{(-2.36)}{-8.66\%} + \underset{(5.58)}{3.74}\text{D/P(Fit)} - \underset{(-2.08)}{0.29}\text{D/P(Error)} \quad (39)$$

(with an adjusted R^2 of 42.2 percent).

The estimated coefficients of D/P(Fit) and D/P(Error) for each of the forecast horizons (regression Equations 35–39) are plotted in **Figure 9**. Although annual predictability (Equation 35) is weak, the short-term predictability present is clearly driven by D/P(Error). The story changes dramatically as horizon increases, until at long

Figure 9. Estimated Coefficients of D/P(Fit) and D/P(Error) for Each Forecast Horizon, 1891–1998



Note: All returns are annualized compound returns.

horizons (15 years and 20 years), D/P(Fit) is clearly adding considerable predictive power whereas D/P(Error) is adding none. Figure 9 tells a clear story that at short horizons, D/P(Error) is what counts but at long horizons, what counts is D/P(Fit). (Analogous results held for E/P.)

To sum up, the forecasting power of D/P can be decomposed into the forecasting power of D/P(Fit) and D/P(Error). In the model, D/P(Fit) is the normal or expected dividend yield, and D/P(Error) is interpreted as the D/P in excess (or deficit) of normal. Evidence presented here indicates that D/P itself forecasts stock returns at both long and short horizons but for different reasons. D/P(Fit) forecasts long-horizon stock returns but has almost no power for the short term. D/P(Error) forecasts short-horizon stock returns but has little power for the long term.

Do Stock Yields Have Farther to Fall?

Many have wondered lately why the market is currently selling at such a historically low D/P and E/P (or high P/D and P/E). In particular, in the book *Dow 36,000*, Glassman and Hassett came to an extreme conclusion. They argued that the reason stock prices seem so high relative to measures such as dividends and earnings is that the expected (or required) return on the stock market is going down as investors realize that the stock market is less risky in relation to the bond market than previously thought. Furthermore, they reasoned that this fall in expected returns is not over yet and concluded that it will not stop until stock and bond market expected returns are equal (a point at which, by their calculations, the Dow will reach approximately 36,000). Part of their reasoning sounds much like the arguments advanced here. Well, part of it is, and part of it is not.

Their first conclusion is 100 percent consistent with this article: the conclusion that stocks have low yields now because they are perceived to be less risky versus bonds than historically normal. In fact, my central thesis is that the return required by investors to own stocks versus bonds varies directly with the perceived relative risk of the two assets (for which I used their respective rolling 20-year volatilities as proxies). I believe that my model, coupled with currently low bond market yields and a low perceived risk of stocks versus bonds, entirely explains, within the bounds of statistical error, today's low yields on stocks (and, according to the model, the low long-term expected returns that come with low yields). Thus, my work strongly supports one aspect of the argument in *Dow 36,000*, namely, that stock market expected returns versus bonds have come down as investor perceptions of the relative risk of stocks versus bonds have changed.

My conclusions differ, however, from the next conclusion of *Dow 36,000*. Glassman and Hassett extrapolated the trend in lowered return-premium expectations to continue, but my model offers them no support. The authors of *Dow 36,000* stated that the fall in stock expected returns is not over yet and will not be complete until the expected return on stocks is the same as bonds (presumably not yet the case) because the authors believe that stocks are no riskier than bonds in the long term. This hypothesis is quite provocative. If stocks are no riskier than bonds, then stock prices should rise as investors realize stocks are currently priced as if they are more risky. Now, much debate involves the long-run risk of stocks versus bonds, and to review or settle this matter is not the province of this paper.[23] However, much of the reason behind the current prominence of this debate in the first place is how different today appears from the past (i.e., today's historically high stock prices versus dividends or earnings). My conclusion is that, in fact, the structure of the world really is not much different today; only the inputs to the model have changed. In other words, stock yields (and required returns) have always moved with bond yields, and the relative difference between them has always been a function of their relative perceived volatility. In fact, when I directly estimated this relationship, I found that it fits well for the long term and fits well today.

The reason the study reported here is a problem for theories like those proposed in *Dow 36,000* is that I say the rise in stock prices today, rather than simply beginning as investors start to perceive how safe stocks really are, is actually proceeding much as it has throughout financial history. According to the model, investors have repriced stocks to reflect a lower perception of stock market risk, but any farther drop in the required return on stocks (and concurrent rise in stock prices) must come from a further reduction in actual stock volatility (versus bond volatility) or a reduction in bond yields. If investors have been all along implicitly using the relationship hypothesized here to price stocks (as the data strongly support they have since at least 1891), then they have acted consistently in recently raising the price of equities. But we can expect no more such rises unless either interest rates or realized relative volatility change.[24] The model discussed here suggests that unless the inputs to the model change, any repricing of equities is approximately complete.

Finally, if the model is accurate, a belief that a near-term windfall profit of about three times your money is currently available in the broad stock market, a belief held by Glassman and Hassett, is dangerous. First, investors who believe in the windfall possibility may overallocate to stocks.[25] Second, short-term pricing errors induced by believers in this argument (or "bubbles") can be dangerous to the real economy. Third, and perhaps most worrisome, if the model presented and tested in this paper is correct, the belief that stocks stand to receive a one-time enormous windfall profit is not simply wrong, it is backward. The low stock yields of today are fully explained by the model, meaning that the forecast of short-term stock returns is about average.[26] Moreover, if the conclusion here is true that the best forecasting variable for long-term stock returns is the absolute level of stock yields, then today's low yields (both D/P and E/P) point to a poor forecast for the long-term return on stocks.

Conclusion

Each of the puzzles stated at the beginning of this article can be resolved by using the model provided in Equation 6 for the required yield on stocks. Consider the first question: Why did the stock market strongly outyield bonds for so long only to now consistently underyield bonds? The model states that (1) the higher bond yields are and (2) the higher perceived stock market volatility versus bond market volatility is, then the higher stock yields must be. For a long time (before the 1950s), stocks outyielded bonds because the realized volatility of stocks versus bonds was much higher than in modern times.

Consider the second question: Why did stock and bond yields move relatively independently, or even perversely, in the 1927–98 period but strongly move together in the later 40 years of this period? Stock and bond yields appear to move independently or even perversely over long periods (e.g., 1926–1998), but this appearance is an artifact of missing a part of their structural relationship. If the impact of changing volatility is taken into account, stock and bond yields are strongly positively correlated over the entire period for which we have data, which many strategists and economists would have hypothesized.

Finally, consider the third question: Why are today's stock market yields so low and what does that fact mean for the future? Today's stock market yields are so low simply because bond yields are low and recent realized stock market volatility has been low when compared with bond market volatility. I do not need to resort to "the world has changed" types of arguments to explain today's low yields. The model fully explains them. And the model indicates that they will not go much lower unless realized stock versus bond volatility or interest rates fall farther.

Although testing a long-term, slowly changing relationship has statistical difficulties, the model easily survived every reasonable robustness check, including out-of-sample testing of a previously untouched period (1871–1945) and the formation of completely nonoverlapping, nonserially correlated independent and dependent variables for the entire 1871–1998 period.

This work has strong theoretical implications. A link between volatility and expected return is one of the strongest implications of modern finance.[27] Researchers have found compelling evidence of this phenomenon in comparing asset classes (i.e., stocks versus bonds), but evidence of a link *within* asset classes (e.g., testing the capital asset pricing model for stocks) or an intertemporal link within one asset class has been weak. This article addresses the intertemporal link. Past studies failed to convincingly link expected stock returns to *ex ante* volatility through realized stock returns.[28] However, realized stock returns are very noisy. I hypothesized that D/P (or E/P) is a proxy for expected stock returns and that Y is a proxy for expected bond returns and found strong confirmation that the difference between these proxies is a positive function of differences in experienced volatility. In other words, unlike many other studies, I have documented a strong positive intertemporal relationship between expected return and perceived risk.

This article demonstrated that the relative long-term volatility experienced by investors is a strong driver of the relative yields they require on stocks versus bonds; it did *not* show that these long-term realized volatility figures are accurate forecasts of future volatility. Thus, I have clearly identified a behavioral relationship that I believe is important, but I offer no verdict on market efficiency.[29]

The bottom line is that today's stock market (as of May 1998) has very low yields (D/P and E/P) for the simple reason that bond yields are low and stock volatility has been low as compared with bond volatility. These conditions historically lead investors to accept a low yield (and expected return) on stocks. If one is a short-term investor, knowing that these low yields are not abnormal may be comforting. A long-term investor, however, might be very nervous, because raw stock yields (D/P and E/P) are the best predictors of long-term stock market returns and these raw yields are currently at very low levels.

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Notes

1. A set of assumptions sufficient for this equality to hold for coupon-bearing bonds is that the yield curve be flat and unchanging.
2. The sources and/or construction of the data for this article are as follows: For stocks, return and earnings yield data on the S&P 500 came from Datastream and dividend yields from Ibbotson Associates. For bonds, return data for January 1980 to May 1998 are from the J.P. Morgan Government Bond Index levered to a constant duration of 7.0 (i.e., the monthly return used is the T-bill rate plus 7.0 divided by the beginning-of-the-month J.P. Morgan duration times the return on the J.P. Morgan index minus the T-bill return). I constructed a constant-duration bond in the hopes of making my bond return series more homoscedastic. The choice of a duration of 7.0 was arbitrary and had no effect on the results. I performed the regression of this excess return series on the excess monthly return of the Ibbotson Associates long- and intermediate-term bond series for January 1980 to May 1998. For January 1926 to December 1979, I used the fitted values on the Ibbotson return series to approximate the 7.0-year duration J.P. Morgan government bond series. For bond yields, I used the 10-year benchmark yield from Datastream from January 1980 to May 1998. For January 1926 to December 1979, I used the fitted multiple regression forecast (fitted from the regression over the January 1980–May 1998 period) of the 10-year yield on the Ibbotson

short-, intermediate-, and long-term government bond series. The results are not sensitive to precise definitions of the bond yield or return.

3. The earnings yield I used is prior year's earnings over current price. All the economic results in this article are robust to using either a 3- or 10-year moving average of real earnings in the numerator.
4. Equation 3 almost assuredly should be augmented with variables proxying time-varying expected dividend growth (see Fama and French 1988). I have tested such proxies and found them to be statistically significant, but I omitted them from this article because they affect none of the results or conclusions significantly.
5. Bernstein (1993, 1997) examined a related (although slightly different) model and came to some of the same conclusions.
6. The results presented here were insensitive to assuming other reasonable functional forms for this relationship (for example, assuming linearity in the log of the volatilities rather than the levels).
7. Kane, Marcus, and Noh (1996) examined a related model for the first difference of market P/Es (a somewhat different exercise) and came to some conclusions similar to mine.
8. These studies used forms similar to Equation 5.
9. Another logical extension of Equations 3, 4, and 5 is Y = c + eT-bill + $d\sigma$(Bonds). That is, the yield on bonds moves (possibly at a multiple) with the short-term interest rate, and this weighted difference between long-term and short-term yields is a positive function of perceived bond volatility. Although not the focus of this article (but the focus of a future paper), empirical tests of this equation strongly support this specification.
10. This work is not sensitive to the definition of generation as precisely 20 years.
11. Note that I am not attempting to use the best short-term conditional estimate of volatility. Short-term changes in volatility may be mostly transitory. If so, they would have little impact on stock prices and required stock yields (see, for instance, Poterba and Summers 1986).
12. All R^2 values were adjusted for degrees of freedom.
13. Granger and Newbold (1974) found that in regressions of one random walk on another, rejection of the null hypothesis is more the rule than the exception. Also see Kirby (1997) or Goetzmann and Jorion (1993).
14. As mentioned previously, the results of this article are not very sensitive to the choice of a 20-year window for volatility. For instance, using a 10-year window for volatility estimation greatly reduced (but did not eliminate) the degree of autocorrelation in the right-hand variables. When I reestimate Equation 9 using 10-year rolling volatility (which also added 10 more years, 1936–1945, to the regression), the t-statistics did not materially change; the t-statistics on Y, σ(Stocks), and σ(Bonds) were, respectively, +10.00, +14.45, and –14.75. Using a 7-year window (now adding data from 1933–1945 to the regression), the t-statistics were +5.21, +11.54, and –10.93. A later section addresses this issue more directly by using longer-term data and analyzing nonoverlapping 20-year periods.
15. The sources for these data are Robert J. Shiller's Web page (an update of the data in Chapter 26 of Shiller 1989) and the company Global Financial Data.
16. These ratios are somewhat higher than reported in Figure 5 because the duration of the bond used in these annual tests was, on average, somewhat shorter than the duration of 7.0 years used in the monthly tests. Thus, bond volatility is somewhat lower in these annual tests. This change is only a matter of scale and has no economic effect on the tests.
17. For instance, Fama and French (1988) found that the parameters of the Lintner (1956) model for explaining dividend changes changed radically during the 1927–86 period.
18. As a final check, I reestimated Equation 24 using the Cochrane–Orcutt procedure to adjust for first-order auto-

correlation in the residuals. Each coefficient was essentially the same and remained statistically significant, whereas the first-order annual residual autocorrelation was highly statistically significant at 0.55.

19. This low autocorrelation matches the results of Poterba and Summers, who found only very short-term persistence in market volatility. Interestingly, I found that long-term rolling estimates of volatility seem to be crucial in determining the required expected return on the market but do not forecast the next period of long-term volatility itself. Thus, although investor perceptions of volatility drive market expected returns, those perceptions have not necessarily been accurate. My model might correctly describe investor behavior, but reconciling this behavior with market efficiency may be difficult (although not necessarily impossible). I leave this endeavor to future work.
20. In fact, the failure of the one regression (1895–1995) was driven by the 1975 observation (without this observation, the regression had an R^2 of 89.5 percent and a t-statistic of +5.93). Furthermore, by the luck of the draw, this regression did not include values for either the x or y variable as extreme as in Figures 7 and 8, which lowered the power of this test.
21. These regression results should not be considered an accurate test of a short- or long-term trading strategy. First, the regressions used D/P, which because it has price in the denominator, is known to induce a small bias toward finding a positive coefficient. The regressions also used the full-period data to form D/P(Error), which would not have been known prior to the end of the period. Finally, of course, the regressions do not account for trading costs. These regressions are meant to be indicative of the forecasting power of the model versus traditional models. Formal tests of a trading strategy based on these methods are not available from the author; trying to profit from such strategies is what I do for a day job.
22. These tests were carried out on in-sample regression residuals to retain the full 1946–98 period. Analogous significant results (although a bit weaker) were found for 1966–1998 when rolling out-of-sample versions of D/P(Fit) and D/P(Error) were used.
23. Two good sources for a scholarly but readable review of these issues are Siegel (1994) and Cornell (1999).
24. Glassman and Hassett did offer some reasons why stock volatility might be lower in the future than in the past, but their central argument does not need this farther drop to happen because their argument is that stocks are no more risky than bonds right now.
25. In all fairness, the actual practical investment advice in the book *Dow 36,000* appears quite reasonable, although it is still easy to see how an investor who believes in the authors' premise will not act so reasonably.
26. When this article was written, May 1998 data were the latest used. As of November 1999, the model's short-term forecast for stocks had joined the long-term forecast of stocks as below average, although not nearly as severely below average as the long-term forecast. I would be happy to provide a more up-to-date forecast and can be contacted at cliff.asness@aqrcapital.com. Of course, trade on such a forecast at your own risk!
27. This link does not need to hold precisely for inefficient portfolios.
28. An exception is Kane, Marcus, and Noh, who correctly pointed out that this relationship is much clearer in *ex ante* measures than in *ex post* returns.
29. Unfortunately, I also could not determine the rationality of the predictive power of D/P(Error) over short horizons and D/P(Fit) over long horizons. Modigliani and Cohn (1979) argued that when inflation (and presumably bond yields) is low, investors mistakenly (i.e., irrationally or inefficiently) overprice equities (and vice versa). The empirical results of this study support their hypothesis in one way: When volatility is held constant, investors do price stocks at higher P/Es and P/Ds when interest rates are low (and vice versa). This empirical finding is an important contribution, because more-naive tests (which fail to account for relative volatility changes) do not pick up this relationship. However, distinguishing whether the short-term predictive power of D/P(Error) or the long-term predictive power of D/P(Fit) is coming from such mispricing or rational variance in expected return (perhaps caused by changing risk aversion) is beyond the scope of this article.

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Searching for the Equity Premium

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October 2020‡

Abstract

Labor market frictions are crucial for the equity premium in production economies. A dynamic stochastic general equilibrium model with recursive utility, search frictions, and capital accumulation yields a high equity premium of 4.26% per annum, a stock market volatility of 11.8%, and a low average interest rate of 1.59%, while simultaneously retaining plausible business cycle dynamics. The equity premium and stock market volatility are strongly countercyclical, while the interest rate and consumption growth are largely unpredictable. Because of wage inertia, dividends are procyclical despite consumption smoothing via capital investment. The welfare cost of business cycles is huge, 29%.

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1 Introduction

Mehra and Prescott (1985) show that the equity premium (the average difference between the stock market return and risk-free interest rate) in the Arrow-Debreu economy is negligible relative to its historical average. Subsequent studies have largely succeeded in specifying preferences and cash flow dynamics to explain the equity premium in endowment economies (Campbell and Cochrane 1999; Bansal and Yaron 2004; Barro 2006). Unfortunately, explaining the equity premium in general equilibrium production economies, in which cash flows are endogenously determined, has proven more challenging.[1] To date, no consensus general equilibrium framework has emerged. Consequently, finance and macroeconomics have largely developed in a dichotomic fashion. Finance specifies "exotic" preferences and exogenous cash flow dynamics to match asset prices but ignore firms, whereas macroeconomics analyzes full-fledged general equilibrium production economies but ignore asset prices with simple preferences (Christiano, Eichenbaum, and Evans 2005; Smets and Wouters 2007).

This macro-finance dichotomy has left many important questions unanswered. What are the microeconomic foundations underlying the exogenously specified, often complicated cash flow dynamics in finance models (Bansal, Kiku, and Yaron 2012; Nakamura, Steinsson, Barro, and Ursua 2013)? What are the essential ingredients in the production side that can endogenize the key elements of cash flow dynamics necessary to explain the equity premium? To what extent do time-varying risk premiums matter quantitatively for macroeconomic dynamics? How large is the welfare cost of business cycles in an equilibrium production economy that replicates the equity premium?

Our long-term objective is to formulate a unified equilibrium theory that explains the equity

[1]Rouwenhorst (1995) shows that the standard real business cycle model cannot explain the equity premium because optimal investment of firms provides a powerful mechanism for the representative household to smooth consumption, yielding little consumption risks. With internal habit preferences, Jermann (1998) and Boldrin, Christiano, and Fisher (2001) adopt capital adjustment costs and cross-sector immobility, respectively, to restrict consumption smoothing to match the equity premium. However, both models struggle with excessively high interest rate volatilities because of low elasticities of intertemporal substitution. Using recursive utility, Tallarini (2000) shows that increasing risk aversion in a real business cycle model improves its fit with the market Sharpe ratio but does not materially affect macro quantities. However, the model fails to match the equity premium and its volatility. Kaltenbrunner and Lochstoer (2010) show that long-run consumption risks arise endogenously from consumption smoothing in a real business cycle model, but the model falls short in explaining the equity premium and stock market volatility.

premium puzzle, while simultaneously retaining plausible business cycle dynamics. We embed the standard Diamond-Mortensen-Pissarides search model of equilibrium unemployment into a dynamic stochastic general equilibrium framework with recursive utility and capital accumulation. When calibrated to the consumption growth volatility in the Jordà-Schularick-Taylor macrohistory database, the model succeeds in yielding an equity premium (adjusted for financial leverage) of 4.26% per annum, which is close to 4.36% in the historical data. The average interest rate is 1.59%, which is not far from 0.82% in the data (the difference is insignificant). However, the stock market volatility is 11.8% in the model, which, although sizeable, is still significantly lower than 16% in the data. Also, the model implies strong time series predictability for stock market excess returns and volatilities, some predictability for consumption volatility, and weak to no predictability for consumption growth and the real interest rate. Quantitatively, the model explains stock market predictability but somewhat overstates consumption growth predictability in the historical data.

Wage inertia plays a key role in our model. To keep the model parsimonious, we work with the Nash wage that features a low bargaining weight of workers and a high flow value of unemployment. This calibration implies a wage elasticity to labor productivity of 0.256 in the model. Hagedorn and Manovskii (2008) estimate this elasticity to be 0.449 in the U.S. postwar 1951–2004 sample. Drawing from historical sources (Kendrick 1961; Officer 2009), we extend the Hagedorn-Manovskii evidence and estimate the wage elasticity to be 0.267 in the historical 1890–2015 sample.

Unlike endowment economies, in which cash flows can be exogenously specified to fit the equity premium, the main challenge facing general equilibrium production economies is that cash flows are often endogenously countercyclical. With frictionless labor market, wages equal the marginal product of labor, which is almost as procyclical as output and profits (output minus wages). Alas, investment is more procyclical than output because of consumption smoothing, making dividends (profits minus investment) countercyclical (Kaltenbrunner and Lochstoer 2010). With wage inertia, profits are more procyclical than output. The magnified procyclicality of profits is sufficient to overcome the procyclicality of investment (and vacancy costs) to render dividends procyclical. In addition, wage inertia is stronger in bad times, with smaller profits. This time-varying wage inertia amplifies risks and risk premiums in bad times, giving rise to time series predictability of the equity premium and stock market volatility. Finally, despite adjustment costs, investment still absorbs a large amount of shocks, making consumption growth and the interest rate largely unpredictable.

Risk aversion strongly affects quantity dynamics, in contrast to Tallarini (2000). In comparative statics, reducing risk aversion from 10 to 5 lowers the equity premium to 0.54% per annum. More important, consumption volatility falls from 5.13% to 3.93%, and consumption disaster probability from 5.83% to 3.82%. A lower discount rate raises the marginal benefit of hiring and reduces the unemployment rate from 8.63% to 4.63%. Echoing Hall's (2017) partial equilibrium analysis, our general equilibrium results indicate that it is imperative to study quantity and price dynamics jointly.

Our model predicts downward-sloping term structures of the equity premium and equity volatility, consistent with Binsbergen, Brandt, and Koijen (2012). Intuitively, when the search economy slides into a disaster, short-maturity dividend strips take a big hit because of inertial wages. In contrast, long-maturity strips are less impacted because disasters are followed by subsequent recoveries. Also, despite recursive utility calibrated to feature the early resolution of uncertainty, the timing premium (the fraction of the consumption stream that the investor is willing to trade for the early resolution) is only 15.3% in our model. Intuitively, the expected consumption growth and conditional consumption volatility in our search economy are much less persistent than those typically calibrated in the long-run risks model, thereby avoiding its pitfall of implausibly high timing premiums.

Finally, the average welfare cost of business cycles is huge, 29.1%, which is more than 580 times of 0.05% in Lucas (2003). More important, the welfare cost is countercyclical with a long, right tail. In simulations, its 5th percentile of 18.4% is not far below its median of 24.4%, but its 95th percentile is substantially higher, 56.3%. As such, countercyclical policies aimed to dampen disaster risks are even more important than what the average welfare cost estimate of 29.1% would suggest.

We view this work as a solid progress report toward a unified theory of asset prices and business cycles. This holy grail of macro-finance has proven elusive for decades. Petrosky-Nadeau, Zhang, and Kuehn (2018) show that the standard search model exhibits disaster dynamics. However, their asset pricing results are very limited because of no capital. Capital is particularly important for asset prices because it represents the core challenge of endogenizing procyclical dividends in production economies (Jermann 1998). We embed capital and recursive utility simultaneously to study asset prices with production, while overcoming ensuing heavy computational burden. Bai (2020) incorporates defaultable bonds to study the credit spread. We instead focus on the equity premium puzzle.

Embedding rare disasters per Rietz (1988) and Barro (2006) into a real business cycle model, Gourio (2012) shows that aggregate risks significantly affect quantity dynamics. Echoing Gourio, we show that Tallarini's (2000) separation between prices and quantities does not hold under more general settings. However, we differ from Gourio in that disasters arise endogenously from labor market frictions. We also endogenize operating leverage via wage inertia to explain the equity premium and stock market volatility. In contrast, Gourio relies on exogenous leverage to generate volatile cash flows but "does not address the volatility of the unlevered return on capital (p. 2737)." Kilic and Wachter (2018) embed the exogenous Rietz-Barro diasters into the search model of unemployment to yield a high unemployment volatility and examine its relation with a high stock market volatility. While our work differs from Kilic and Wachter's in many details, the most important distinction is, again, the endogenous nature of disasters in our setting.[2]

The rest of the paper is organized as follows. Section 2 constructs the general equilibrium model. Section 3 presents the model's key quantitative results, including the equity premium, stock market volatility, and their predictability. Section 4 examines several additional implications of the model, including the welfare cost of business cycles. Section 5 concludes. Appendix A describes

[2]Several recent studies have examined the equity premium in general equilibrium production economies but outside the disasters framework. Croce (2014) embeds exogenous long-run productivity risks into a production model. While long-run risks increase the equity premium, the return volatility is only about one quarter of that in the data. Kung and Schmid (2015) endogenize long-run productivity risks via firms' research and development in an endogenous growth model. Favilukis and Lin (2016) examine the impact of infrequent wage renegotiations in a stochastic growth model with long-run productivity risks. Finally, Chen (2017) examines a general equilibrium production model with external habit and emphasizes the role of endogenous consumption volatility risks.

our algorithm. A separate Internet Appendix details data, derivations, and supplementary results.

2 A General Equilibrium Production Economy

The economy is populated by a representative household and a representative firm. Following Merz (1995), we assume that the household has perfect consumption insurance. A continuum of mass one of members is either employed or unemployed at any point in time. The fractions of employed and unemployed workers are representative of the population at large. The household pools the income of all the members together before choosing per capita consumption.

The household maximizes recursive utility, denoted J_t, given by:

$$J_t = \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left(E_t\left[J_{t+1}^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \qquad (1)$$

in which C_t is consumption, β time preference, ψ the elasticity of intertemporal substitution, and γ relative risk aversion (Epstein and Zin 1989; Weil 1990). The consumption Euler equation is:

$$1 = E_t[M_{t+1} r_{St+1}], \qquad (2)$$

in which r_{St+1} is the firm's stock return, and M_{t+1} the household's stochastic discount factor:

$$M_{t+1} \equiv \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{J_{t+1}}{E_t\left[J_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}. \qquad (3)$$

The riskfree rate is $r_{ft+1} = 1/E_t[M_{t+1}]$, which is known at the beginning of t.

The representative firm uses capital, K_t, and labor, N_t, to product output, Y_t, with a constant elasticity of substitution (CES) production technology (Arrow et al. 1961):

$$Y_t = X_t\left[\alpha\left(\frac{K_t}{K_0}\right)^{\omega} + (1-\alpha)N_t^{\omega}\right]^{\frac{1}{\omega}}, \qquad (4)$$

in which α is the distribution parameter, and $e \equiv 1/(1-\omega)$ the elasticity of substitution between capital and labor. When ω approaches zero in the limit, equation (4) reduces to the special case of the Cobb-Douglas production function with a unitary elasticity. To facilitate the model's calibration, we work with the "normalized" CES function in equation (4), in which $K_0 > 0$ is a scaler that makes the unit of K_t/K_0 comparable to the unit of N_t (Klump and La Grandville 2000). Specifically, we calibrate K_0 to ensure that $1-\alpha$ matches the average labor share in the data (Section 3.2). Doing so eliminates the distribution parameter, α, as a free parameter.[3] Finally, the CES production function is of constant returns to scale, $Y_t = K_t \partial Y_t/\partial K_t + N_t \partial Y_t/\partial N_t$ (the Internet Appendix).

The firm takes the aggregate productivity, X_t, as given, with $x_t \equiv \log(X_t)$ governed by:

$$x_{t+1} = (1-\rho_x)\overline{x} + \rho_x x_t + \sigma_x \epsilon_{t+1}, \tag{5}$$

in which $\overline{x}$ is unconditional mean, $0 < \rho_x < 1$ persistence, $\sigma_x > 0$ conditional volatility, and ϵ_{t+1} an independently and identically distributed (i.i.d.) standard normal shock. We scale $\overline{x}$ to make the average marginal product of labor around one in simulations to ease the interpretation of parameters.

The representative firm posts a number of job vacancies, V_t, to attract unemployed workers, U_t. Vacancies are filled via the Den Haan-Ramey-Watson (2000) matching function:

$$G(U_t, V_t) = \frac{U_t V_t}{(U_t^{\iota} + V_t^{\iota})^{1/\iota}}, \tag{6}$$

in which $\iota > 0$. This matching function has the desirable property that matching probabilities fall between zero and one. In particular, define $\theta_t \equiv V_t/U_t$ as the vacancy-unemployment (V/U) ratio. The probability for an unemployed worker to find a job per unit of time (the job finding rate) is $f(\theta_t) \equiv G(U_t, V_t)/U_t = \left(1 + \theta_t^{-\iota}\right)^{-1/\iota}$. The probability for a vacancy to be filled per unit of time (the vacancy filling rate) is $q(\theta_t) \equiv G(U_t, V_t)/V_t = (1 + \theta_t^{\iota})^{-1/\iota}$. It follows that $f(\theta_t) = \theta_t q(\theta_t)$ and $q'(\theta_t) < 0$. An increase in the scarcity of unemployed workers relative to vacancies makes it harder to fill a vacancy. As such, θ_t is labor market tightness, and $1/q(\theta_t)$ the average duration of vacancies.

The representative firm incurs costs in posting vacancies. The unit cost per vacancy is given by

[3] In contrast, in prior applications of the CES production function in asset pricing, the distribution parameter, α, is largely treated as a free parameter (Favilukis and Lin 2016; Kilic and Wachter 2018; Bai 2020).

$\kappa > 0$. The marginal cost of hiring, $\kappa/q(\theta_t)$, increases with the mean duration of vacancies, $1/q(\theta_t)$. In expansions, the labor market is tighter for the firm (θ_t is higher), and the vacancy filling rate, $q(\theta_t)$, is lower. As such, the marginal cost of hiring is procyclical.

Jobs are destroyed at a constant rate of s per period. Employment, N_t, evolves as:

$$N_{t+1} = (1 - s)N_t + q(\theta_t)V_t, \tag{7}$$

in which $q(\theta_t)V_t$ is the number of new hires. Population is normalized to be one, $U_t + N_t = 1$, meaning that N_t and U_t are also the rates of employment and unemployment, respectively.

The firm incurs adjustment costs when investing. Capital accumulates as:

$$K_{t+1} = (1 - \delta)K_t + \Phi(I_t, K_t), \tag{8}$$

in which δ is the capital depreciation rate, I_t is investment, and

$$\Phi_t \equiv \Phi(I_t, K_t) = \left[a_1 + \frac{a_2}{1 - 1/\nu}\left(\frac{I_t}{K_t}\right)^{1-1/\nu}\right]K_t, \tag{9}$$

is the installation function with the supply elasticity of capital $\nu > 0$. We set $a_1 = \delta/(1 - \nu)$ and $a_2 = \delta^{1/\nu}$ to ensure no adjustment costs in the deterministic steady state (Jermann 1998). This parsimonious parametrization involves only one free parameter, ν.

The dividends to the firm's shareholders are given by:

$$D_t = Y_t - W_t N_t - \kappa V_t - I_t, \tag{10}$$

in which W_t is the equilibrium wage rate. Taking W_t, the household's stochastic discount factor, M_{t+1}, and the vacancy filling rate, $q(\theta_t)$, as given, the firm chooses optimal investment and the optimal number of vacancies to maximize the cum-dividend market value of equity, S_t:

$$S_t \equiv \max_{\{V_{t+\tau}, N_{t+\tau+1}, I_{t+\tau}, K_{t+\tau+1}\}_{\tau=0}^{\infty}} E_t\left[\sum_{\tau=0}^{\infty} M_{t+\tau} D_{t+\tau}\right], \tag{11}$$

subject to equations (7) and (8) as well as a nonnegativity constraint on vacancies, $V_t \geq 0$. Because $q(\theta_t) > 0$, $V_t \geq 0$ is equivalent to $q(\theta_t)V_t \geq 0$. In contrast, equation (9) implies that $\partial\Phi_t/\partial I_t = a_2(I_t/K_t)^{-1/\nu}$, which goes to infinity as investment, I_t, goes to zero. As such, I_t is always positive.

From the first-order conditions for I_t and K_{t+1}, we obtain the investment Euler equation:

$$\frac{1}{a_2}\left(\frac{I_t}{K_t}\right)^{1/\nu} = E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + \frac{1}{a_2}\left(\frac{I_{t+1}}{K_{t+1}}\right)^{1/\nu}(1-\delta+a_1) + \frac{1}{\nu-1}\frac{I_{t+1}}{K_{t+1}}\right]\right]. \quad (12)$$

Equivalently, $E_t[M_{t+1}r_{Kt+1}] = 1$, in which r_{Kt+1} is the investment return:

$$r_{Kt+1} \equiv \frac{\partial Y_{t+1}/\partial K_{t+1} + (1/a_2)(1-\delta+a_1)\left(I_{t+1}/K_{t+1}\right)^{1/\nu} + (1/(\nu-1))(I_{t+1}/K_{t+1})}{(1/a_2)\left(I_t/K_t\right)^{1/\nu}}. \quad (13)$$

Let λ_t be the multiplier on $q(\theta_t)V_t \geq 0$. From the first-order conditions with respect to V_t and N_{t+1}, we obtain the intertemporal job creation condition:

$$\frac{\kappa}{q(\theta_t)} - \lambda_t = E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial N_{t+1}} - W_{t+1} + (1-s)\left(\frac{\kappa}{q(\theta_{t+1})} - \lambda_{t+1}\right)\right]\right]. \quad (14)$$

Equation (14) implies that $E_t[M_{t+1}r_{Nt+1}] = 1$, in which r_{Nt+1} is the hiring return:

$$r_{Nt+1} \equiv \frac{\partial Y_{t+1}/\partial N_{t+1} - W_{t+1} + (1-s)\left(\kappa/q(\theta_{t+1}) - \lambda_{t+1}\right)}{\kappa/q(\theta_t) - \lambda_t}. \quad (15)$$

Finally, the optimal vacancy policy also satisfies the Kuhn-Tucker conditions:

$$q(\theta_t)V_t \geq 0, \quad \lambda_t \geq 0, \quad \text{and} \quad \lambda_t q(\theta_t)V_t = 0. \quad (16)$$

Under constant returns to scale, the stock return of the representative firm, r_{St+1}, is a weighted average of the investment return and the hiring return (the Internet Appendix):

$$r_{St+1} = \frac{\mu_{Kt}K_{t+1}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}}r_{Kt+1} + \frac{\mu_{Nt}N_{t+1}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}}r_{Nt+1}, \quad (17)$$

in which the shadow value of capital, μ_{Kt}, equals the marginal cost of investment, $(1/a_2)(I_t/K_t)^{(1/\nu)}$, and the shadow value of labor, μ_{Nt}, equals the marginal cost of hiring, $\kappa/q(\theta_t) - \lambda_t$.

The equilibrium wage rate is determined endogenously by applying the sharing rule per the outcome of a generalized Nash bargaining process between employed workers and the firm (Pissarides 2000). Let $\eta \in (0, 1)$ be the workers' relative bargaining weight and b the workers' flow value of unemployment. The equilibrium wage rate is given by (the Internet Appendix):

$$W_t = \eta \left(\frac{\partial Y_t}{\partial N_t} + \kappa \theta_t \right) + (1 - \eta) b. \tag{18}$$

The wage rate increases with the marginal product of labor, $\partial Y_t / \partial N_t$, and the vacancy cost per unemployed worker, $\kappa \theta_t$. Intuitively, the more productive the workers are, and the more costly for the firm to fill a vacancy, the higher the wage rate is for the employed workers. In addition, the workers' bargaining weight, η, affects the wage elasticity to labor productivity. The lower η is, the more the equilibrium wage is tied with the constant b, reducing the wage elasticity to productivity.

The competitive equilibrium consists of optimal investment, I_t, vacancy posting, V_t, multiplier, λ_t, and consumption, C_t, such that (i) C_t satisfies the consumption Euler equation (2); (ii) I_t satisfies the investment Euler equation (12), and V_t and λ_t satisfy the intertemporal job creation condition (14) and the Kuhn-Tucker conditions (16), while taking the stochastic discount factor, M_{t+1}, in equation (3), and the equilibrium wage in equation (18) as given; and (iii) the goods market clears:

$$C_t + \kappa V_t + I_t = Y_t. \tag{19}$$

Solving for the competitive equilibrium is computationally challenging. We adapt Petrosky-Nadeau and Zhang's (2017) globally nonlinear projection method with parameterized expectations to our setting (Appendix A). The state space consists of employment, capital, and productivity. We parameterize the conditional expectation in the right-hand side of equation (14) and solve for the indirect utility, investment, and conditional expectation functions from equations (1), (12), and (14). We use Rouwenhorst's (1995) discrete state method to approximate the log productivity with 17 grid points. We use the finite element method with cubic splines on 50 nodes on the employment space and 50 nodes on the capital space and take their tensor product on each grid point of productivity. To solve the resulting system of 127,500 equations, we use the derivative-free fixed point iteration with a small damping parameter (Judd, Maliar, Maliar, and Valero 2014).

3 Quantitative Results

We describe our data in Section 3.1 and calibrate the model in Section 3.2. We examine the model's unconditional moments in Section 3.3, sources of the equity premium in Section 3.4, and time-varying risks and risk premiums in Section 3.5. Finally, we report comparative statics in Section 3.6.

3.1 Data

For business cycle moments, we use the historical cross-country panel of output, consumption, and investment from Jordà, Schularick, and Taylor (2017), who in turn build on Barro and Ursúa (2008). For asset pricing moments, we use the Jordà et al. (2019) cross-country panel. We obtain the data from the Jordà-Schularick-Taylor macrohistory database.[4] The database contains macro and return series for 17 developed countries. The only missing series are returns for Canada, which we supplement from the Dimson-Marsh-Staunton (2002) database purchased from Morningstar. Although the Dimson-Marsh-Staunton database contains asset prices and the Barro-Ursúa database provides consumption and output series for more countries, we mainly rely on the Jordà-Schularick-Taylor database because it provides quantities and asset prices for the same set of countries. More important, it also contains investment series. The sample starts as early as 1871 and ends in 2015.[5]

Table 1 shows the properties of log growth rates of real consumption, output, and investment per capita in the historical panel. From Panel A, the consumption growth is on average 1.62% per annum, with a volatility of 5.45%, and a skewness of −0.67, all averaged across 17 countries. The first-order autocorrelation is 0.12. The consumption volatility exhibits a substantial amount

[4]http://www.macrohistory.net/data.

[5]More precisely, in the Jordà-Schularick-Taylor database, the consumption, output, and investment series start in 1870, meaning that their growth rates start in 1871. The quantities series end in 2016, but asset prices end in 2015.

of cross-country variation, ranging from 2.76% in UK to 8.72% in Belgium. The first-order autocorrelations also varies widely across countries, ranging from −0.2 in Switzerland to 0.39 in France.

From Panel B, averaged across countries, the output growth has a mean of 1.78% per annum, a volatility of 5.1%, a skewness of −1.06, and a first-order autocorrelation of 0.18. The output volatility of 5.1% is lower than the consumption volatility of 5.45%.[6] Finally, Panel C shows that the investment growth volatility is high on average, 13.5% per annum, varying from 8.2% in Netherlands to 24.4% in the United States. Its first-order autocorrelation is 0.13.

Following Barro (2006), we calculate leverage-adjusted equity premium as one minus financial leverage times the unadjusted equity premium and calculate leverage-adjusted market volatility as the standard deviation of the leverage-weighted average of stock market and bill returns. We set leverage to be 0.29, which is the mean market leverage ratio in a cross-country panel reported in Fan, Titman, and Twite (2012). From Panel D, the leverage-adjusted equity premium is 4.36% per annum on average, varying from 2.71% in Portugal to 6.8% in Finland. The leverage-adjusted stock market volatility is on average 16%, ranging from 11.9% in Denmark to 23% in Finland. For the real interest rate, the mean is only 0.82% across countries. Finland has the lowest mean interest rate of −0.74%, whereas Denmark has the highest of 3.08%. Finally, the real interest rate volatility is on average 7.3%, ranging from 4.32% in Australia to 13.22% in Germany.[7]

The asset pricing literature has traditionally focused only on the postwar U.S. data. Table S2 in the Internet Appendix reports basic macro and asset pricing moments in the 1950–2015 cross-country sample. The real consumption, output, and investment growth rates are less volatile, with standard deviations of 2.4%, 2.47%, and 7.06% per annum, respectively, averaged across countries. The U.S. macro volatilities are lower still at 1.73%, 2.21%, and 4.98%, respectively. Relatedly,

[6] As explained in Barro and Ursúa (2008), government purchases rise sharply in wartime, decrease consumption relative to output, and raise the consumption volatility relative to the output volatility.

[7] When calculating the return moments, we require stock, bond, and bill returns to be nonmissing for a given year in a given country. Relaxing this restriction has little impact on the moments. In Table S1 in the Internet Appendix, we recalculate the moments with the longest sample possible for each series. The leverage-adjusted equity premium remains at 4.36% per annum, and the leverage-adjusted stock market volatility rises lightly from 16.04% to 16.08%. The mean real interest rate increases somewhat from 0.82% to 1.05%, and its volatility from 7.3% to 7.53%.

the consumption, output, and investment growth rates are more persistent in the postwar sample, with the first-order autocorrelations of 0.46, 0.39, and 0.29, respectively. However, the postwar leverage-adjusted equity premium is higher than the historical equity premium, 5.38% versus 4.36%. The leverage-adjusted stock market volatility is also higher in the postwar sample, 17.15% versus 16.04%. The evidence indicates that the postwar U.S. sample might not be representative. As such, we mostly rely on the historical cross-country panel to calibrate our model.

For labor market moments, to our knowledge, a historical cross-country panel is unavailable. As such, we work with the U.S. historical monthly series compiled by Petrosky-Nadeau and Zhang (2020).[8] Following Weir (1992), in addition to civilian unemployment rates, Petrosky-Nadeau and Zhang construct a separate series of private nonfarm unemployment rates, by subtracting farm and government employment from both civilian labor force and civilian employment. Because this unemployment series better depicts the functioning of the private economy (Lebergott 1964), we focus our calibration on this series. This series dates back to 1890, and the vacancy rate series to 1919.

From January 1890 to December 2015, the mean private nonfarm unemployment rate is 8.94%. The skewness and kurtosis of the unemployment rates are 2.13 and 9.5, respectively. In the postwar sample from January 1950 to December 2015, the mean unemployment rate is lower, 7.65%. Skewness is also smaller, 0.55, and kurtosis is close to that of the normal distribution, 2.92.

To calculate the second moments, we follow Shimer (2005) to take quarterly averages of monthly unemployment and vacancy rates to convert to quarterly series, which are detrended as Hodrick-Prescott (1997, HP) filtered proportional deviations from the mean with a smoothing parameter of 1,600. We do not take log deviations from the HP trend because the $V \geq 0$ constraint can be occasionally binding in the model. From 1890 onward, the private nonfarm unemployment volatility is 24.43% per quarter (25.9% with log deviations). From 1919 onward, the vacancy rate volatility is 18.98% (17.36% with log deviations). For labor market tightness (the ratio of the vacancy rate over the private nonfarm unemployment rate), the volatility is 61.62% (but only 38.38% with log devia-

[8] The series are available at https://ars.els-cdn.com/content/image/1-s2.0-S0304393220300064-mmc2.csv.

tions). The U-V correlations are -0.57 and -0.79 across the two detrending methods, respectively.[9]

3.2 Calibration

We calibrate the model in monthly frequency. We set the time discount factor $\beta = 0.9976$ to help match the mean real interest rate. We set the risk aversion, γ, to 10 per the long-run risks literature (Bansal and Yaron 2004). We set the elasticity of intertemporal substitution, ψ, to 2 per Barro (2009), who is in part based on Gruber's (2013) microeconomic estimates. Following Gertler and Trigari (2009), we set the persistence of the log productivity, ρ_x, to be $0.95^{1/3}$, and set its conditional volatility, σ_x, to match the consumption growth volatility in the data. Instead of the output volatility, we target the consumption volatility, which is more important for the model's asset pricing properties. This procedure yields a value of 0.015 for σ_x. This value implies a consumption volatility of 5.13% per annum, which is close to but lower than 5.45% in the data (Table 1). However, the output volatility is 6.43%, which is higher than 5.1% in the data.

For the CES production function, we set $\omega = -1.5$. This ω value implies an elasticity of capital-labor substitution of 0.4, which is the point estimate in Chirinko and Mallick (2017). When calibrating the distribution parameter, α, we target the average labor share. Gollin (2002) shows that factor shares are approximately constant across time and space. Table S3 in the Internet Appendix reports the labor shares for the 12 countries that are in both the Gollin and the Jordà-Schularick-Taylor databases. The average labor shares across the countries from Gollin's first two adjustment methods are 0.765 and 0.72, respectively, with an average of 0.743. Gollin emphasizes that these two adjustments "give estimated labor shares that are essentially flat across countries and over time (p. 471)." As such, we set $\alpha = 0.25$, which yields an average labor share of 0.746 in simulations.

The distribution parameter, α, is close to one minus the average labor share only in the "normalized" CES production function, in which the capital unit is comparable to the labor unit

[9] Labor market volatilities are lower in the postwar sample. From 1950 onward, the private nonfarm unemployment volatility is 13.81% per quarter, and the vacancy rate volatility is 13.49%. The market tightness volatility is 26.17%, and the U-V correlation -0.9. Detrending with log deviations from the HP trend yields very similar estimates.

(Klump and La Grandville 2000). We calibrate the capital scaler, K_0, at 13.75 to set the labor share at the deterministic steady state at 0.75. For comparison, the value of capital at the deterministic steady state is 16.14. Despite the model's nonlinearity, the labor share is very close across the deterministic and stochastic steady states. We calibrate the long-run mean of the productivity, $\overline{x} = 0.1887$, to target the marginal product of labor, $\partial Y_t / \partial N_t$, around one on average in simulations.[10]

The supply elasticity of capital, ν, governs the magnitude of adjustment costs. A lower ν implies higher adjustment costs, which reduce the investment volatility but raise the consumption volatility. Alas, direct estimates of ν seem scarce. We set ν to 1.25 and the depreciation rate, δ, to 1.25%. We set the separation rate, s, to 0.035, which is the average total nonfarm separation rate in the Job Openings and Labor Turnover Survey (JOLTS) at Bureau of Labor Statistics (BLS). The curvature of the matching function, ι, is 1.25, which is based on Den Haan, Ramey, and Watson (2000).

3.2.1 Wage Inertia

We are left with the bargaining weight of workers, η, the flow value of unemployment activities, b, and the unit cost of vacancy posting, κ. To match the equity premium without overshooting the mean unemployment rate, we combine inertial wages and low vacancy costs. Specifically, we set $\eta = 0.015$ and $b = 0.91$, which yield a wage elasticity to labor productivity of 0.256 in the model. We set the unit vacancy cost, κ, to 0.01, to obtain a mean unemployment rate of 8.63%, which is close to the average private nonfarm unemployment rate of 8.94% in the 1890–2015 sample.

Is the model implied wage elasticity to labor productivity empirically plausible? Hagedorn and Manovskii (2008), for example, estimate the wage elasticity to labor productivity to be 0.449 in the postwar 1951–2004 quarterly sample from BLS.[11] However, a voluminous literature on economic history documents severe wage inertia and quantifies its large impact during the Great Depression.[12]

[10]Setting $\partial Y_t / \partial N_t = 1$ at the deterministic steady state yields $\overline{x} = 0.1787$. However, $\partial Y_t / \partial N_t$ at the stochastic steady state is somewhat lower than one. As such, we manually adjust $\overline{x}$ to 0.1887 to yield the desired outcome.

[11]Both real wages and labor productivity are in logs and HP-filtered with a smoothing parameter of 1,600.

[12]Prominent examples include Eichengreen and Sachs (1985), Bernanke and Powell (1986), Bernanke and Carey (1996), Hanes (1996), Dighe (1997), Bordo, Erceg, and Evans (2000), Cole and Ohanian (2004), and Ohanian (2009).

As such, we extend the Hagedorn-Manovskii evidence to a historical U.S. sample.

To construct a historical series of real wages, we draw elements from Gordon (2016). From 1929 to 2015, we obtain compensation of employees from National Income and Product Accounts (NIPA) Tables 6.2A–D (line 3, private industries, minus line 5, farms) at Bureau of Economic Analysis. We obtain the number of full-time equivalent employees from NIPA Tables 6.5A–D (line 3, private industries, minus line 5, farms). Dividing the compensation of employees by the number of employees yields nominal wage rates (compensation per person). We deflate nominal wage rates with the personal consumption deflator from NIPA Table 1.1.4 (line 2) to obtain real wage rates.

From 1890 to 1929, we obtain the average (nominal) hourly compensation of production workers in manufacturing and consumer price index from measuringworth.com (Officer and Williamson 2020a, 2020b). The nominal compensation series from their Web site only has two digits after the decimal. We instead use the average hourly compensation series, with three digits after the decimal, from Officer (2009, Table 7.1). To obtain an index of hours, we divide the index of manhours by the index of persons engaged in manufacturing from Kendrick (1961, Table D-II). We multiply the average hourly compensation series with the hours index to obtain the nominal compensation per person, which we then deflate with the Officer-Williamson consumer price index to obtain the series of real wages. Finally, we splice this series in 1929 to the NIPA series from 1929 onward to yield an uninterrupted series from 1890 to 2015. Splicing means that we rescale the pre-1929 series so that its value in 1929 is identical to that for the NIPA post-1929 series.[13] Finally, for labor productivity, we use the historical 1890–2015 series from Petrosky-Nadeau and Zhang (2020).[14] We

[13]We differ from Gordon (2016) in two aspects. First, Gordon measures real wages as real compensation per manhour. We instead use real compensation per person that better fits our model with no hours. This practice seems standard in the macro labor literature (Shimer 2005). Second, Gordon measures nominal compensation as total compensation of employees from NIPA Table 1.10 (line 2), which includes government and farm employees. We instead use employee compensation for the private nonfarm sector, which matches the measurement of labor productivity.

[14]The monthly series is the ratio of a nonfarm business real output series over a private nonfarm employment series. The real output series draws from Kendrick (1961) and NIPA (from 1929 onward) as well as monthly industrial production series (as monthly indicators) from Miron and Romer (1990) and Federal Reserve Bank of St. Louis (from 1919 onward). The private nonfarm employment series draws from Weir (1992) and Current Employment Statistics at BLS as well as monthly employment indicators from NBER macrohistory files. From January 1947 onward, the monthly labor productivity series is benchmarked to the quarterly nonfarm business real output per job series from BLS.

time-aggregate their monthly series into annual by taking the monthly average within a given year.

We detrend the annual real wages and labor productivity series as log deviations from their HP-trends with a smoothing parameter of 6.25, which is equivalent to a quarterly smoothing parameter of 1,600.[15] In our postwar 1950–2015 annual sample, regressing the log real wages on the log labor productivity yields a wage elasticity of 0.406, with a standard error of 0.081. The elasticity estimate is not far from the Hagedorn-Manovskii estimate of 0.449 in their 1951–2004 quarterly sample.

More important, in our 1890–2015 historical sample, the wage elasticity to labor productivity is estimated to be 0.267, with a standard error of 0.066. Deflating the pre-1929 nominal compensation series with the Johnston-Williamson (2020) implicit GDP deflator, as opposed to the Officer-Williamson (2020b) consumer price index, yields a similar wage elasticity of 0.263, with a standard error of 0.062. Our evidence that real wages are more inertial in the historical sample accords well with the economic history literature (footnote 12). In particular, the low wage elasticity to labor productivity, 0.256, in our model is empirically plausible.

Our value of $b = 0.91$ might seem high, as the marginal product of labor is around one in the model's simulations. However, the value of b includes unemployment benefits, the value of home production, self-employment, leisure, and disutility of work. Hagedorn and Manovskii (2008) argue that b should equal the marginal product of capital in a perfectly competitive labor market. Ljungqvist and Sargent (2017) show that to explain the unemployment volatility, a search model must diminish the fundamental surplus, which is the fraction of output allocated to the firm by the labor market. We view our high-b calibration as perhaps the simplest way to achieve this goal. More important, we view our high-b-low-η calibration as a parsimonious metaphor for real wage inertia. More explicit structures of wage inertia, such as alternating offer bargaining in Hall and Milgrom (2008) or staggered multiperiod Nash bargainng in Gertler and Trigari (2009), are likely to deliver similar quantitative results but would complicate our model greatly.[16]

[15]Ravn and Uhlig (2002) show that the smoothing parameter should be adjusted by the fourth power of the observation frequency ratio, which equals four going from the quarterly to annual frequency. In particular, $1600/4^4 = 6.25$.

[16]The high-b calibration is also of contemporary interest. Ganong, Noel, and Vavra (2020) document that under

3.3 Unconditional Moments

We report basic business cycle, labor market, and asset pricing moments from the model economy.

3.3.1 Business Cycle Moments

From the model's stationary distribution (after a burn-in period of 1,200 months), we repeatedly simulate 10,000 artificial samples, each with 1,740 months (145 years). The length of each sample matches the length of the Jordà-Schularick-Taylor database (1871–2015). On each artificial sample, we time-aggregate monthly consumption, output, and investment into annual observations. We add up 12 monthly observations within a given year and treat the sum as the year's annual observation. For each annual series, we compute its volatility, skewness, kurtosis, and autocorrelations of up to five lags of log growth rates. For each moment, we report the mean as well as the 5th, 50th, and 95th percentiles across the 10,000 simulations. We also report the p-value that is the fraction with which a given moment in the model is higher than its matching moment in the data. The fraction can be interpreted as the p-value for a one-sided test of our model using the moment in question.

Panel A of Table 2 shows that the model does a good job in matching consumption moments. None of the p-values for one-sided tests are significant at the 5% level. The consumption growth volatility in the model is 5.13% per annum, which is close to 5.45% in the data ($p = 0.41$). Kurtosis is 8.09 in the model, which is close to 10.34 in the data ($p = 0.18$). The first-order autocorrelation is 0.21 in the model, which is higher than 0.12 in the data, but the difference is insignificant ($p = 0.78$). The autocorrelations at higher orders are close to zero in the model as in the data.

From Panel B, the output volatility in the model is 6.43% per annum, which is higher than 5.1% in the data, but the difference is insignificant ($p = 0.86$). The model falls short in explaining the skewness, 0.09 versus −1.06, and kurtosis, 5.45 versus 14.09, of the output growth. Both differences are significant. The model comes close to match the first-order autocorrelation, 0.2 versus 0.18.

the 2020 Coronavirus Aid, Relief, and Economic Security Act, the ratio of mean benefits to mean earnings in the data is roughly 100%. The median replacement ratio is even higher at 134%. Finally, 68% of eligible unemployed workers have replacement ratios higher than 100%, and 20% of the workers have replacement ratios higher than 200%.

From Panel C, the investment volatility in the model is only 8.59% per annum, which is lower than 13.53% in the data. The difference is significant, but none of the p-values for other investment moments are significant. The kurtosis in the model is 7.12, relative to 10.75 in the data ($p = 0.08$). The first-order autocorrelation is 0.15 in the model, which is close to 0.13 in the data.

3.3.2 Labor Market Moments

Panel D of Table 2 shows that the model does a good job matching the first four moments of the unemployment rate. The mean unemployment rate is 8.63% in the model, which is close to 8.94% in the data ($p = 0.37$). The skewness is 2.64, relative to 2.13 in the data ($p = 0.53$), and the kurtosis, 13.45 versus 9.5 ($p = 0.35$). The unemployment volatility is 32.2% per quarter, which is higher than 24.43% in the data. However, the difference is not significant ($p = 0.76$).

The vacancy rate volatility is 33.73% per quarter in the model, which is significantly higher than 18.98% in the data. The volatility of labor market tightness is 33.98%, which is significantly lower than 61.62% in the data. However, as noted, this data moment is sensitive to detrending method and is only 38.38% with log deviations from the HP-trend. The unemployment-vacancy correlation is only -0.07 in the model, which is lower in magnitude than -0.57 in the data. However, this moment is also sensitive to detrending method. Using the monthly data simulated from the model with no detrending yields a U-V correlation of -0.475, which is close to the matching data moment of -0.517, and the difference is insignificant ($p = 0.66$). Finally, the wage elasticity to labor productivity is 0.256, and the data moment of 0.267 yields an insignificant p-value of 0.23.

3.3.3 Asset Pricing Moments

Most important, Panel E shows that our general equilibrium production economy succeeds in yielding an equity premium of 4.26% per annum, which is close to 4.36% in the data. The data moment lies comfortably within the model's 90% confidence interval, with a p-value of 0.34. The mean interest rate is 1.59% in the model, which is not far from 0.82% in the data. The data moment is again lies within the model's 90% confidence interval ($p = 0.87$).

The model implies a stock market volatility of 11.77% per annum, which is significantly lower than the data moment of 16.04%, although the U.S. volatility of 13.66% (Table 1) falls within the model's 90% confidence interval. The model's performance in matching stock market volatility improves over prior attempts in general equilibrium production economies (Gourio 2012).

The interest rate volatility in the model is 3.13% per annum, which is significantly lower than 7.3% in the data. The most likely reason is that we do not model sovereign default and hyperinflation that are the driving forces behind the historically high interest rate volatilities in Germany, Italy, and Japan. These destructive forces play only a limited role in the U.S., which has an interest rate volatility of only 4.65% (Table 1). It is well within the model's 90% confidence interval.

3.4 Sources of the Equity Premium

In this subsection we examine the driving forces behind the model's equity premium.

3.4.1 Dividend Dynamics

Rouwenhorst (1995) points out the difficulty in explaining the equity premium in production economies. Unlike endowment economies, in which dividends are exogenously specified to fit the data, dividends are often endogenously countercyclical in production economies. Dividends equal profits (output minus wages) minus investment. Intuitively, with frictionless labor market, wages equal the marginal product of labor, which is almost as procyclical as output. With the Cobb-Douglous production function, the marginal product of labor is exactly proportional to output. As such, profits are no more procyclical than output. However, due to consumption smoothing, investment is more procyclical than output and profits, rendering dividends countercyclical. Kaltenbrunner and Lochstoer (2010) demonstrate this insight in a stochastic growth model.

In contrast, dividends are endogenously procyclical in our search economy. Under the benchmark calibration, wages are more inertial than the marginal product of labor, making profits more procyclical than output. The magnified procyclical dynamics of profits then overpower the procyclical dynamics of vacancy costs and capital investment to make dividends procyclical.[17]

To what extent are the model's implied dividend dynamics empirically plausible? For each country, the Jordà-Schularick-Taylor macrohistory database provides separate capital gain, dividend-to-price, and consumer price index series, from which we construct the real dividend series (the Internet Appendix). Table S4 shows that dividends are procyclical in the historical cross-country panel. The correlation between the cyclical components of annual dividends and output is on average 0.11 across the countries, ranging from -0.02 from Portugal to 0.47 in the U.S. Only 3 out of 17 countries have negative correlations, all of which are small in magnitude. The relative volatility of dividends (the ratio of the dividend volatility over the output volatility) is 8.61 across the countries, varying from 3.06 from Portugal to 16.81 in Netherlands (3.18 in the U.S.).[18] Time-aggregating annual observations into 3- and 5-year observations raises the dividend-output correlation to 0.31 and 0.35 and lowers the relative volatility of dividends to 6.54 and 5.69, respectively.

The model explains procyclical dividends but overshoots the dividend-output correlation, 0.947. The model also underestimates the relative volatility of dividends at 2.89. Both differ significantly from their data moments. Time-aggregating does not materially affect the model's estimates. The dividend-output correlations are 0.954 and 0.952, and the relative volatility of dividends 2.83 and 2.74 at the 3- and 5-year frequencies, respectively. In the historical data, there are likely measurement errors in real dividends, which tend to average out over time, yielding higher dividend-output correlations at longer horizons. In contrast, no such measurement errors exist within the model.

A possible reason why the model overshoots the dividend-output correlation is that dividends in the data refer only to cash dividends, but dividends in the model match more closely to net payouts. Net payouts in the data include not only cash dividends but also share repurchases net of

[17]Petrosky-Nadeau, Zhang, and Kuehn (2018) examine this mechanism in a baseline search model without capital. However, with capital, consumption smoothing via investment strengthens the countercyclicality of dividends. We overcome this core challenge via wage inertia, for which we also provide new, supportive evidence (Section 3.2.1).

[18]Due to a few zero-dividend observations (7 out of 2,034), we detrend dividend and output series with HP-filtered proportional deviations from the mean. Using HP-filtered log deviations after discarding the 7 observations yields a higher dividend-output correlation of 0.24 and a relative dividend volatility of 7.92 averaged across the countries.

equity issuances (Boudoukh et al. 2007). Alas, to our knowledge, a historical sample of net payouts is not available. Perhaps more important, our model has only one shock, which drives the high dividend-output correlation, but there exist most likely multiple shocks in the data.

3.4.2 Disaster Dynamics

As shown in Petrosky-Nadeau, Zhang, and Kuehn (2018), the search model of equilibrium unemployment gives rise endogenously to rare disasters. To explain the equity premium, we formulate a more general model by incorporating both recursive utility and capital accumulation. Disaster risks in consumption play a key role in explaining the equity premium in our framework.

To characterize disasters in the data, we apply the Barro-Ursúa (2008) peak-to-trough method on the Jordà-Schularick-Taylor cross-country panel of consumption and output. Disasters are identified as episodes, in which the cumulative fractional decline in consumption or output exceeds a predetermined hurdle rate. We adopt two such hurdle rates, 10% and 15%.[19] We adjust for trend growth in the data because our model abstracts from growth. We subtract the mean log annual consumption growth of 1.62% from each consumption growth observation and subtract the mean log annual output growth of 1.78% from each output growth in the historical data (Table 1).

Table 3 shows that with a disaster hurdle rate of 10%, the consumption disaster probability is 6.4%, and the output disaster probability 5.78% in the cross-country panel. With a higher hurdle rate of 15%, the probabilities drop to 3.51% and 2.62%, respectively. The disaster size is 23.2% and 22.3% for consumption and output with a hurdle rate of 10%, but higher, 30.4 and 32.9, respectively, with a higher hurdle rate of 15%. The duration for consumption and output disasters lasts 4.2 and 4.1 years with a hurdle rate of 10%, but 4.5 and 5 years with a hurdle rate of 15%.

The model implied consumption disaster dynamics, which are crucial for the equity premium,

[19] Suppose there are two states, normalcy and disaster, in a given period. The number of disaster years is the number of years in the interval between peak and trough for each disaster event. The number of normalcy years is the total number of years in the sample minus the number of disaster years. The disaster probability is the likelihood with which the economy switches from normalcy to disaster in a given year. We calculate this probability as the ratio of the number of disasters over the number of normalcy years. For each disaster event, the disaster size is the cumulative fractional decline in consumption or output from peak to trough. Duration is the number of years from peak to trough.

are empirically plausible. We simulate 10,000 artificial samples from the model's stationary distribution, each with 1,740 months, matching the 1871–2015 sample length. On each sample, we time-aggregate monthly into annual consumption and apply the exact peak-to-trough method as in the data. From Panel A of Table 3, the disaster probabilities are 5.83% and 3.64%, which are relatively close to 6.4% and 3.51% in the data, with the hurdle rates of 10% and 15%, respectively. The size and duration of consumption disasters in the model are also close to those in the data, 23.4% versus 23.2% for size, and 4.1 versus 4.2 years for duration, with a hurdle rate of 10%, for example. The p-values all indicate that the differences between the model and data moments are insignificant.

As noted, consumption is more volatile than output in the cross-country panel, likely due to government purchases during wartime (Barro and Ursúa 2008). In contrast, consumption is naturally less volatile than output in production economies because of consumption smoothing. We focus on matching consumption dynamics because of their paramount importance for the equity premium. Consequently, the model overshoots output disasters. From Panel B, the output disaster probability is 10.9%, which is higher than 5.78% in the data ($p = 0.97$), with a hurdle rate of 10%. With a higher hurdle of 15%, the disaster probability is 6.1% in the model, which is still higher than 2.62% in the data ($p = 0.94$). However, disaster size and duration are relatively close to their data moments.

3.4.3 Consumption Dynamics

We dig deeper by comparing consumption dynamics in the search economy with those specified in the long-run risks literature (Bansal and Yaron 2004). Kaltenbrunner and Lochstoer (2010) show that long-run risks (high persistence in expected consumption growth) arise endogenously in production economies with frictionless labor market via consumption smoothing. Because of persistent aggregate productivity and consumption smoothing, long-run risks might also be present in our model. What is the relative role of long-run risks compared with disaster risks in our model? This economic question is important because different specifications of consumption dynamics can largely accord with observed moments of consumption growth, such as volatilities and autocorrelations, in the data. However, different specifications imply vastly different economic mechanisms.

We calculate the expected consumption growth and conditional consumption growth volatility in the model's state space. We use these solutions to simulate one million monthly periods from the model's stationary distribution. Fitting the consumption growth process specified by Bansal and Yaron (2004) on the simulated data yields:

$$g_{Ct+1} = E_t[g_{Ct+1}] + \sigma_{Ct}\,\epsilon^g_{t+1} \tag{20}$$

$$E_{t+1}[g_{Ct+2}] = 0.288\,E_t[g_{Ct+1}] + 0.705\,\sigma_{Ct}\,\epsilon^e_{t+1} \tag{21}$$

$$\sigma^2_{Ct+1} = 0.008^2 + 0.964(\sigma^2_{Ct} - 0.008^2) + 0.421\times 10^{-5}\epsilon^V_{t+1}, \tag{22}$$

in which g_{Ct+1} is realized consumption growth, $E_t[g_{Ct+1}]$ expected consumption growth, σ_{Ct} conditional volatility of g_{Ct+1}, and ϵ^g_{t+1}, ϵ^e_{t+1}, and ϵ^V_{t+1} are i.i.d. standard normal shocks. In addition, the unconditional correlation between ϵ^g_{t+1} and ϵ^e_{t+1} is 0.048, the unconditional correlation between ϵ^e_{t+1} and ϵ^V_{t+1} is 0.024, and the unconditional correlation between ϵ^g_{t+1} and ϵ^V_{t+1} is 0.079 in simulations.

Equation (21) shows that the persistence in expected consumption growth is only 0.288 in our model, which is substantially lower than 0.979 in Bansal and Yaron (2004).[20] However, our expected consumption growth is more volatile, with its conditional volatility about 70.5% of the conditional volatility of realized consumption growth. This fraction is much higher than 4.4% in Bansal and Yaron. Similarly, our persistence of expected consumption growth, 0.288, is also much lower than that implied by baseline production economies in Kaltenbrunner and Lochstoer (2010).[21] As such, despite recursive utility and autoregressive productivity shocks, long-run risks (in the sense of highly persistent expected consumption growth) do not play an important role in our economy.

Equation (22) shows that the search economy gives rise endogenously to time-varying volatil-

[20]Bansal and Yaron (2004) specify the monthly consumption growth process to be $E_{t+1}[g_{Ct+2}] = 0.979\,E_t[g_{Ct+1}] + 0.044\,\sigma_{Ct}\,\epsilon^e_{t+1}$, $g_{Ct+1} = 0.0015 + E_t[g_{Ct+1}] + \sigma_{Ct}\,\epsilon^g_{t+1}$, and $\sigma^2_{Ct+1} = 0.0078^2 + 0.987(\sigma^2_{Ct} - 0.0078^2) + 0.23\times 10^{-5}\epsilon^V_{t+1}$, in which $\epsilon^e_{t+1}, \epsilon^g_{t+1}$, and ϵ^V_{t+1}, are i.i.d. and mutually uncorrelated standard normal shocks.

[21]Kaltenbrunner and Lochstoer (2010, Table 6) show that the consumption growth follows $E_{t+1}[g_{Ct+2}] = 0.986\,E_t[g_{Ct+1}] + 0.093\,\sigma_{Ct}\,\epsilon^e_{t+1}$ and $g_{Ct+1} = 0.0013 + E_t[g_{Ct+1}] + \sigma_{Ct}\,\epsilon^g_{t+1}$, with transitory productivity shocks. With permanent shocks, $E_{t+1}[g_{Ct+2}] = 0.99\,E_t[g_{Ct+1}] + 0.247\,\sigma_{Ct}\epsilon^e_{t+1}$. However, σ_{Ct} is largely constant in both models.

ities (Bloom 2009). The consumption conditional variance appears "stochastic" in our model. Its persistence is 0.964, which is lower than 0.987 calibrated in Bansal and Yaron (2004) and 0.999 in Bansal, Kiku, and Yaron (2012). However, the volatility of our stochastic variance is 0.42×10^{-5}, which is higher than 0.23×10^{-5} in Bansal and Yaron and 0.28×10^{-5} in Bansal, Kiku, and Yaron. The time-variation of volatilities is another important dimension along which our search economy differs from stochastic growth models. These models with frictionless labor market yield largely constant volatilities (Kaltenbrunner and Lochstoer 2010). Perhaps more important, our quantitative results in equation (22) suggest that long-run risks in consumption volatility can be observationally equivalent to consumption disaster risks, potentially lending support to disaster models.

3.5 Time-varying Risks and Risk Premiums

We quantify the model's implications on time-varying equity premium and stock market volatility.

3.5.1 Equilibrium Properties

We first evaluate qualitative implications of the model's competitive equilibrium. From the model's stationary distribution (after a burn-in period of 1,200 months), we simulate a long sample of one million months. Figure 1 shows the scatterplots of key conditional moments against productivity. From Panel A, the price-to-consumption ratio, P_t/C_t, increases with productivity. In the 1-million-month sample, the correlations of P_t/C_t with productivity, output, unemployment, vacancy, and the investment rate are 0.97, 0.78, −0.48, 0.9, and 0.6, respectively. Clearly, P_t/C_t is procyclical.

In contrast, Panel B shows that the expected equity premium, $E_t[r_{St+1}] - r_{ft+1}$, is countercyclical. Its correlations with productivity, output, unemployment, vacancy, and the investment rate are −0.84, −0.86, 0.66, −0.87, and −0.36, respectively. In addition, the correlation between the expected equity premium and price-to-consumption is −0.88. Stock market volatility, σ_{St}, is also countercyclical (Panel C). Its correlations with productivity, output, unemployment, vacancy, and the investment rate are −0.91, −0.83, 0.57, −0.92, and −0.42, respectively. In addition, its correlations with the expected equity premium and price-to-consumption are 0.98 and −0.95, respectively.

Panel D shows that the riskfree rate, r_{ft+1}, is weakly procyclical in the model. Its correlations with productivity, output, unemployment, vacancy, and the investment rate are 0.23, 0.22, −0.2, 0.1, and 0.27, respectively. In addition, its correlations with the expected equity premium, stock market volatility, and price-to-consumption are $-0.15, -0.13$, and 0.28, respectively. Panel E shows that expected consumption growth, $E_t[g_{Ct+1}]$, behaves similarly as the risk-free rate. The correlation between $E_t[g_{Ct+1}]$ and r_{ft+1} is 0.998. Panel F shows that consumption volatility, σ_{Ct}, is weakly countercyclical. Although its correlations with output and unemployment are high, −0.48 and 0.74, its correlations with productivity and investment rate are low, −0.05 and 0.1, respectively.

In all, the model implies strong predictability for stock market excess return and volatility, some predictability for consumption volatility, and weak to no predictability for consumption growth and the interest rate. Intuitively, wage inertia yields operating leverage. In bad times, output falls, but wage inertia causes profits to drop disproportionately more than output, thereby magnifying the procyclical covariation of profits and dividends, causing the expected equity premium to rise.

More important, the impact of wage inertia is stronger in bad times, when the profits are even smaller because of low productivity. This time-varying wage inertia amplifies the risks and risk premiums, making the expected equity premium and stock market volatility countercyclical.[22] In contrast, consumption growth and consumption volatility are less predictable because of consumption smoothing via capital investment. Despite adjustment costs, investment absorbs a large amount of shocks to render the first two moments of consumption growth less predictable.

3.5.2 Data

Before quantifying the model's implications on time-varying risks and risk premiums, Table 4 shows long-horizon regressions of stock market excess returns and log consumption growth on log price-to-consumption in the historical data. We follow Beeler and Campbell (2012) but implement the

[22]Relatedly, Favilukis and Lin (2016) study this time-varying mechanism in a general equilibrium production economy with (exogenously specified) infrequent wage renegotiation, long-run risks, and labor adjustment costs. In contrast, wage inertia arises endogenously in our economy, and the equity premium arises from endogenous disaster risks.

tests on the Jordá-Schularick-Taylor historical cross-country panel. We perform the regressions on log price-to-consumption, as opposed to log price-to-dividend, because dividends (net payouts) can be negative in the model. To align the data moments with the model moments, we adjust excess returns in the data for financial leverage (by multiplying unadjusted excess returns with 0.71).

Panel A shows long-horizon predictive regressions of market excess returns:

$$\sum_{h=1}^{H} [\log(r_{St+h}) - \log(r_{ft+h})] = a + b \log(P_t/C_t) + u_{t+H}, \tag{23}$$

in which H is the forecast horizon, P_t real market index, C_t real consumption at the beginning of period t, and u_{t+H} the residual. Panel B shows long-horizon regressions of log consumption growth:

$$\sum_{h=1}^{H} \log(C_{t+h}/C_t) = a + b \log(P_t/C_t) + v_{t+H}, \tag{24}$$

in which v_{t+H} is the residual. In both long-horizon regressions, $\log(P_t/C_t)$ is standardized to have a mean of zero and a volatility of one. H ranges from one to five years. Finally, the t-values are adjusted for heteroscedasticity and autocorrelations of $2(H-1)$ lags.

Panel A shows some evidence of predictability of market excess returns. The slopes are largely negative across the countries and forecast horizons from one to five years, and their t-values are often significant, especially at the longer horizons. The R-squares averaged across the countries vary from 1.87% to 9% as the forecast horizon goes from one to five years. The prior asset pricing literature has mostly focused on the U.S. sample, which is an outlier in Panel A. In particular, the U.S. features the strongest evidence of predictability in terms of the t-values of slopes and R-squares. For example, in the 5-year horizon, the R^2 is 33.6% in the U.S. and 28% in the U.K., in contrast to 0% in Germany, 1% in Italy and Portugal, and 2% in France.

In the Internet Appendix (Table S5, Panel A), we document stronger stock market return predictability in the post-1950 sample. The slopes are all negative and mostly significant across the countries and forecast horizons. On average, the slopes are significant for all horizons except year one. The R-squares range from 4.9% in year one to 17.8% in year five.

Panel B of Table 4 shows that consumption growth is largely unpredictable. In the historical sample, the slopes averaged across the countries are all negative but insignificant. Even at the 5-year horizon, the R^2 is only 5.77% on average. In the post-1950 sample, the average slopes all flip to positive but remain insignificant, although the average R-squares increase somewhat, for example, to 9.1% in year five (Table S5, Panel B, the Internet Appendix).

Table 5 shows long-horizon regressions of excess return and consumption growth volatilities on log price-to-consumption. For a given forecast horizon, H, we measure excess return volatility as $\sigma_{St,t+H-1} = \sum_{h=0}^{H-1} |\epsilon_{St+h}|$, in which ϵ_{St+h} is the h-period-ahead residual from the first-order autoregression of log excess returns, $\log(r_{St+1}) - \log(r_{ft+1})$ (again adjusted for financial leverage). Panel A performs long-horizon predictive regressions of excess return volatilities:

$$\log \sigma_{St+1,t+H} = a + b \log(P_t/C_t) + u^{\sigma}_{t+H}. \tag{25}$$

In Panel B, the consumption volatility is $\sigma_{Ct,t+H-1} = \sum_{h=0}^{H-1} |\epsilon_{Ct+h}|$, in which ϵ_{Ct+h} is the h-period-ahead residual from the first-order autoregression of log consumption growth, $\log(C_{t+1}/C_t)$. We then perform long-horizon predictive regressions of consumption volatilities:

$$\log \sigma_{Ct+1,t+H} = a + b \log(P_t/C_t) + v^{\sigma}_{t+H}. \tag{26}$$

Panel A of Table 5 shows weak predictability for excess return volatilities. The average slopes are all negative and marginally significant in the first two years. The average R-squares range from 6.3% in year one to 19% in year five. However, the evidence is sensitive to sample period. In the post-1950 sample, the average slopes are all insignificant, with mixed signs (Table S6, Panel A, the Internet Appendix). Consumption volatilities are essentially unpredictable with log price-to-consumption. In the historical sample, the average slopes are all positive and, in long horizons, marginally significant. However, in the post-1950 sample, the slopes all flip to negative and insignificant.

3.5.3 The Model's Performance

We simulate 10,000 samples from the model's stationary distribution, each with 1,740 months. On each sample, we time-aggregate monthly returns and consumption into annual observations and implement the same procedures as in the data. Overall, the model succeeds in explaining stock market predictability but somewhat overstates consumption growth predictability, especially its volatility.

Table 6 shows the details. From Panel A, market excess returns are predictable in the model. The slopes are all significantly negative, and the R-squares range from 3.9% in year one to 13.5% in year five. None of the p-values for the slopes, their t-values, and R-squares are significant at the 5% level. From Panel B, the model overstates somewhat the consumption growth predictability. The slopes are all significantly negative. However, except for year one, the p-values for the slopes and their t-values indicate only insignificant differences between the model and data moments.

Panel C shows that stock market volatility is weakly predictable with log price-to-consumption in the model. As in the data, the slopes are all negative but insignificant. None of the p-values for slopes and their t-values suggest that the model moments deviate significantly from their data counterparts. However, the R-squares in the model are significantly lower than those in the data. More important, from Panel D, the model overstates the predictability of consumption growth volatility. While the slopes are mostly insignificant and positive in the data, the slopes in the model are significantly negative, and the p-values for the slopes and their t-values are significant.

3.6 Comparative Statics

In this subsection, we conduct comparative statics to shed light on the inner workings of our model. In each experiment, we vary one parameter only, while keeping all the other parameters identical to those in the benchmark calibration. (For log utility, we set both the risk aversion and intertemporal elasticity of substitution to one.) In all experiments, we recalibrate the capital scalar, K_0, to ensure the average labor share is unchanged from the benchmark calibration. Otherwise, the impact from changing a given parameter would be confounded with the impact of changing the labor share. The only exception is the $\alpha = 0.3$ experiment, in which we recalibrate K_0 to match the average labor share of 0.7. The simulations follow the same design as in the benchmark model.

3.6.1 Preference Parameters

Table 7 details the results. Not surprisingly, the risk aversion, γ, has a quantitatively important impact on the equity premium. Reducing γ from 10 to 7.5 and further to 5 lowers the equity premium from 4.26% per annum in the benchmark calibration to 1.55% and further to 0.54%. Stock market volatility also falls from 11.8% to 9.5% and further to 8%.

Most important, risk aversion also affects quantities. Reducing γ from 10 to 7.5 and further to 5 lowers consumption volatility from 5.13% to 4.24% and further to 3.93%. The probability of consumption disasters falls from 5.83% to 4.28% and further to 3.82%, and the disaster size also drops somewhat. A lower discount rate (the equity premium plus the interest rate) raises the marginal benefit of hiring, stimulating employment. Consequently, the mean unemployment rate falls from 8.63% to 5.71% and further to 4.63%. Although the unemployment volatility remains stable, the vacancy and labor market tightness volatilities both fall by about one-third. As such, echoing Gourio (2012) and Hall (2017) but differing from Tallarini (2000), our results indicate the necessity to jointly study macro quantities and asset prices, which do not seem to be determined separately.

The intertemporal elasticity of substitution, ψ, governs the willingness of the representative investor to substitute consumption over time. A lower elasticity indicates stronger incentives for consumption smoothing. Consequently, reducing ψ from 2 to 1.5 and further to 1 lowers the consumption volatility from 5.13% per annum to 4.89% and further to 4.51%. The consumption disaster probability falls from 5.83% to 5.4% and further to 4.77%. The disaster size also drops somewhat. The lower consumption risks reduce the equity premium from 4.26% to 3.82% and further to 3.17%. The lower discount rate again raises the marginal benefit of hiring to reduce the unemployment rate to 7.9% and further to 6.87%. However, labor market volatilities remain largely unchanged.

Finally, the log utility ($\gamma = \psi = 1$) implies lower consumption, output, and investment volatilities, 3.83%, 5.21%, and 5.32% per annum, than the benchmark calibration with recursive utility, 5.13%, 6.43%, and 8.59%, respectively. Although the unemployment volatility is largely unaffected, the vacancy and labor market tightness volatilities both fall by about one-third. The equity premium drops from 4.26% to only 0.53%, and stock market volatility from 11.77% to 8.68%.

3.6.2 Labor Market Parameters

The flow value of unemployment, b, plays an important role in driving our results. Lowering its value from 0.91 to 0.85 is sufficient to reduce the unemployment rate from 8.63% to 3.45% and the unemployment volatility from 0.32 to 0.07. Intuitively, a lower b reduces wages and raises profits, stimulating hiring incentives. A lower b also enlarges the fundamental surplus allocated to the firm, dampening the unemployment volatility (Hagedorn and Manovskii 2008; Ljungqvist and Sargent 2017). This mechanism also reduces the consumption volatility from 5.13% per annum to 2.62% and the consumption disaster probability from 5.83% to 2.36%. The smaller consumption risks then reduce the equity premium to only 0.45% and stock market volatility to 7.33%.

The bargaining weight of workers, η, also plays an important role in driving our results. Raising η from 0.01 to 0.025 makes wages more sensitive to shocks. The wage elasticity to labor productivity rises from 0.26 to 0.37. Because wages become more cyclical, profits and dividends become less cyclical, and the equity premium falls to 3.98% per annum. In addition, because workers gain a larger fraction of bargaining surplus, the unemployment rate rises somewhat from 8.63% to 8.81%. However, business cycle and labor market volatilities are largely unchanged.

The results are relatively insensitive to the separation rate, s. Reducing s from 3.5% to 3.25% lowers the unemployment rate slightly from 8.63% to 8.51%. The impact on business cycle and labor market volatilities is also small. The equity premium rises slightly from 4.26% per annum to 4.41%, and stock market volatility from 11.77% to 11.91%. The results are also relatively insensitive to the curvature parameter in the matching function, ι. Raising ι from 1.25 to 1.35 makes the matching process less frictional. The unemployment rate falls slightly from 8.63% to 8.5%. The impact on business cycle and labor market volatilities is also small. The equity premium rises slightly from 4.26% per annum to 4.3%, but stock market volatility falls slightly from 11.77% to 11.72%.

Raising the unit cost of vacancy posting, κ, from 0.01 to 0.025 increases the marginal cost of hiring, causing the unemployment rate to rise from 8.63% to 8.9%. The consumption, output, and investment volatilities all go up, but labor market volatilities remain largely unchanged. The equity premium falls somewhat from 4.26% per annum to 4.02%, but stock market volatility remains stable. From equation (18), a higher κ makes wages more sensitive to procyclical labor market tightness, θ_t. Consequently, profits and dividends become less procyclical, dampening the equity premium.

3.6.3 Technology Parameters

The supply elasticity of capital, ν, governs the magnitude of capital adjustment costs. A rising ν from 1.25 to 1.5 means falling adjustment costs, which in turn imply a stronger mechanism of consumption smoothing via investment. Consequently, the consumption volatility falls from 5.13% per annum to 4.98%, but the investment volatility rises from 8.59% to 9.41%, even though the output volatility remains largely unchanged at 6.45% (6.43% in the benchmark calibration). The lower consumption risks give rise to a lower equity premium, 4.03%, echoing Jermann (1998). A lower discount rate then raises the marginal benefit of hiring, reducing the unemployment rate to 8.54%. However, similar to the output volatility, labor market volatilities are largely unchanged.

Lowering the rate of capital depreciation, δ, from 1.25% to 1% per month reduces the consumption volatility from 5.13% to 4.71% per annum and the consumption disaster probability from 5.83% to 5.26%. The output volatility also falls to 5.98%, and the investment volatility to 7.3%. The lower amount of consumption risk reduces the equity premium from 4.26% to 2.56%. The lower discount rate provides stronger hiring incentives and reduces the unemployment rate to 6.86%. Intuitively, a lower δ gives rise to a larger stochastic steady state capital than the benchmark calibration, 18.2 versus 14.7. The larger capital stock helps stabilize the economy in the presence of shocks.[23]

[23]This effect of δ on the capital stock is distinct from the impact of the capital share. As note, we recalibrate the capital scalar, K_0, to keep the average labor share unchanged. Scaling by their respective K_0 values still yields a

Raising the elasticity of capital-labor substitution, $e = 1/(1-\omega)$, from 0.4 to 0.5 increases the business cycle and labor market volatilities. The consumption volatility rises from 5.13% per annum to 5.78%, and the consumption disaster probability from 5.83% to 6.31%. From the CES production function in equation (4), $\partial Y_t / \partial X_t$ increases with ω (and e). The higher amount of consumption risk implies a higher equity premium of 4.72% and a higher stock market volatility of 12.13%. Finally, a higher discount rate in turn implies a higher unemployment rate of 9.06%.

Finally, we change the distribution parameter, α, from 0.25 to 0.3. The average labor share falls to 0.7 in simulations. Although the stochastic steady state capital rises to 20.64, its value scaled by K_0 remains at 1.07, which is identical to the benchmark calibration. Because of a smaller labor share, labor market frictions play a less prominent role in this economy. Consequently, the business cycle and labor market volatilities all fall. The consumption volatility declines to 4.26% per annum, and the consumption disaster probability to 5.1%. As a result of the lower consumption risk, the equity premium falls to only 2.27%, and stock market volatility to 9.15%. The lower discount rate raises the marginal benefit of hiring, reducing the unemployment rate to 7.2%.

4 Additional Predictions

In this section, we quantify several additional implications from the model, including the term structure of the equity premium (Section 4.1), the term structure of real interest rates (Section 4.2), the timing premium (Section 4.3), and the welfare cost of business cycles (Section 4.4).

4.1 The Term Structure of the Equity Premium

Binsbergen, Brandt, and Koijen (2012) show that short-maturity dividend strips on the aggregate stock market have higher expected returns and volatilities than long-maturity dividend strips. This downward-sloping pattern seems difficult to reconcile with leading consumption-based models.[24]

somewhat higher stochastic steady state capital for the low-δ economy than the benchmark economy, 1.1 versus 1.07.

[24]Intuitively, in the Campbell-Cochrane (1999) external habit model, the impact of shocks on slow-moving surplus consumption is more pronounced for long-maturity dividend strips than for short-maturity strips, giving rise to an upward-sloping term structure of equity returns. In the Bansal-Yaron (2004) long-run risks model, small shocks on highly persistent expected consumption growth and to stochastic consumption volatility gradually build up

Our model yields a downward-sloping equity term structure. Let P^D_{nt} denote the price of an n-period dividend strip. For $n = 1$, $P^D_{1t} = E_t[M_{t+1}D_{t+1}]$. For $n > 1$, we solve for P^D_{nt} recursively from $P^D_{nt} = E_t[M_{t+1}P^D_{n-1,t+1}]$. We calculate $r^D_{n,t+1} \equiv P^D_{n-1,t+1}/P^D_{nt}$ as the return of buying the n-period dividend strip at time t and selling it at $t+1$. However, as noted, dividends in the model are net payouts, which can be negative in certain states of the world. Negative prices on these dividend strips then render their returns undefined. In practice, dividends are all positive when $n \geq 67$ months. As such, we calculate the equity term structure from year 6 to 40. In contrast, consumption in the model is always positive in all states of the world. Accordingly, we also calculate the term structure of consumption strips from year 1 to 40. The definitions of price of an n-period consumption strip, P^C_{nt}, and its return, $r^C_{n,t+1}$, are exactly analogous to those of the n-period dividend strip.

Figure 2 shows that risk premiums, volatilities, and Sharpe ratios on dividend and consumption strips are largely downward-sloping in our model. From Panel A, the dividend risk premium falls from 7.91% per annum in year 6 to 6.64% in year 10 and further to 1.26% in year 40. The volatility of the dividend strip falls from 22.54% in year 6 to 18.6% in year 10 and further to 3.86% in year 40 (Panel B). The Sharpe ratio of the dividend strip starts at 0.35 in year 6, rises slightly to 0.36 in year 10, and then falls steadily to 0.32 in year 40 (Panel C). For the consumption strip, the risk premium starts at 2.37% in year 1, rises to 2.52% in year 6, and then falls gradually to 0.59% in year 40 (Panel D). Its volatility starts at 6.82% in year 1, rises to 7.04% in year 4, and then drops to 2.49% in year 40 (Panel E). The Sharpe ratio starts at 0.348 in year 1, rises slightly to 0.358 in year 8, and falls to 0.237 in year 40 (Panel F). Finally, for the wealth portfolio that pays the consumption stream as its dividends, its risk premium is 2.23%, and its volatility 5.17%.

Intuitively, short-maturity dividend and consumption strips are riskier in our model because of their higher exposures to disaster risks. When the economy slides into a disaster, short-maturity

over longer horizons to make long-maturity dividend strips riskier than short-maturity strips, again yielding an upward-sloping equity term structure. In the Rietz-Barro baseline disaster model, dividend strips of all maturities are exposed to the same amount of disaster risks, which are specified to be i.i.d., yielding a flat equity term structure. Finally, in the Wachter (2013) model with time-varying, but highly persistent disaster probabilities, small shocks on the disaster probabilities build up over time to yield an upward-sloping equity term structure.

dividends and consumption take a big hit because of inertial wages. Long-maturity dividend and consumption strips are less impacted because disasters are followed by subsequent recoveries.[25]

4.2 The Term Structure of Real Interest Rates

We calculate the prices of real zero-coupon bonds for maturities ranging from 1 month to 10 years. Let P_{nt} denote the price of an n-period zero-coupon bond. For $n = 1$, $P_{1t} = E_t[M_{t+1}]$. For $n > 1$, we solve for P_{nt} recursively from $P_{nt} = E_t[M_{t+1}P_{n-1,t+1}]$. The log yield-to-maturity is $y_{nt} \equiv -\log(P_{nt})/n$. Let $r_{n,t+1} \equiv P_{n-1,t+1}/P_{nt}$ be the return of buying the n-period zero-coupon bond at time t and selling it at $t+1$. Excess returns are in excess of the 1-month interest rate, $r_{n,t+1} - r_{ft+1}$.

To calculate the term structure, we simulate one million months from the model's stationary distribution. The real yield curve is downward sloping in the model. The yield-to-maturity starts at 1.53% per annum for 1-month zero-coupon bond but falls to 1.29% for 1-year, 0.95% for 5-year, and further to 0.72% for 10-year zero-coupon bond. The average yield spread is −0.81% for the 10-year zero-coupon bond relative to the 1-month bond. The real term premium is also negative, −1.11%, for the 10-year zero-coupon bond. Intuitively, long-term bonds earn lower average returns because these bonds are hedges against disaster risks. Disasters stimulate precautionary savings, which in turn drive down real interest rates and push up real bond prices. Because the prices of long-term bonds tend to rise at the onset of disasters, these bonds provide hedges against disaster risks and, consequently, earn lower average returns (Nakamura et al. 2013; Wachter 2013).

Evidence on the slope of the real yield curve seems mixed. A large and liquid market for inflation-indexed bonds (index-linked gilts) has existed in the UK since 1982. Evans (1998) and Piazzesi and Schneider (2007) document that real yield curve is downward sloping in the U.K. In the U.S., Treasury inflation-protected securities (TIPS) start trading in 1997. Piazzesi and Schneider show that the TIPS yield curve appears to be upward sloping but caution that interpreting the evidence

[25]Nakamura et al. (2013) show that a model with (exogenous) multiperiod disasters and subsequent recoveries also yields a downward-sloping equity term structure. Our work differs in that disasters and recoveries are endogenous.

might be complicated by the relatively short sample and poor liquidity in the TIPS market.[26]

4.3 The Timing Premium

Epstein, Farhi, and Strzalecki (2014) show that the representative investor in the Bansal-Yaron (2004) model would give up an implausibly high fraction, 31%, of its consumption stream for the early resolution of consumption risks. In the Wachter (2013) model with time-varying disaster probabilities, this fraction is even higher at 42%. Epstein et al. argue that the fractions (dubbed the timing premium) seem too high because the household cannot use the information from the early resolution to modify its risky consumption stream. Because we follow Bansal and Yaron when calibrating preference parameters, with risk aversion higher than the inverse of the elasticity of intertemporal substitution (10 > 1/2), it is natural to ask what the timing premium is in our model.

The timing premium is defined as $\pi \equiv 1 - J_0/J_0^\star$, in which J_0 is the household's utility with risks resolved gradually, and $J_0^\star$ is the utility with risks resolved in the next period. Formally,

$$J_0^\star = \left[(1-\beta)C_0^{1-\frac{1}{\psi}} + \beta\left(E_t\left[(J_1^\star)^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}}, \tag{27}$$

in which the continuation utility $J_1^\star$ is given by

$$J_1^\star = \left[(1-\beta)\sum_{t=1}^{\infty}\beta^{t-1}C_t^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-1/\psi}}. \tag{28}$$

Following Epstein, Farhi, and Strzalecki (2014), we calculate $J_0^\star$ via Monte Carlo simulations, with the economy's stochastic steady state ($N_t = 0.9137$, $K_t = 14.6909$, and $x_t = 0.1887$) as the initial condition. Specifically, we simulate in total 100,000 sample paths, each with $T = 2,500$ months, while pasting J_0 as the continuation value at T. J_0 is available from our projection algorithm. On each path, we calculate one realization of $J_1^\star$ using equation (28). The expectation in equation (27), $E_t\left[(J_1^\star)^{1-\gamma}\right]$, is calculated as the cross-simulation average.

[26]We wish to point out that the downward sloping real yield curve in our model does not necessarily contradict the upward sloping nominal yield curve in the data. Nominal bonds are subject to inflation risks, which are left outside our model. Because long-term bonds are more exposed to persistent inflation risks, a positive inflation risk premium would give rise to an upward sloping nominal yield curve. We leave such an extension of our model to future work.

The timing premium in our model is only 15.3%. We view this estimate to be empirically plausible. For comparison, Epstein, Farhi, and Strzalecki (2014) calculate the timing premium to be 9.5% with i.i.d. consumption growth, a risk aversion of 10, and an elasticity of intertemporal substitution of 1.5. In the Barro (2009) model with a constant disaster probability, a risk aversion of 4, and an elasticity of intertemporal substitution of 2, the timing premium is 18%.

Intuitively, the long-run risks model assumes extremely high persistence in expected consumption growth (Bansal and Yaron 2004) or in conditional consumption volatility (Bansal, Kiku, and Yaron 2012). Analogously, the Wachter (2013) model assumes very high persistence in time-varying disaster probabilities. Because the risks are not resolved until much later, the investor that prefers early resolution of uncertainty would pay a high timing premium for the risks to be resolved early. In contrast, in our model, the expected consumption growth and conditional consumption volatility are much less persistent, as shown in equations (21) and (22), yielding a relatively low timing premium.

4.4 The Welfare Cost of Business Cycles

Lucas (1987, 2003) argues that the welfare cost of business cycles is negligible. Assuming log utility for the representative household and log-normal distribution for consumption growth, Lucas (2003) calculates that the agent would sacrifice a mere 0.05% of their consumption in perpetuity to eliminate consumption fluctuations. However, Lucas assumes log utility that fails to explain the equity premium puzzle. Atkeson and Phelan (1994), for example, argue that welfare cost calculations should be carried out within models that at least roughly replicate how asset markets price consumption risks. Because our model replicates the equity premium, we quantify its implied welfare cost.

Following Lucas (1987, 2003), we define the welfare cost of business cycles as the permanent percentage of the consumption stream that the representative household would sacrifice to eliminate aggregate consumption fluctuations. Formally, let ${}_tC \equiv \{C_t, C_{t+1}, \ldots\}$ be the consumption stream starting at time t. For a given state of the economy, (N_t, K_t, x_t), at date t, we calculate the welfare cost, denoted $\chi_t \equiv \chi(N_t, K_t, x_t)$, implicitly from:

$$J\left({}_tC(1+\chi_t)\right) = \overline{J}, \tag{29}$$

in which $\overline{J}$ is the recursive utility derived from the constant consumption at the deterministic steady state, $\overline{C}$. We solve for $\overline{J}$ by iterating on $\overline{J} = \left[(1-\beta)\overline{C}^{1-\frac{1}{\psi}} + \beta\overline{J}^{1-\frac{1}{\psi}}\right]^{\frac{1}{1-1/\psi}}$. Because the recursive utility J_t is linear homogeneous, $J\left({}_tC(1+\chi_t)\right) = (1+\chi_t)J\left({}_tC\right)$, solving for χ_t yields:

$$\chi_t = \frac{\overline{J}}{J_t} - 1. \tag{30}$$

We calculate the welfare cost, χ_t, on the state space, (N_t, K_t, x_t). To evaluate its magnitude, we simulate one million months of χ_t from the model's stationary distribution. The average welfare cost in simulations is 29.1%, which is more than 580 times of the Lucas estimate of 0.05%. The consumption in the stochastic steady state is 3.13% lower than the deterministic steady state consumption.

Perhaps more important, the welfare cost is time-varying and strongly countercyclical. In simulation, its median is 24.4%, and the 2.5th, 5th, and 25th percentiles are 17.3%, 18.4%, and 21.5%, whereas the 75th, 95th, and 97.5th percentiles are 31.7%, 56.3%, and 66.1%, respectively. Figure 3 shows the scatterplot of the welfare cost against the productivity in simulations. The welfare cost is clearly countercyclical. Its correlations with productivity, output, unemployment, vacancy, and the investment rate are $-0.76, -0.97$, 0.94, -0.66, and -0.46, respectively. The countercyclicality of the welfare cost imply that optimal fiscal and monetary policies aimed to dampen disaster risks are even more important than what the average welfare cost of 29.1% would suggest.

5 Conclusion

Labor market frictions are crucial for explaining the equity premium puzzle in general equilibrium. A dynamic stochastic general equilibrium economy with recursive utility, search frictions, and capital accumulation yields a high equity premium of 4.26% per annum and a low average interest rate of 1.59%, while simultaneously obtaining plausible quantity dynamics. The equity premium and stock market volatility are both countercyclical, and the real interest rate and consumption growth are largely unpredictable. The welfare cost of business cycles is huge, 29%. Wage inertia plays a key role by amplifying the procyclical dynamics of profits, which in turn overcome the procyclical dynamics of investment and vacancy costs to make dividends endogenously procyclical.

Several directions arise for future research. First, one can embed our model into a New Keynesian framework to examine the nominal yield curve and the interaction between the equity premium and fiscal and monetary policies. Second, one can extend our model to a multi-country setting to study international asset prices and business cycles. Finally, one can incorporate heterogeneous firms to study how the cross-sectional distribution impacts on aggregate quantities and asset prices.

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A Computational Algorithm

We adapt the globally nonlinear projection method with parameterized expectations in Petrosky-Nadeau and Zhang (2017) to our more general setting.

We approximate the x_t process with the discrete state space method of Rouwenhorst (1995) with 17 grid points, which are sufficient to cover the values of x_t within four unconditional standard deviations from its unconditional mean, $\overline{x}$. The Rouwenhorst grid is symmetric around $\overline{x}$. The grid is also even-spaced, with the distance between any two adjacent grid points, d_x, given by:

$$d_x \equiv 2\sigma/\sqrt{(1-\rho^2)(n_x-1)}, \tag{A.1}$$

in which ρ is the persistence, σ the conditional volatility of x_t, and $n_x = 17$. We still need to construct the transition matrix, Π, in which the (i,j) element, Π_{ij}, is the probability of $x_{t+1} = x_j$ conditional on $x_t = x_i$. To this end, we set $p = (\rho+1)/2$, and:

$$\Pi^{(3)} \equiv \begin{bmatrix} p^2 & 2p(1-p) & (1-p)^2 \\ p(1-p) & p^2+(1-p)^2 & p(1-p) \\ (1-p)^2 & 2p(1-p) & p^2 \end{bmatrix}, \tag{A.2}$$

which is the transition matrix for $n_x = 3$. To obtain $\Pi^{(17)}$, we use the following recursion:

$$p\begin{bmatrix} \Pi^{(n_x)} & \mathbf{0} \\ \mathbf{0}' & 0 \end{bmatrix} + (1-p)\begin{bmatrix} \mathbf{0} & \Pi^{(n_x)} \\ 0 & \mathbf{0}' \end{bmatrix} + (1-p)\begin{bmatrix} \mathbf{0}' & 0 \\ \Pi^{(n_x)} & \mathbf{0} \end{bmatrix} + p\begin{bmatrix} 0 & \mathbf{0}' \\ \mathbf{0} & \Pi^{(n_x)} \end{bmatrix}, \tag{A.3}$$

in which $\mathbf{0}$ is a $n_x \times 1$ column vector of zeros. We then divide all but the top and bottom rows by two to ensure that the conditional probabilities sum up to one in the resulting transition matrix, $\Pi^{(n_x+1)}$. Rouwenhorst (p. 306–307; p. 325–329) contains more details.

The state space of our model consists of employment, capital, and productivity, (N_t, K_t, x_t). The goal is to solve for the indirect utility function, $J(N_t, K_t, x_t)$, the optimal vacancy function, $V(N_t, K_t, x_t)$, the multiplier function, $\lambda(N_t, K_t, x_t)$, and the optimal investment function, $I(N_t, K_t, x_t)$, from the following three functional equations:

$$J(N_t, K_t, x_t) = \left[(1-\beta)C(N_t, K_t, x_t)^{1-\frac{1}{\psi}} + \beta\left(E_t\left[J(N_{t+1}, K_{t+1}, x_{t+1})^{1-\gamma}\right]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} \tag{A.4}$$

$$\begin{aligned} \frac{1}{a_2}\left(\frac{I(N_t, K_t, x_t)}{K_t}\right)^{1/\nu} &= E_t\left[M_{t+1}\left[\frac{Y(N_{t+1}, K_{t+1}, x_{t+1})}{K_{t+1}}\frac{\alpha\left(K_{t+1}/K_0\right)^{\omega}}{\alpha\left(K_{t+1}/K_0\right)^{\omega} + (1-\alpha)N_{t+1}^{\omega}}\right.\right. \\ &\left.\left. + \frac{1}{a_2}\left(\frac{I(N_{t+1}, K_{t+1}, x_{t+1})}{K_{t+1}}\right)^{1/\nu}(1-\delta+a_1) + \frac{1}{\nu-1}\frac{I(N_{t+1}, K_{t+1}, x_{t+1})}{K_{t+1}}\right]\right] \end{aligned} \tag{A.5}$$

$$\begin{aligned} \frac{\kappa}{q(\theta_t)} - \lambda(N_t, K_t, x_t) &= E_t\left[M_{t+1}\left[\frac{Y(N_{t+1}, K_{t+1}, x_{t+1})}{N_{t+1}}\frac{(1-\alpha)N_{t+1}^{\omega}}{\alpha\left(K_{t+1}/K_0\right)^{\omega} + (1-\alpha)N_{t+1}^{\omega}} - W_{t+1}\right.\right. \\ &\left.\left. + (1-s)\left[\frac{\kappa}{q(\theta(N_{t+1}, K_{t+1}, x_{t+1}))} - \lambda(N_{t+1}, K_{t+1}, x_{t+1})\right]\right]\right], \end{aligned} \tag{A.6}$$

in which

$$M_{t+1} = \beta \left[\frac{C(N_{t+1}, K_{t+1}, x_{t+1})}{C(N_t, K_t, x_t)} \right]^{-\frac{1}{\psi}} \left[\frac{J(N_{t+1}, K_{t+1}, x_{t+1})}{E_t[J(N_{t+1}, K_{t+1}, x_{t+1})^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right]^{\frac{1}{\psi}-\gamma}. \tag{A.7}$$

Also, $V(N_t, K_t, x_t)$ and $\lambda(N_t, K_t, x_t)$ must satisfy the Kuhn-Tucker conditions.

Following Petrosky-Nadeau and Zhang (2017), we deal with $V_t \geq 0$ by exploiting a convenient mapping from the conditional expectation function, $\mathcal{E}_t \equiv \mathcal{E}(N_t, K_t, x_t)$, defined as the right-hand side of equation (A.6), to policy and multiplier functions to eliminate the need to parameterize the multiplier separately. After obtaining $\mathcal{E}_t$, we first calculate $\tilde{q}(\theta_t) = \kappa_0 / (\mathcal{E}_t - \kappa_1)$. If $\tilde{q}(\theta_t) < 1$, the $V_t \geq 0$ constraint is not binding, we set $\lambda_t = 0$ and $q(\theta_t) = \tilde{q}(\theta_t)$. We then solve $\theta_t = q^{-1}(\tilde{q}(\theta_t))$, in which $q^{-1}(\cdot)$ is the inverse function of $q(\theta_t)$, and $V_t = \theta_t(1 - N_t)$. If $\tilde{q}(\theta_t) \geq 1$, the $V_t \geq 0$ constraint is binding, we set $V_t = 0$, $\theta_t = 0$, $q(\theta_t) = 1$, and $\lambda_t = \kappa_0 + \kappa_1 - \mathcal{E}_t$. An advantage of the installation function, Φ_t, is that when investment goes to zero, the marginal benefit of investment, $\partial\Phi(I_t, K_t)/\partial I_t = a_2(I_t/K_t)^{-1/\nu}$, goes to infinity. As such, the optimal investment is always positive, with no need to impose the $I_t \geq 0$ constraint. We approximate $I(N_t, K_t, x_t)$ directly.

We approximate $J(N_t, K_t, x_t)$, $I(N_t, K_t, x_t)$, and $\mathcal{E}(N_t, K_t, x_t)$ on each grid point of x_t. We use the finite element method with cubic splines on 50 nodes on the N_t space, $[0.245, 0.975]$, and 50 nodes on the K_t space, $[5, 20]$. We experiment to ensure that the bounds are not binding at a frequency higher than 0.01% in the model's simulations. We take the tensor product of N_t and K_t for each grid point of x_t. We use the Miranda-Fackler (2002) CompEcon toolbox for function approximation and interpolation. With three functional equations on the 17-point x_t grid, the 50-point N_t grid, and the 50-point K_t grid, we must solve a system of 127,500 nonlinear equations. we use such a large system to ensure the accuracy of our numerical solution. Following Judd et al. (2014), we use derivative-free fixed point iteration with a damping parameter of 0.00325. The convergence criterion is set to be 10^{-4} for the maximum absolute value of the errors across the nonlinear functional equations.

Table 1 : Basic Properties of the Real Consumption, Output, and Investment Growth and Asset Prices in the Historical Sample

The historical cross-country panel is from the Jordà-Schularick-Taylor macrohistory database, except for Canada's asset prices, which we obtain from the Dimson-Marsh-Staunton (2002) database purchased from Morningstar. All (annual) series end in 2015. In Panels A and B, the column "Sample" indicates the sample's starting year. In Panels C and D, besides the starting year, the "Sample" column also reports the missing years in parentheses. For example, the real investment growth series for Australia starts in 1871 but is missing from 1947 to 1949. Other than Italy, which has missing asset prices from 1872 to 1884, in Panel D, all other missing years are in the 20th century. In Panel A, $\overline{g}_C$, σ_C, S_C, K_C, and $\rho_C^{(i)}$ denote the mean (in percent), volatility (in percent), skewness, kurtosis, and ith-order autocorrelation, for $i = 1, 2, \ldots, 5$, of log real per capita consumption growth. In Panel B, $\overline{g}_Y$, σ_Y, S_Y, K_Y, and $\rho_Y^{(i)}$ denote the mean (in percent), volatility (in percent), skewness, kurtosis, and ith-order autocorrelation for log real per capita output growth. In Panel C, $\overline{g}_I$, σ_I, S_I, K_I, and $\rho_I^{(i)}$ denote the mean (in percent), volatility (in percent), skewness, kurtosis, and ith-order autocorrelation for log real per capita investment growth. Finally, in Panel D, $E[\widetilde{r}_S]$, $\widetilde{\sigma}_S$, and $E[\widetilde{r}_S - r_f]$ are the average stock market return, stock market volatility, and the equity premium, respectively, without adjusting for financial leverage. $E[r_S - r_f]$ and σ_S are the equity premium and stock market volatility, respectively, after adjusting for financial leverage. $E[r_f]$ is the mean real interest rate, and σ_f the interest rate volatility. All asset pricing moments are in annual percent. We require nonmissing stocks, bonds, and bills.

	Panel A: Real consumption growth										Panel B: Real output growth									
	Sample	$\overline{g}_C$	σ_C	S_C	K_C	$\rho_C^{(1)}$	$\rho_C^{(2)}$	$\rho_C^{(3)}$	$\rho_C^{(4)}$	$\rho_C^{(5)}$	Sample	$\overline{g}_Y$	σ_Y	S_Y	K_Y	$\rho_Y^{(1)}$	$\rho_Y^{(2)}$	$\rho_Y^{(3)}$	$\rho_Y^{(4)}$	$\rho_Y^{(5)}$
Australia	1871	1.11	5.76	−0.77	6.35	−0.04	0.22	−0.03	0.03	−0.09	1871	1.45	4.11	−0.90	5.49	0.04	0.27	−0.10	−0.03	−0.05
Belgium	1914	1.35	8.72	−1.14	13.18	0.26	0.19	0.00	−0.40	−0.22	1871	1.63	7.45	1.26	19.01	0.33	0.05	0.00	0.03	−0.29
Canada	1872	1.77	4.62	−1.04	6.27	0.00	0.16	−0.16	−0.04	−0.14	1871	1.87	4.97	−0.78	5.11	0.26	0.11	−0.07	−0.15	−0.15
Denmark	1871	1.38	5.27	−0.83	11.44	−0.01	−0.41	0.06	0.18	−0.23	1871	1.68	3.66	−1.03	8.13	0.05	−0.17	0.08	0.08	−0.08
Finland	1871	2.07	5.54	−1.13	9.01	0.16	−0.08	0.02	−0.04	−0.23	1871	2.06	4.47	−0.78	7.15	0.25	−0.11	0.10	−0.12	−0.17
France	1871	1.37	6.57	−1.06	13.69	0.39	0.19	−0.06	−0.28	−0.14	1871	1.64	6.20	−0.60	10.30	0.09	−0.09	0.10	0.19	−0.09
Germany	1871	1.67	5.51	−0.57	7.11	0.25	0.24	0.28	−0.07	0.00	1871	1.62	10.66	−7.62	78.70	0.30	−0.04	−0.11	−0.16	−0.13
Italy	1871	1.47	3.63	0.14	7.62	0.38	0.32	0.10	0.08	0.11	1871	1.80	4.71	−1.32	13.34	0.27	−0.06	−0.03	0.14	0.01
Japan	1875	2.11	6.74	−1.53	20.90	0.21	0.10	0.18	0.20	0.20	1871	2.40	6.18	−2.23	15.50	0.27	0.03	0.16	0.09	0.01
Netherlands	1871	1.41	8.18	−0.83	19.86	0.17	0.13	−0.21	−0.21	−0.19	1871	1.54	6.75	0.97	32.58	0.25	−0.12	−0.02	−0.07	−0.16
Norway	1871	1.83	3.65	−0.32	12.65	−0.06	−0.34	0.26	0.07	−0.24	1871	2.10	3.53	−0.72	7.21	0.11	−0.08	0.12	0.06	−0.15
Portugal	1911	2.36	4.36	−0.49	3.30	0.22	0.23	−0.02	0.09	−0.16	1871	1.84	4.16	−0.01	4.23	0.01	0.18	0.02	0.18	0.04
Spain	1871	1.56	7.92	−2.20	17.20	0.00	−0.02	−0.13	−0.05	0.08	1871	1.86	4.98	−1.58	10.94	0.18	0.05	0.03	0.04	0.14
Sweden	1871	1.80	4.20	0.44	7.04	−0.15	−0.17	0.05	0.07	−0.20	1871	2.02	3.39	−1.32	7.30	−0.08	−0.04	0.02	0.18	−0.17
Switzerland	1871	1.22	5.85	0.35	7.34	−0.20	−0.10	−0.11	−0.10	0.04	1871	1.41	3.84	−0.41	4.02	0.13	−0.14	−0.05	0.09	0.05
UK	1871	1.33	2.76	−0.34	8.90	0.33	0.02	−0.06	−0.01	−0.11	1871	1.40	2.86	−0.89	5.62	0.35	0.03	−0.18	−0.22	−0.09
USA	1871	1.75	3.42	−0.07	3.99	0.08	0.09	−0.11	0.00	−0.10	1871	1.91	4.77	−0.08	4.83	0.25	0.08	−0.13	−0.19	−0.19
Mean		1.62	5.45	−0.67	10.34	0.12	0.04	0.00	−0.03	−0.09		1.78	5.10	−1.06	14.09	0.18	0.00	0.00	0.01	−0.09
Median		1.56	5.51	−0.77	8.90	0.16	0.10	−0.02	−0.01	−0.14		1.80	4.71	−0.78	7.30	0.25	−0.04	0.00	0.04	−0.09

	Panel C: Real investment growth									
	Sample	$\overline{g}_I$	σ_I	S_I	K_I	$\rho_I^{(1)}$	$\rho_I^{(2)}$	$\rho_I^{(3)}$	$\rho_I^{(4)}$	$\rho_I^{(5)}$
Australia	1871 (47–49)	1.60	13.56	−0.72	5.06	0.15	0.09	−0.07	−0.16	−0.07
Belgium	1901 (14–20, 40–46)	1.68	10.74	−0.20	3.44	−0.09	−0.06	−0.02	−0.23	0.14
Canada	1872	2.17	18.12	−0.18	10.68	0.27	0.02	−0.18	−0.19	−0.16
Denmark	1871 (15–22)	1.96	10.10	−0.52	6.63	0.21	−0.11	−0.05	0.00	−0.17
Finland	1871	2.40	13.24	−1.49	11.14	0.19	0.01	0.06	−0.27	−0.28
France	1871 (19–20, 45–46)	1.98	19.23	−1.33	16.16	−0.07	−0.31	−0.04	−0.08	0.15
Germany	1871 (14–20, 40–48)	2.69	14.42	−0.56	5.40	0.06	−0.01	−0.10	−0.11	−0.23
Italy	1871	2.50	12.42	1.82	23.10	0.11	−0.14	0.12	0.03	−0.08
Japan	1886 (45–46)	4.21	14.36	−0.77	13.61	0.14	−0.04	−0.07	0.00	0.08
Netherlands	1871 (14–21, 40–48)	1.78	8.23	−0.28	3.70	0.03	0.01	−0.15	−0.04	−0.21
Norway	1871 (40–46)	2.69	13.33	2.08	21.86	−0.13	−0.16	0.02	−0.04	−0.05
Portugal	1954	2.64	9.58	−0.22	3.08	0.22	0.21	0.06	−0.13	0.08
Spain	1871	2.85	13.23	−0.41	4.01	0.23	0.02	−0.23	−0.13	−0.12
Sweden	1871	2.65	12.43	0.10	4.88	0.07	−0.27	−0.08	0.01	−0.11
Switzerland	1871 (14–48)	2.58	11.02	0.69	5.33	0.37	0.17	−0.11	−0.33	−0.22
UK	1871	1.98	11.68	2.82	26.62	0.35	−0.14	−0.12	−0.03	−0.08
USA	1871	2.04	24.37	−1.71	18.02	0.17	−0.11	−0.32	−0.13	−0.02
Mean		2.38	13.53	−0.05	10.75	0.13	−0.05	−0.07	−0.11	−0.08
Median		2.40	13.23	−0.28	6.63	0.15	−0.04	−0.07	−0.11	−0.08

	Panel D: Asset prices							
	Sample	$E[\widetilde{r}_S]$	$\widetilde{\sigma}_S$	$E[r_f]$	σ_f	$E[\widetilde{r}_S - r_f]$	$E[r_S - r_f]$	σ_S
Australia	1900 (45–47)	7.75	17.08	1.29	4.32	6.46	4.58	12.55
Belgium	1871 (14–19)	6.31	19.88	1.21	8.43	5.10	3.62	14.62
Canada	1900	7.01	17.00	1.60	4.79	5.41	3.84	12.26
Denmark	1875 (15)	7.47	16.43	3.08	5.68	4.39	3.12	11.91
Finland	1896	8.83	30.57	−0.74	10.93	9.57	6.80	22.98
France	1871 (15–21)	3.99	22.22	−0.47	7.78	4.45	3.16	16.75
Germany	1871 (23, 44–49)	8.83	27.59	−0.23	13.22	9.05	6.43	20.22
Italy	1871 (1872–84, 15–21)	6.63	27.21	0.58	10.50	6.05	4.29	20.41
Japan	1886 (46–47)	8.86	27.69	0.00	11.20	8.87	6.29	21.10
Netherlands	1900	6.96	21.44	0.78	4.91	6.19	4.39	15.32
Norway	1881	5.67	19.82	0.90	5.98	4.77	3.39	14.53
Portugal	1880	3.81	25.68	−0.01	9.43	3.82	2.71	19.29
Spain	1900 (36–40)	6.25	21.41	−0.04	6.90	6.29	4.47	15.94
Sweden	1871	8.00	19.54	1.77	5.60	6.23	4.42	14.26
Switzerland	1900 (15)	6.69	19.08	0.89	5.00	5.79	4.11	14.00
UK	1871	6.86	17.77	1.16	4.82	5.70	4.05	12.96
USA	1872	8.40	18.68	2.17	4.65	6.23	4.43	13.66
Mean		6.96	21.71	0.82	7.30	6.14	4.36	16.04
Median		6.96	19.88	0.89	5.98	6.05	4.29	14.62

Table 2 : Basic Moments in the Model Under the Benchmark Calibration

The model moments are based on 10,000 simulated samples, each with 1,740 months. On each artificial sample, we calculate the moments and report the mean as well as the 5th, 50th, and 95th percentiles across the 10,000 simulations. p-value is the fraction with which a model moment is higher than its data moment. The data moments are from Table 1. In Panel A, σ_C, S_C, K_C, and ρ_{Ci}, for $i = 1, 2, \ldots, 5$, denote the volatility (in percent), skewness, kurtosis, and ith-order autocorrelation of the log consumption growth. The symbols in Panels B and C are defined analogously. In Panel D, $E[U]$, S_U, and K_U are the mean, skewness, and kurtosis of monthly unemployment rates, σ_U, σ_V, and σ_θ are the volatilities of quarterly unemployment, vacancy, and labor market tightness, respectively. ρ_{UV} is the cross-correlation of quarterly unemployment and vacancy rates, and $e_{w,y/n}$ the wage elasticity to labor productivity. In Panel E, $E[r_S - r_f]$, $E[r_f]$, σ_S, and σ_f are the average equity premium, average real interest rate, stock market volatility, and interest rate volatility, respectively, all of which are in annual percent.

	Data	Mean	5th	50th	95th	p		Data	Mean	5th	50th	95th	p
Panel A: Real consumption growth							Panel B: Real output growth						
σ_C	5.45	5.13	2.87	5.13	7.39	0.41	σ_Y	5.10	6.43	4.46	6.40	8.48	0.86
S_C	−0.67	0.03	−1.03	0.03	1.10	0.89	S_Y	−1.06	0.09	−0.62	0.08	0.81	0.99
K_C	10.34	8.09	4.38	7.30	14.44	0.18	K_Y	14.09	5.45	3.50	5.09	8.64	0.00
ρ_{C1}	0.12	0.21	−0.01	0.22	0.40	0.78	ρ_{Y1}	0.18	0.20	0.03	0.21	0.36	0.60
ρ_{C2}	0.04	−0.05	−0.26	−0.05	0.17	0.24	ρ_{Y2}	0.00	−0.06	−0.23	−0.06	0.12	0.31
ρ_{C3}	0.00	−0.04	−0.24	−0.04	0.16	0.35	ρ_{Y3}	0.00	−0.05	−0.22	−0.05	0.12	0.31
ρ_{C4}	−0.03	−0.04	−0.23	−0.04	0.15	0.44	ρ_{Y4}	0.01	−0.05	−0.21	−0.05	0.12	0.29
ρ_{C5}	−0.09	−0.04	−0.23	−0.04	0.14	0.67	ρ_{Y5}	−0.09	−0.05	−0.21	−0.05	0.12	0.65
Panel C: Real investment growth							Panel D: Labor market moments						
σ_I	13.53	8.59	5.29	8.61	11.83	0.00	$E[U]$	8.94	8.63	3.81	7.45	17.63	0.37
S_I	−0.05	0.31	−0.57	0.28	1.26	0.76	S_U	2.13	2.64	0.76	2.20	5.85	0.53
K_I	10.75	7.12	4.12	6.47	12.17	0.08	K_U	9.50	13.45	2.11	6.77	39.06	0.35
ρ_{I1}	0.13	0.15	−0.04	0.16	0.33	0.58	σ_U	0.24	0.32	0.16	0.32	0.48	0.76
ρ_{I2}	−0.05	−0.11	−0.29	−0.11	0.08	0.30	σ_V	0.19	0.34	0.23	0.32	0.49	1.00
ρ_{I3}	−0.07	−0.09	−0.27	−0.09	0.10	0.45	σ_θ	0.62	0.34	0.23	0.32	0.50	0.01
ρ_{I4}	−0.11	−0.07	−0.25	−0.07	0.11	0.62	ρ_{UV}	−0.57	−0.07	−0.16	−0.07	0.01	1.00
ρ_{I5}	−0.08	−0.06	−0.24	−0.06	0.12	0.56	$e_{w,y/n}$	0.27	0.26	0.23	0.26	0.27	0.22
Panel E: Asset pricing moments													
$E[r_S - r_f]$	4.36	4.26	3.52	4.12	5.49	0.34							
$E[r_f]$	0.82	1.59	0.07	1.83	2.26	0.87							
σ_S	16.04	11.77	9.19	11.74	14.46	0.00							
σ_f	7.30	3.13	1.13	3.05	5.37	0.00							

Table 3 : Disaster Moments in the Data and in the Model

The data moments are obtained by applying the Barro-Ursúa (2008) peak-to-trough method on the Jordà-Schularick-Taylor cross-country panel. We adjust for trend growth in the data (no growth in our model). We subtract each log annual consumption growth observation with its mean of 1.62% and subtract each log annual output growth with the mean of 1.78% in the historical panel. For model moments, we simulate 10,000 artificial samples from the model's stationary distribution under the benchmark calibration, each with 1,740 months, matching the number of years, 145, from 1871 to 2015. On each artificial sample, we time-aggregate consumption and output into annual observations and apply the peak-to-trough method to identify disasters as cumulative fractional declines of consumption or output of at least 10% or 15%. We report the mean, 5th, 50th, and 95th percentiles across the simulations. If no disaster appears in an artificial sample, we set its disaster probability to zero and calculate the model's disaster probability moments across all the 10,000 simulations. However, we calculate disaster size and duration across samples with at least one disaster. The disaster probability and size are in percent, and duration in the number of years.

	Data	Mean	5th	50th	95th	p	Data	Mean	5th	50th	95th	p
	Disaster hurdle = 10%						Disaster hurdle = 15%					
	Panel A: Consumption disasters											
Probability	6.40	5.83	1.55	5.31	11.32	0.37	3.51	3.64	0.71	3.20	7.69	0.45
Size	23.16	23.41	14.52	22.83	33.84	0.48	30.36	29.51	18.81	28.49	43.31	0.38
Duration	4.19	4.10	2.80	4.00	5.80	0.40	4.50	4.49	3.00	4.33	6.81	0.39
	Panel B: Output disasters											
Probability	5.78	10.9	6.14	10.58	16.48	0.97	2.62	6.10	2.33	5.88	10.68	0.94
Size	22.34	22.31	15.91	21.89	30.13	0.46	32.9	28.50	20.07	27.88	38.86	0.20
Duration	4.14	3.73	2.89	3.67	4.78	0.23	5.04	4.25	3.00	4.17	5.75	0.15

Table 4 : Predicting Excess Returns and Consumption Growth with Log Price-to-consumption in the Historical Sample

The cross-country panel is from the Jordà-Schularick-Taylor macrohistory database, except for Canada. The annuals series start as early as 1870 and end in 2015 (the Internet Appendix). Panel A performs predictive regressions of stock market excess returns on log price-to-consumption, $\sum_{h=1}^{H}[\log(r_{St+h}) - \log(r_{ft+h})] = a + b\log(P_t/C_t) + u_{t+H}$, in which H is the forecast horizon, r_{St+1} the real stock market return, r_{ft+1} the real interest rate, P_t the real stock market index, and C_t real consumption. r_{St+1} and r_{ft+1} are over the course of period t, and P_t and C_t are at the beginning of period t (the end of period $t-1$). Excess returns are adjusted for a financial leverage ratio of 0.29. Panel B performs long-horizon predictive regressions of log consumption growth on $\log(P_t/C_t)$, $\sum_{h=1}^{H}\log(C_{t+h}/C_t) = c + d\log(P_t/C_t) + v_{t+H}$. In both regressions, $\log(P_t/C_t)$ is standardized to have a mean of zero and a standard deviation of one. H ranges from one year (1y) to five years (5y). The t-values of the slopes are adjusted for heteroscedasticity and autocorrelations of $2(H-1)$ lags. The slopes and R-squares are in percent.

	Slopes					t-values					R-squares				
	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
	Panel A: Predicting stock market excess returns														
Australia	−1.42	−2.49	−2.92	−3.53	−3.77	−1.97	−1.96	−1.80	−1.74	−1.62	1.80	3.14	3.56	4.20	4.29
Belgium	−1.30	−3.26	−4.79	−5.48	−5.16	−0.82	−0.98	−1.00	−0.91	−0.76	0.58	1.62	2.47	2.58	2.01
Denmark	−0.81	−1.94	−2.87	−3.74	−4.24	−0.85	−1.18	−1.43	−1.81	−2.14	0.50	1.35	2.13	3.04	3.76
Finland	−1.38	−3.79	−5.40	−6.40	−7.36	−0.77	−1.05	−1.06	−1.07	−1.22	0.55	1.78	2.55	3.05	3.78
France	−0.12	−0.34	−0.52	−0.63	−0.43	−0.11	−0.18	−0.21	−0.20	−0.11	0.01	0.03	0.05	0.05	0.02
Germany	−1.04	−2.11	−2.06	−1.54	0.13	−0.75	−0.91	−0.54	−0.28	0.02	0.19	0.34	0.21	0.08	0.00
Italy	−0.36	−0.58	−0.36	−0.07	0.38	−0.25	−0.22	−0.10	−0.01	0.07	0.04	0.05	0.01	0.00	0.01
Japan	−0.70	−1.40	−1.60	−1.73	−1.77	−0.45	−0.56	−0.45	−0.41	−0.36	0.19	0.36	0.35	0.30	0.24
Netherlands	−3.03	−6.45	−8.88	−11.06	−13.35	−1.68	−1.88	−2.11	−2.48	−2.98	4.15	9.00	12.73	16.34	20.25
Norway	−1.77	−3.59	−5.13	−6.52	−7.92	−1.55	−2.07	−2.41	−2.76	−3.24	1.75	3.61	5.60	7.68	9.84
Portugal	−0.24	−2.39	−3.87	−3.41	0.53	−0.08	−0.39	−0.50	−0.37	0.06	0.02	0.55	0.83	0.48	0.01
Spain	−1.02	−2.77	−4.90	−6.80	−8.13	−0.74	−0.92	−1.21	−1.66	−2.38	0.59	1.68	3.13	4.42	5.25
Sweden	−1.56	−3.81	−6.04	−8.31	−10.50	−1.63	−2.29	−2.91	−3.19	−3.20	1.42	3.74	6.47	9.64	13.08
Switzerland	−3.09	−6.51	−8.50	−10.67	−12.95	−1.70	−2.30	−2.85	−3.89	−4.17	4.02	8.50	11.76	15.72	20.05
UK	−2.95	−5.64	−7.62	−8.91	−10.51	−2.33	−4.92	−5.43	−5.84	−5.92	6.35	12.49	18.14	23.18	28.03
USA	−3.50	−7.45	−9.89	−12.98	−15.75	−3.83	−4.50	−4.35	−4.59	−5.16	7.71	16.13	21.01	27.48	33.59
Mean	−1.52	−3.41	−4.71	−5.74	−6.30	−1.22	−1.64	−1.77	−1.95	−2.07	1.87	4.02	5.69	7.39	9.01
Median	−1.34	−3.01	−4.84	−5.94	−6.26	−0.83	−1.11	−1.32	−1.70	−1.88	0.59	1.73	2.84	3.63	4.03
	Panel B: Predicting consumption growth														
Australia	0.75	0.98	1.14	1.50	1.85	1.40	0.88	0.65	0.67	0.71	1.69	1.52	1.21	1.49	1.75
Belgium	−1.03	−1.38	−0.94	−0.68	−0.10	−0.91	−0.73	−0.41	−0.26	−0.04	1.41	1.05	0.30	0.11	0.00
Denmark	0.23	0.32	0.28	0.24	0.20	0.71	0.73	0.52	0.40	0.29	0.18	0.18	0.13	0.08	0.05
Finland	−0.91	−2.10	−2.90	−3.62	−4.07	−1.14	−1.46	−1.54	−1.67	−1.68	2.30	5.20	6.56	7.56	7.66
France	−0.84	−1.47	−2.02	−2.55	−3.18	−2.12	−1.81	−1.81	−1.85	−1.95	1.64	1.79	1.89	2.11	2.67
Germany	−0.95	−1.87	−2.88	−3.79	−4.70	−2.15	−1.85	−1.81	−1.74	−1.74	2.97	4.64	6.17	6.84	7.79
Italy	−0.60	−1.22	−1.74	−2.28	−2.91	−2.71	−2.21	−1.96	−1.87	−1.89	2.74	4.02	4.32	4.84	5.79
Japan	−1.76	−3.59	−5.38	−7.12	−8.78	−4.04	−3.35	−2.89	−2.60	−2.40	8.22	11.84	14.23	15.81	16.95
Netherlands	0.66	1.10	1.43	1.83	2.32	2.41	1.47	1.22	1.17	1.14	7.27	6.03	5.50	6.17	7.48
Norway	−0.35	−0.77	−1.21	−1.68	−2.10	−1.36	−1.80	−2.11	−2.40	−2.54	0.91	2.40	5.58	8.09	9.68
Portugal	−1.05	−2.20	−3.26	−4.08	−4.95	−2.18	−1.72	−1.67	−1.61	−1.53	4.82	8.98	10.91	11.55	11.55
Spain	−0.10	−0.18	−0.41	−0.67	−1.10	−0.14	−0.14	−0.27	−0.38	−0.55	0.02	0.02	0.08	0.17	0.40
Sweden	0.18	0.22	0.20	0.02	−0.17	0.56	0.44	0.28	0.02	−0.17	0.18	0.15	0.10	0.00	0.05
Switzerland	0.22	0.31	0.36	0.35	0.34	1.32	0.84	0.61	0.43	0.33	2.52	1.40	1.00	0.64	0.44
UK	−0.33	−0.89	−1.53	−2.32	−3.15	−1.78	−2.22	−2.77	−3.53	−4.16	1.44	3.94	7.06	11.86	17.32
USA	0.48	−0.09	−0.64	−1.05	−1.40	1.86	−0.18	−0.85	−1.08	−1.23	1.89	0.03	0.94	1.92	2.70
Mean	−0.34	−0.80	−1.22	−1.62	−1.99	−0.64	−0.82	−0.93	−1.02	−1.09	2.51	3.32	4.12	4.95	5.77
Median	−0.34	−0.83	−1.07	−1.36	−1.75	−1.02	−1.09	−1.20	−1.35	−1.38	1.79	2.09	3.10	3.48	4.25

Table 5 : Predicting Volatilities of Stock Market Excess Returns and Consumption Growth with Log Price-to-consumption in the Historical Sample

The cross-country panel is from the Jordà-Schularick-Taylor macrohistory database, except for Canada. The annuals series start in 1870 and end in 2015. For a given forecast horizon, H, we measure excess return volatility as $\sigma_{St,t+H-1} = \sum_{h=0}^{H-1} |\epsilon_{St+h}|$, in which ϵ_{St+h} is the h-period-ahead residual from the first-order autoregression of excess returns, $\log(r_{St+1}) - \log(r_{ft+1})$. Excess returns are adjusted for a financial leverage ratio of 0.29. Panel A performs long-horizon predictive regressions of excess return volatilities, $\log \sigma_{St+1,t+H} = a + b \log(P_t/C_t) + u^{\sigma}_{t+H}$. Consumption growth volatility is $\sigma_{Ct,t+H-1} = \sum_{h=0}^{H-1} |\epsilon_{Ct+h}|$, in which ϵ_{Ct+h} is the h-period-ahead residual from the first-order autoregression of log consumption growth, $\log(C_{t+1}/C_t)$. Panel B performs long-horizon predictive regressions of consumption growth volatilities, $\log \sigma_{Ct+1,t+H} = c + d \log(P_t/C_t) + v^{\sigma}_{t+H}$. $\log(P_t/C_t)$ is standardized to have a mean of zero and a standard deviation of one. H ranges from one year (1y) to five years (5y). The t-values are adjusted for heteroscedasticity and autocorrelations of $2(H-1)$ lags. The slopes and R-squares are in percent.

	Slopes					t-values					R-squares				
	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
	Panel A: Predicting stock market volatility														
Australia	20.04	16.84	15.73	16.20	15.82	1.89	1.91	1.81	1.81	1.80	2.15	3.55	4.28	5.51	6.55
Belgium	11.85	12.70	12.20	11.69	11.61	1.28	2.11	2.05	1.99	2.21	1.42	5.12	7.26	8.84	10.71
Denmark	−35.30	−37.64	−38.17	−37.40	−36.44	−3.70	−4.02	−3.95	−3.74	−3.43	7.90	16.11	21.11	24.13	25.79
Finland	6.94	3.22	5.56	5.49	3.54	0.66	0.45	0.97	1.02	0.65	0.42	0.23	1.26	1.56	0.76
France	−58.81	−60.73	−60.03	−58.54	−57.57	−6.19	−6.50	−5.93	−5.51	−5.17	20.49	37.29	42.82	45.17	46.37
Germany	−31.59	−35.33	−35.20	−34.06	−32.90	−2.89	−3.61	−3.42	−3.02	−2.67	5.44	12.60	16.61	17.96	18.71
Italy	−17.60	−23.51	−24.80	−24.34	−24.27	−1.92	−2.94	−2.86	−2.59	−2.41	2.45	8.22	12.62	15.43	17.69
Japan	8.99	7.32	9.12	10.91	11.75	0.80	0.74	0.94	1.15	1.26	0.48	0.72	1.89	3.41	4.74
Netherlands	7.49	8.46	11.28	10.93	8.97	0.50	0.67	1.03	1.18	1.15	0.48	1.43	4.60	5.52	5.30
Norway	−51.27	−54.63	−54.25	−53.32	−52.54	−5.44	−7.48	−7.26	−7.41	−7.81	20.22	39.57	51.02	56.54	60.54
Portugal	−50.20	−45.97	−44.35	−43.46	−39.43	−4.10	−3.57	−3.50	−3.56	−3.39	14.11	23.07	27.71	28.72	25.37
Spain	−37.40	−34.97	−34.23	−33.42	−32.51	−4.00	−5.24	−4.81	−4.51	−3.96	10.86	18.97	26.06	30.91	33.48
Sweden	−23.98	−22.89	−21.83	−21.84	−21.98	−2.75	−2.62	−2.16	−1.93	−1.79	4.88	8.45	9.78	10.82	11.85
Switzerland	7.05	11.57	9.51	11.11	11.03	0.39	0.87	0.90	1.18	1.30	0.27	2.01	3.01	5.79	7.64
UK	−35.31	−34.28	−33.22	−32.10	−31.62	−4.99	−4.69	−4.10	−3.58	−3.23	9.59	18.29	21.60	22.69	23.91
USA	0.30	5.54	6.58	7.08	8.06	0.03	0.87	1.57	2.13	2.51	0.00	0.65	1.77	2.89	4.86
Mean	−17.43	−17.77	−17.26	−16.57	−16.16	−1.90	−2.07	−1.80	−1.59	−1.44	6.32	12.27	15.84	17.87	19.02
Median	−20.79	−23.20	−23.32	−23.09	−23.12	−2.34	−2.78	−2.51	−2.26	−2.10	3.67	8.34	11.20	13.12	14.77
	Panel B: Predicting consumption growth volatility														
Australia	3.23	−3.95	−3.07	−4.13	−5.52	0.28	−0.39	−0.27	−0.32	−0.40	0.06	0.18	0.14	0.28	0.55
Belgium	48.77	54.27	55.42	58.66	59.15	2.88	3.74	3.41	3.50	3.50	11.11	23.57	29.61	36.63	40.03
Denmark	−2.11	−1.62	0.21	0.64	1.13	−0.17	−0.15	0.02	0.05	0.09	0.02	0.03	0.00	0.01	0.03
Finland	32.87	35.10	38.93	40.82	41.42	2.38	2.84	3.27	3.61	3.79	6.84	16.21	25.65	30.35	33.31
France	84.04	78.92	77.39	76.70	76.69	9.11	9.59	9.25	8.72	8.35	37.34	53.32	60.94	65.91	68.76
Germany	11.37	11.62	13.29	14.96	16.28	1.14	0.98	0.95	0.96	0.95	0.77	1.42	2.15	3.01	3.75
Italy	6.73	7.80	8.51	9.88	11.70	0.78	1.01	1.06	1.15	1.30	0.36	0.89	1.60	2.71	4.48
Japan	37.88	39.76	39.78	39.88	39.93	3.66	3.72	3.54	3.26	3.07	8.50	15.96	21.30	23.11	24.82
Netherlands	7.04	7.92	9.68	9.26	8.42	0.60	0.73	0.85	0.89	0.90	0.58	1.51	3.20	3.47	3.56
Norway	3.69	5.69	4.34	3.81	3.63	0.34	0.50	0.37	0.33	0.32	0.09	0.38	0.28	0.27	0.29
Portugal	13.68	15.62	16.08	18.09	19.63	1.43	2.51	3.57	5.20	5.93	2.03	6.49	10.05	15.19	18.62
Spain	64.78	61.73	59.39	57.46	56.29	6.29	6.16	5.80	5.51	5.46	25.68	40.34	49.05	51.18	54.04
Sweden	−1.44	1.56	3.93	6.22	7.43	−0.14	0.17	0.39	0.57	0.64	0.01	0.03	0.26	0.79	1.32
Switzerland	−13.49	−13.67	−13.37	−9.64	−6.97	−1.01	−1.06	−1.11	−0.84	−0.69	1.40	2.71	3.78	2.63	1.61
UK	0.76	0.75	1.50	2.03	2.31	0.07	0.11	0.22	0.27	0.30	0.00	0.01	0.06	0.14	0.23
USA	−18.05	−19.66	−18.25	−17.46	−15.82	−1.89	−2.05	−1.87	−1.71	−1.45	2.44	5.14	6.30	6.70	6.07
Mean	17.49	17.62	18.36	19.20	19.73	1.61	1.78	1.84	1.95	2.00	6.08	10.51	13.40	15.15	16.34
Median	6.88	7.86	9.10	9.57	10.06	0.69	0.85	0.90	0.93	0.92	1.09	2.11	3.49	3.24	4.12

Table 6 : Predicting Excess Returns, Consumption Growth, and Their Volatilities with Log Price-to-consumption in the Model

The data moments are the mean estimates in Tables 4 and 5 on the Jordà-Schularick-Taylor database. For the model moments, we simulate 10,000 artificial samples from the model's stationary distribution (with a burn-in of 1,200 months), each with 1,740 months. On each artificial sample, we time-aggregate monthly market excess returns and consumption growth into annual observations and implement the exactly same procedures as in Tables 4 and 5. We report the mean, 5th, 50th, and 95th percentiles across the simulations as well as the p-value that is the fraction of simulations with which a given model moment is higher than its data moment. In all the long-horizon regressions, the log price-to-consumption ratio, $\log(P_t/C_t)$, is standardized to have a mean of zero and a standard deviation of one. The forecast horizon, H, ranges from one year (1y) to five years (5y). The t-values of the slopes are adjusted for heteroscedasticity and autocorrelations of $2(H-1)$ lags. The slopes and R-squares are in percent.

	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
	Panel A: Predicting stock market excess returns														
	Data					Mean					p				
b	−1.52	−3.41	−4.71	−5.74	−6.30	−1.82	−3.45	−4.91	−6.22	−7.40	0.33	0.50	0.47	0.42	0.34
t	−1.22	−1.64	−1.77	−1.95	−2.07	−2.36	−2.86	−3.17	−3.39	−3.57	0.09	0.09	0.08	0.08	0.09
R^2	1.87	4.02	5.69	7.39	9.01	3.86	6.83	9.40	11.59	13.52	0.77	0.76	0.78	0.77	0.76
	5th					50th					95th				
b	−2.96	−5.40	−7.61	−9.66	−11.55	−1.81	−3.41	−4.84	−6.13	−7.29	−0.74	−1.64	−2.41	−3.10	−3.64
t	−3.87	−4.49	−4.99	−5.33	−5.68	−2.33	−2.81	−3.10	−3.31	−3.48	−0.93	−1.36	−1.59	−1.71	−1.79
R^2	0.58	1.67	2.62	3.55	4.31	3.44	6.35	8.86	11.00	13.09	8.64	13.70	17.94	21.34	24.49
	Panel B: Predicting consumption growth														
	Data					Mean					p				
b	−0.34	−0.80	−1.22	−1.62	−1.99	−1.27	−1.86	−2.44	−3.00	−3.52	0.01	0.06	0.09	0.12	0.13
t	−0.64	−0.82	−0.93	−1.02	−1.09	−2.69	−2.41	−2.49	−2.64	−2.79	0.01	0.07	0.10	0.13	0.13
R^2	2.51	3.32	4.12	4.95	5.77	7.34	7.20	8.44	9.86	11.27	0.88	0.74	0.70	0.68	0.68
	5th					50th					95th				
b	−2.02	−3.06	−4.09	−5.07	−6.03	−1.24	−1.83	−2.41	−2.95	−3.47	−0.61	−0.71	−0.83	−0.99	−1.12
t	−4.47	−4.57	−4.96	−5.41	−5.74	−2.63	−2.29	−2.32	−2.44	−2.60	−1.15	−0.69	−0.55	−0.53	−0.51
R^2	1.36	0.65	0.53	0.54	0.58	6.68	6.11	6.95	8.25	9.55	15.51	17.63	21.02	24.69	27.92
	Panel C: Predicting excess return volatilities														
	Data					Mean					p				
b	−17.43	−17.77	−17.26	−16.57	−16.16	−15.94	−13.55	−12.03	−11.01	−10.15	0.55	0.68	0.75	0.78	0.81
t	−1.90	−2.07	−1.80	−1.59	−1.44	−1.48	−1.73	−1.84	−1.89	−1.88	0.64	0.62	0.49	0.41	0.37
R^2	6.32	12.27	15.84	17.87	19.02	2.12	3.30	4.54	5.61	6.34	0.06	0.02	0.02	0.03	0.04
	5th					50th					95th				
b	−36.81	−28.96	−25.35	−23.21	−21.78	−15.80	−13.46	−12.02	−10.95	−10.04	4.60	1.70	0.99	0.68	0.91
t	−3.43	−3.69	−3.90	−4.03	−4.06	−1.46	−1.72	−1.82	−1.85	−1.84	0.42	0.20	0.15	0.11	0.17
R^2	0.02	0.04	0.06	0.07	0.08	1.37	2.37	3.38	4.24	4.80	6.73	9.73	13.01	15.98	18.01
	Panel D: Predicting consumption growth volatilities														
	Data					Mean					p				
b	17.49	17.72	18.36	19.20	19.73	−34.67	−32.89	−31.47	−30.14	−28.85	0.00	0.00	0.00	0.00	0.00
t	1.61	1.78	1.84	1.95	2.00	−3.36	−3.98	−4.03	−3.95	−3.82	0.00	0.00	0.00	0.00	0.00
R^2	6.08	10.51	13.40	15.15	16.34	7.69	13.16	15.89	17.19	17.73	0.58	0.62	0.60	0.57	0.54
	5th					50th					95th				
b	−56.53	−51.99	−49.84	−47.99	−46.64	−35.58	−34.16	−32.69	−31.32	−29.94	−8.95	−9.24	−8.75	−7.93	−7.04
t	−5.81	−6.88	−7.22	−7.23	−7.15	−3.39	−4.02	−4.01	−3.88	−3.72	−0.77	−0.98	−0.98	−0.96	−0.91
R^2	0.58	1.31	1.80	1.94	1.88	7.06	12.80	15.58	16.84	17.38	16.72	26.19	30.99	33.55	34.79

Table 7 : Comparative Statics

The first column of numbers shows the model moments from the benchmark calibration. The remaining columns show the model moments from 14 comparative statics. γ is relative risk aversion; ψ the intertemporal elasticity of substitution; b the flow value of unemployment; η the bargaining weight for workers; s the separation rate; ι the curvature of the matching function; κ the unit cost of vacancy posting; ν the adjustment cost parameter; δ the capital depreciation rate; $1/(1-\omega)$ the elasticity of capital-labor substitution; and α the distribution parameter. In each experiment, all the other parameters are identical to those in the benchmark calibration. For the model moments, σ_C is the consumption growth volatility per annum, ρ_{C1} the first-order autocorrelation of consumption growth, and Prob_C, Size_C, and Dur_C the probability, size, and duration of consumption disasters with a cumulative decline hurdle rate of 10%. σ_Y is the output growth volatility, ρ_{Y1} the first-order autocorrelation of output growth, and Prob_Y, Size_Y, and Dur_Y the probability, size, and duration of output disasters with a cumulative decline hurdle rate of 10%. σ_I is the investment growth volatility, ρ_{I1} the first-order autocorrelation of investment growth. The consumption, output, and investment volatilities, and the probability and size of consumption and output disasters are in percent. Their durations are in years. $E[U]$ is mean unemployment rate, σ_U, σ_V, and σ_θ the quarterly volatilities of unemployment, vacancy, and labor market tightness, respectively, ρ_{UV} the cross-correlation of unemployment and vacancy, and $e_{w,y/n}$ the wage elasticity to labor productivity. Finally, $E[r_S - r_f]$ is the average equity premium, $E[r_f]$ the average interest rate, σ_S stock market volatility, and σ_f the interest rate volatility, all of which are in annual percent.

	Benchmark	γ	γ	ψ	ψ	γ, ψ	b	η	s	ι	κ	ν	δ	ω	α
		7.5	5	1.5	1	1	0.85	0.025	0.0325	1.35	0.025	1.5	0.01	−1	0.3
σ_C	5.13	4.24	3.94	4.89	4.51	3.83	2.62	5.19	5.17	5.09	5.24	4.98	4.71	5.78	4.26
ρ_{C1}	0.21	0.18	0.15	0.20	0.19	0.16	0.14	0.22	0.21	0.21	0.22	0.23	0.17	0.19	0.21
Prob_C	5.83	4.28	3.82	5.40	4.77	3.54	2.36	6.42	5.89	5.74	5.93	5.46	5.26	6.31	5.10
Size_C	23.41	20.69	19.36	22.68	21.68	19.70	13.80	22.65	23.36	23.23	23.70	23.68	21.34	25.04	20.27
Dur_C	4.10	4.46	4.46	4.16	4.29	4.52	4.98	4.12	4.12	4.11	4.10	4.21	4.12	3.95	4.34
σ_Y	6.43	5.58	5.17	6.23	5.91	5.21	4.11	6.37	6.45	6.40	6.52	6.45	5.98	6.97	5.62
ρ_{Y1}	0.20	0.18	0.16	0.20	0.19	0.17	0.15	0.21	0.21	0.20	0.21	0.21	0.17	0.20	0.20
Prob_Y	10.90	9.37	8.61	10.47	9.99	8.66	7.44	10.91	10.85	10.80	10.99	10.76	10.17	11.31	9.99
Size_Y	22.31	20.03	18.94	21.76	20.91	19.13	16.00	22.12	22.32	22.15	22.50	22.44	20.84	23.38	20.27
Dur_Y	3.73	3.84	3.88	3.74	3.78	3.88	4.00	3.73	3.73	3.73	3.72	3.75	3.72	3.66	3.81
σ_I	8.59	6.27	4.56	8.13	7.36	5.32	2.55	8.45	8.67	8.54	8.66	9.41	7.30	8.91	6.71
ρ_{I1}	0.15	0.13	0.11	0.15	0.14	0.11	0.09	0.16	0.16	0.15	0.16	0.15	0.14	0.16	0.15
$E[U]$	8.63	5.71	4.63	7.90	6.87	4.90	3.45	8.81	8.51	8.50	8.90	8.54	6.86	9.06	7.20
σ_U	0.32	0.35	0.35	0.33	0.34	0.34	0.07	0.31	0.33	0.32	0.32	0.32	0.35	0.36	0.30
σ_V	0.34	0.27	0.24	0.32	0.30	0.24	0.16	0.34	0.34	0.34	0.33	0.33	0.30	0.35	0.31
σ_θ	0.34	0.27	0.25	0.32	0.30	0.24	0.16	0.34	0.34	0.34	0.34	0.33	0.31	0.35	0.31
ρ_{UV}	−0.07	−0.08	−0.09	−0.08	−0.08	−0.09	−0.30	−0.07	−0.07	−0.07	−0.08	−0.07	−0.08	−0.08	−0.08
$e_{w,y/n}$	0.26	0.26	0.26	0.26	0.26	0.26	0.27	0.37	0.26	0.26	0.26	0.26	0.26	0.25	0.26
$E[r_S - r_f]$	4.26	1.55	0.54	3.82	3.17	0.53	0.45	3.98	4.41	4.30	4.02	4.03	2.57	4.72	2.27
$E[r_f]$	1.59	2.45	2.75	1.58	1.54	2.68	2.82	1.67	1.49	1.51	1.83	1.62	2.26	1.38	2.29
σ_S	11.77	9.50	7.99	11.32	10.61	8.68	7.33	11.13	11.91	11.72	11.79	11.05	10.01	12.13	9.15
σ_f	3.13	2.27	1.78	3.74	4.60	3.32	0.64	2.95	3.09	3.25	2.81	3.11	2.36	3.46	2.23

Figure 1 : Scatterplots of Key Moments Against Productivity

From the model's stationary distribution with the benchmark calibration (after a burn-in period of 1,200 monthly periods), we simulate a long sample path with one million months. The equity premium, stock market volatility, and consumption volatility are in percent.

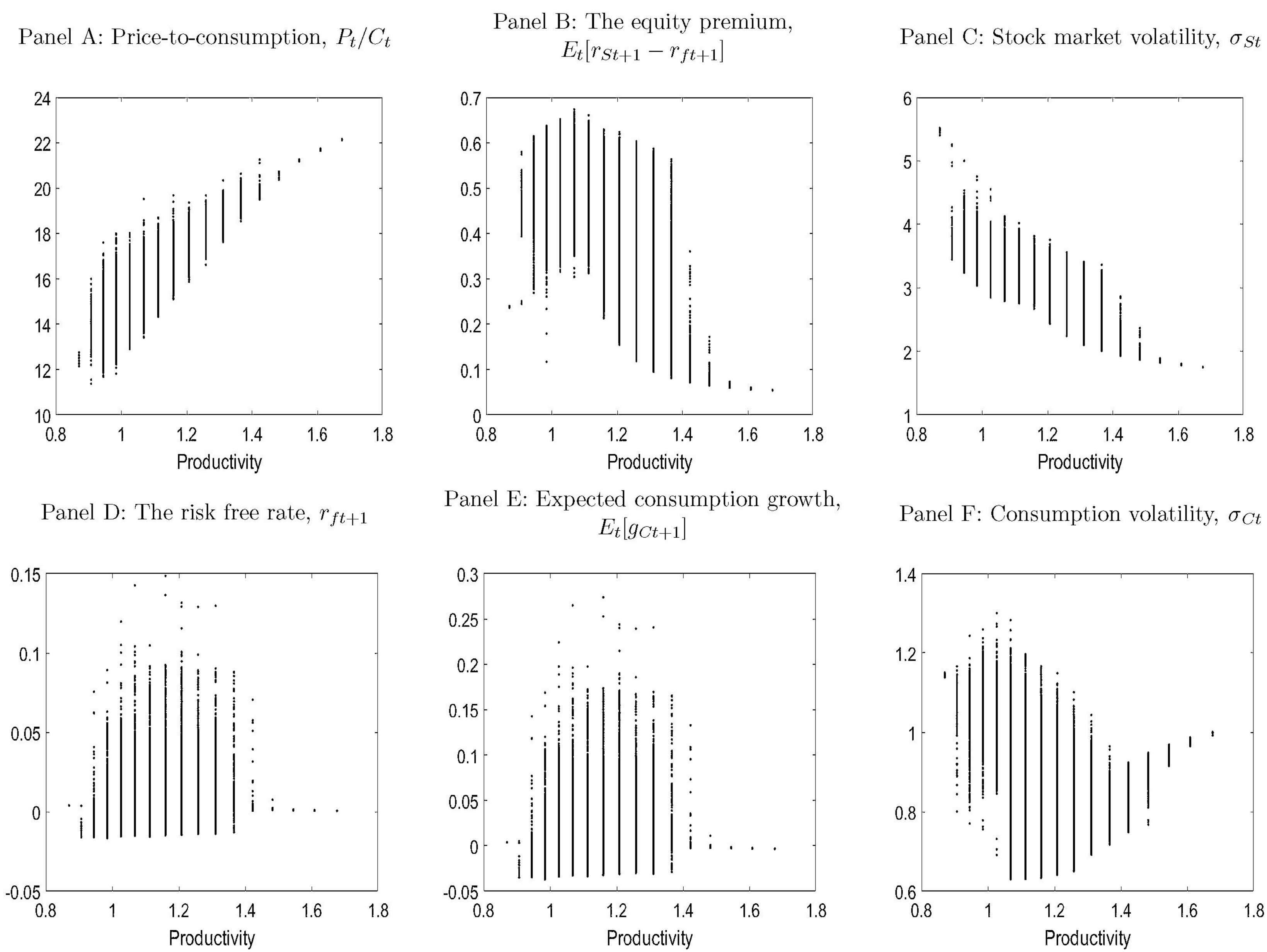


Figure 2 : The Term Structure of the Equity Premium

From the model's stationary distribution with the benchmark calibration (after a burn-in period of 1,200 monthly periods), we simulate a long sample path with one million months. Risk premiums and volatilities are in annualized percentage. Maturity is in years.

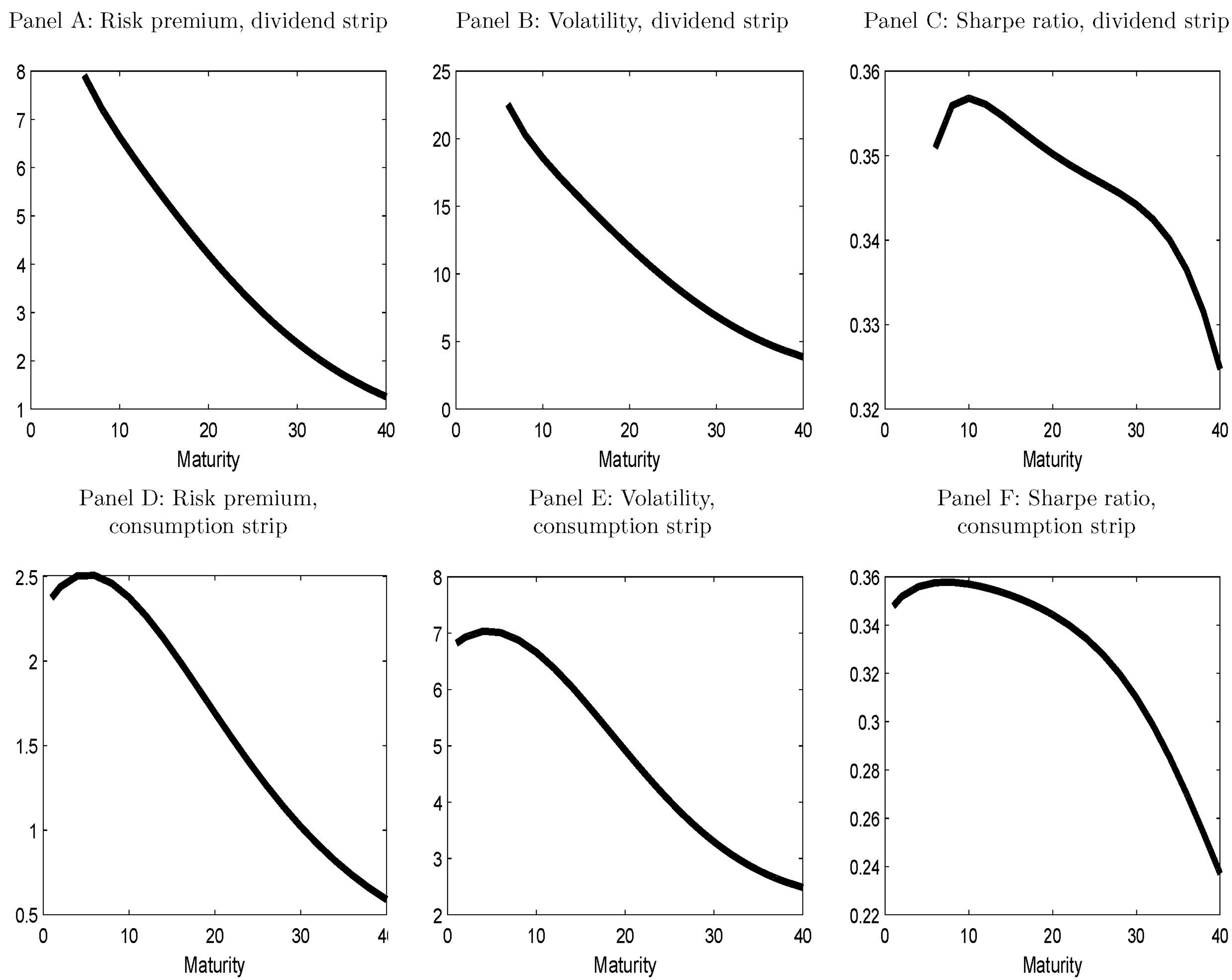
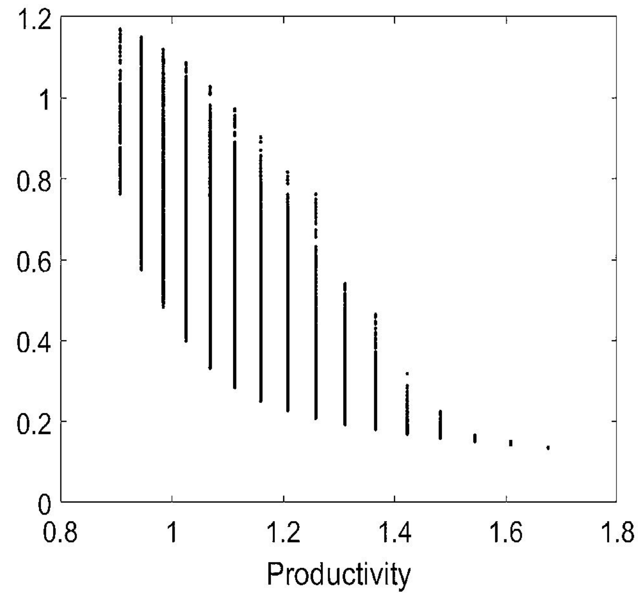


Figure 3 : Scatterplot of the Welfare Cost Against Productivity

From the model's stationary distribution with the benchmark calibration (after a burn-in period of 1,200 monthly periods), we simulate a long sample path with one million months. The vertical axis is the welfare cost, χ_t, and the horizonal axis is the productivity, $\exp(x_t)$.



Internet Appendix (for Online Publication Only): "Searching for the Equity Premium"

A Data

For each country, we construct its dividend index based on three series in the Jordà-Schularick-Taylor macrohistory database, including capital gain (P_t/P_{t-1}), in which P_t is the nominal price level of a stock market index; dividend-to-price (D_t/P_t), in which D_t is nominal dividends delivered by the index; and consumer price index. We first back out the P_t series by cumulating the capital gain series and then construct the D_t series by multiplying P_t with the dividend-to-price series. We scale nominal dividends by consumer price index to yield real dividends. The total number of nonmissing dividends between 1870 and 2015 in the Jordà-Schularick-Taylor dataset is 2,034. Three countries have in total seven dividend observations that equal zero, Germany, Portugal, and Spain. For Switzerland, the capital gain series runs from 1900 to 2015, with 1926–1959 missing. As such, its constructed dividends series starts in 1960. For Netherlands, both its capital gain and dividend-to-price series are missing from 1918 to 1949. As such, its dividends series starts in 1950.

In predicting market excess returns, consumption growth, and their volatilities, we drop Canada from Jordà-Schularick-Taylor macrohistory database. The reason is that its capital gain series (required to construct the price-to-consumption ratio) is incompatible with its total return series from the Dimson-Marsh-Staunton (2002) database. The implied dividend series are frequently negative, unlike the other countries, all of which have nonnegative dividends.

B Derivations

B.1 The Stock Return

Equation (4) implies that the marginal products of capital and labor are given by, respectively:

$$\frac{\partial Y_t}{\partial K_t} = \frac{Y_t}{K_t}\frac{\alpha\,(K_t/K_0)^{\omega}}{\alpha\,(K_t/K_0)^{\omega}+(1-\alpha)N_t^{\omega}}, \quad \text{(S1)}$$

$$\frac{\partial Y_t}{\partial N_t} = \frac{Y_t}{N_t}\frac{(1-\alpha)N_t^{\omega}}{\alpha\,(K_t/K_0)^{\omega}+(1-\alpha)N_t^{\omega}}, \quad \text{(S2)}$$

As such, Y_t is of constant returns to scale, i.e., $K_t\partial Y_t/\partial K_t + N_t\partial Y_t/\partial N_t = Y_t$. From equation (9):

$$\frac{\partial \Phi_t}{\partial I_t} = a_2\left(\frac{I_t}{K_t}\right)^{-\frac{1}{\nu}} \quad \text{(S3)}$$

$$\frac{\partial \Phi_t}{\partial K_t} = a_1 + \frac{a_2}{\nu-1}\left(\frac{I_t}{K_t}\right)^{1-\frac{1}{\nu}} \quad \text{(S4)}$$

It follows that $\Phi(I_t, K_t)$ is of constant returns to scale, i.e., $I_t\partial\Phi_t/\partial I_t + K_t\partial\Phi_t/\partial K_t = \Phi_t$.

The Lagrangian for the firm's problem is:

$$\begin{aligned}\mathcal{L} &= \cdots + Y_t - W_tN_t - \kappa V_t - I_t - \mu_{Nt}[N_{t+1} - (1-s)N_t - q(\theta_t)V_t] - \mu_{Kt}[K_{t+1} - (1-s)K_t - \Phi(I_t, K_t)] \\ &+\lambda_t q(\theta_t)V_t + E_t\left[M_{t+1}\left(Y_{t+1} - W_{t+1}N_{t+1} - \kappa V_{t+1} - I_{t+1} - \mu_{Nt+1}[N_{t+2} - (1-s)N_{t+1} - q(\theta_{t+1})V_{t+1}]\right.\right. \\ &\left.\left. -\mu_{Kt+1}[K_{t+2} - (1-s)K_{t+1} - \Phi(I_{t+1}, K_{t+1})] + \lambda_{t+1}q(\theta_{t+1})V_{t+1} + \cdots\right)\right] \qquad \text{(S5)}\end{aligned}$$

The first-order conditions with respect to V_t and N_{t+1} are given by, respectively,

$$\mu_{Nt} = \frac{\kappa}{q(\theta_t)} - \lambda_t \qquad \text{(S6)}$$

$$\mu_{Nt} = E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial N_{t+1}} - W_{t+1} + (1-s)\mu_{Nt+1}\right]\right] \qquad \text{(S7)}$$

Combining the two equations yields the intertemporal job creation condition in equation (14). The first-order conditions with respect to I_t and K_{t+1} are given by, respectively,

$$\mu_{Kt} = \frac{1}{\partial\Phi_t/\partial I_t} \qquad \text{(S8)}$$

$$\mu_{Kt} = E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + \left(1 - \delta + \frac{\partial\Phi_{t+1}}{\partial K_{t+1}}\right)\frac{1}{\partial\Phi_{t+1}/\partial I_{t+1}}\right]\right] \qquad \text{(S9)}$$

Combining equations (S3)–(S9) yields equation (12).

We first show $P_t = \mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}$, in which $P_t = S_t - D_t$ is ex-dividend equity value, with a guess-and-verify approach (Goncalves, Xue, and Zhang 2020). We first assume it holds for $t+1$: $P_{t+1} = \mu_{Kt+1}K_{t+2} + \mu_{Nt+1}N_{t+2}$. We then show it also holds for t. It then follows that the equation must hold for all periods. We start with recursively formulating equation (11): $P_t = E_t[M_{t+1}(P_{t+1} + D_{t+1})]$. Using $P_{t+1} = \mu_{Kt+1}K_{t+2} + \mu_{Nt+1}N_{t+2}$ to rewrite the right-hand side yields:

$$\begin{aligned}P_t &= E_t\left[M_{t+1}\left[\mu_{Kt+1}K_{t+2} + \mu_{Nt+1}N_{t+2} + D_{t+1}\right]\right] \\ &= E_t\left[M_{t+1}\left[\mu_{Kt+1}[(1-\delta)K_{t+1} + \Phi_{t+1}] + \mu_{Nt+1}[(1-s)N_{t+1} + q(\theta_{t+1})V_{t+1}]\right.\right. \\ &\quad\left.\left. + Y_{t+1} - W_{t+1}N_{t+1} - \kappa V_{t+1} - I_{t+1}\right]\right] \\ &= E_t\left[M_{t+1}\left[\mu_{Kt+1}\left[(1-\delta)K_{t+1} + \frac{\partial\Phi_{t+1}}{\partial I_{t+1}}I_{t+1} + \frac{\partial\Phi_{t+1}}{\partial K_{t+1}}K_{t+1}\right] + \mu_{Nt+1}[(1-s)N_{t+1} + q(\theta_{t+1})V_{t+1}]\right.\right. \\ &\quad\left.\left. + \frac{\partial Y_{t+1}}{\partial K_{t+1}}K_{t+1} + \frac{\partial Y_{t+1}}{\partial N_{t+1}}N_{t+1} - W_{t+1}N_{t+1} - \kappa V_{t+1} - I_{t+1}\right]\right] \\ &= K_{t+1}E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + \left(1 - \delta + \frac{\partial\Phi_{t+1}}{\partial K_{t+1}}\right)\mu_{Kt+1}\right]\right] + \mu_{Kt+1}\frac{\partial\Phi_{t+1}}{\partial I_{t+1}}I_{t+1} \\ &\quad + N_{t+1}E_t\left[M_{t+1}\left[\frac{\partial Y_{t+1}}{\partial N_{t+1}} - W_{t+1} + (1-s)\mu_{Nt+1}\right]\right] + \mu_{Nt+1}q(\theta_{t+1})V_{t+1} - \kappa V_{t+1} - I_{t+1} \\ &= \mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}, \qquad \text{(S10)}\end{aligned}$$

in which the third equality follows from constant returns to scale for Y_{t+1} and Φ_{t+1}, and the last equality follows from equations (S6), (S7), (S8), (S9), and the Kuhn-Tucker condition (16).

To prove equation (17),

$$
\begin{aligned}
r_{St+1} &= \frac{P_{t+1} + D_{t+1}}{P_t} = \frac{\mu_{Kt+1}K_{t+2} + \mu_{Nt+1}N_{t+2} + D_{t+1}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}} \\
&= \frac{\begin{array}{c}\mu_{Kt+1}\left[(1-\delta)K_{t+1} + \Phi_{t+1}\right] + \mu_{Nt+1}[(1-s)N_{t+1} + q(\theta_{t+1})V_{t+1}] \\ +Y_{t+1} - W_{t+1}N_{t+1} - \kappa V_{t+1} - I_{t+1}\end{array}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}} \\
&= \frac{\begin{array}{c}\mu_{Kt+1}\left[(1-\delta)K_{t+1} + \frac{\partial \Phi_{t+1}}{\partial I_{t+1}}I_{t+1} + \frac{\partial \Phi_{t+1}}{\partial K_{t+1}}K_{t+1}\right] + \mu_{Nt+1}[(1-s)N_{t+1} + q(\theta_{t+1})V_{t+1}] \\ +\frac{\partial Y_{t+1}}{\partial K_{t+1}}K_{t+1} + \frac{\partial Y_{t+1}}{\partial N_{t+1}}N_{t+1} - W_{t+1}N_{t+1} - \kappa V_{t+1} - I_{t+1}\end{array}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}} \\
&= \frac{\left[\frac{\partial Y_{t+1}}{\partial K_{t+1}} + \left(1-\delta+\frac{\partial \Phi_{t+1}}{\partial K_{t+1}}\right)\mu_{Kt+1}\right]K_{t+1}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}} + \frac{\left[\frac{\partial Y_{t+1}}{\partial N_{t+1}} - W_{t+1} + (1-s)\mu_{Nt+1}\right]N_{t+1}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}} \\
&= \frac{\mu_{Kt}K_{t+1}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}}r_{Kt+1} + \frac{\mu_{Nt}N_{t+1}}{\mu_{Kt}K_{t+1} + \mu_{Nt}N_{t+1}}r_{Nt+1}. \qquad \text{(S11)}
\end{aligned}
$$

B.2 Wages

We extend the derivation in Petrosky-Nadeau, Zhang, and Kuehn (2018) to our setting with capital accumulation. Let $\partial J_t/\partial N_t$ be the marginal value of an employed worker to the representative household, $\partial J_t/\partial U_t$ the marginal value of an unemployed worker to the household, ϕ_t the marginal utility of the household, $\partial S_t/\partial N_t$ the marginal value of an employed worker to the representative firm, and $\partial S_t/\partial V_t$ the marginal value of an unfilled vacancy to the firm. A worker-firm match turns an unemployed worker into an employed worker for the household as well as an unfilled vacancy into an employed worker for the firm. As such, the total surplus from the Nash bargain is:

$$H_t \equiv \left(\frac{\partial J_t}{\partial N_t} - \frac{\partial J_t}{\partial U_t}\right)/\phi_t + \frac{\partial S_t}{\partial N_t} - \frac{\partial S_t}{\partial V_t}. \qquad \text{(S12)}$$

The equilibrium wage arises from the Nash worker-firm bargain as follows:

$$\max_{\{W_t\}} \quad \left[\left(\frac{\partial J_t}{\partial N_t} - \frac{\partial J_t}{\partial U_t}\right)/\phi_t\right]^{\eta}\left(\frac{\partial S_t}{\partial N_t} - \frac{\partial S_t}{\partial V_t}\right)^{1-\eta}, \qquad \text{(S13)}$$

in which $0 < \eta < 1$ is the worker's bargaining power. The outcome is the surplus-sharing rule:

$$\left(\frac{\partial J_t}{\partial N_t} - \frac{\partial J_t}{\partial U_t}\right)/\phi_t = \eta H_t = \eta\left[\left(\frac{\partial J_t}{\partial N_t} - \frac{\partial J_t}{\partial U_t}\right)/\phi_t + \frac{\partial S_t}{\partial N_t} - \frac{\partial S_t}{\partial V_t}\right]. \qquad \text{(S14)}$$

As such, the worker receives a fraction of η of the total surplus from the wage bargain.

B.2.1 Workers

Tradeable assets consist of risky shares and a riskfree asset. Let r_{ft+1} denote the risk-free interest rate, ξ_t the household's financial wealth, χ_t the fraction of the household's wealth invested in the risky shares, $r_{\xi t+1} \equiv \chi_t r_{St+1} + (1-\chi_t) r_{ft+1}$ the return on wealth, and T_t the taxes raised by the government. The household's budget constraint is given by:

$$\frac{\xi_{t+1}}{r_{\xi t+1}} = \xi_t - C_t + W_t N_t + U_t b - T_t. \tag{S15}$$

The household's dividends income, D_t, is included in the current financial wealth, ξ_t.

Let ϕ_t denote the Lagrange multiplier for the household's budget constraint (S15). The household's maximization problem is given by:

$$J_t = \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} - \phi_t\left(\frac{\xi_{t+1}}{r_{\xi t+1}} - \xi_t + C_t - W_t N_t - U_t b + T_t\right), \tag{S16}$$

The first-order condition for consumption yields:

$$\phi_t = (1-\beta)C_t^{-\frac{1}{\psi}}\left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1}, \tag{S17}$$

which gives the marginal utility of consumption. Using $N_{t+1} = (1-s)N_t + f(\theta_t)U_t$ and $U_{t+1} = sN_t + (1-f(\theta_t))U_t$, we differentiate J_t in equation (S16) with respect to N_t:

$$\begin{aligned}\frac{\partial J_t}{\partial N_t} \;=\; & \phi_t W_t + \frac{1}{1-\frac{1}{\psi}}\left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1} \\ & \times\frac{1-\frac{1}{\psi}}{1-\gamma}\beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}-1} E_t\left[(1-\gamma)J_{t+1}^{-\gamma}\left[(1-s)\frac{\partial J_{t+1}}{\partial N_{t+1}} + s\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]\right].\end{aligned} \tag{S18}$$

Dividing both sides by ϕ_t:

$$\frac{\partial J_t}{\partial N_t}/\phi_t = W_t + \frac{\beta}{(1-\beta)C_t^{-\frac{1}{\psi}}}\left[\frac{1}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma} E_t\left[J_{t+1}^{-\gamma}\left[(1-s)\frac{\partial J_{t+1}}{\partial N_{t+1}} + s\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]\right]. \tag{S19}$$

Dividing and multiplying by ϕ_{t+1}:

$$\begin{aligned}\frac{\partial J_t}{\partial N_t}/\phi_t &= W_t + E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left[\frac{J_{t+1}}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}\left[(1-s)\frac{\partial J_{t+1}}{\partial N_{t+1}} + s\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]/\phi_{t+1}\right] \\ &= W_t + E_t\left[M_{t+1}\left[(1-s)\frac{\partial J_{t+1}}{\partial N_{t+1}} + s\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]/\phi_{t+1}\right]. \qquad \text{(S20)}\end{aligned}$$

Similarly, differentiating J_t in equation (S16) with respect to U_t yields:

$$\begin{aligned}\frac{\partial J_t}{\partial U_t} &= \phi_t b + \frac{1}{1-\frac{1}{\psi}}\left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}-1} \\ &\times\frac{1-\frac{1}{\psi}}{1-\gamma}\beta\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1-1/\psi}{1-\gamma}-1} E_t\left[(1-\gamma)J_{t+1}^{-\gamma}\left[f(\theta_t)\frac{\partial J_{t+1}}{\partial N_{t+1}} + (1-f(\theta_t))\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]\right]. \qquad \text{(S21)}\end{aligned}$$

Dividing both sides by ϕ_t:

$$\frac{\partial J_t}{\partial U_t}/\phi_t = b + \frac{\beta}{(1-\beta)C_t^{-\frac{1}{\psi}}}\left[\frac{1}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma} E_t\left[J_{t+1}^{-\gamma}\left[f(\theta_t)\frac{\partial J_{t+1}}{\partial N_{t+1}} + (1-f(\theta_t))\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]\right]. \qquad \text{(S22)}$$

Dividing and multiplying by ϕ_{t+1}:

$$\begin{aligned}\frac{\partial J_t}{\partial U_t}/\phi_t &= b + E_t\left[\beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left[\frac{J_{t+1}}{\left[E_t\left(J_{t+1}^{1-\gamma}\right)\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}\left[f(\theta_t)\frac{\partial J_{t+1}}{\partial N_{t+1}} + (1-f(\theta_t))\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]/\phi_{t+1}\right] \\ &= b + E_t\left[M_{t+1}\left[f(\theta_t)\frac{\partial J_{t+1}}{\partial N_{t+1}} + (1-f(\theta_t))\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]/\phi_{t+1}\right]. \qquad \text{(S23)}\end{aligned}$$

B.2.2 The Representative Firm

We start by reformulating the firm's problem recursively as:

$$S_t = Y_t - W_t N_t - \kappa V_t - I_t + \lambda_t q(\theta_t) V_t + E_t[M_{t+1} S_{t+1}], \qquad \text{(S24)}$$

subject to $N_{t+1} = (1-s)N_t + q(\theta_t)V_t$ and $K_{t+1} = (1-\delta)K_t + \Phi(I_t, K_t)$.

The first-order condition with respect to V_t says:

$$\frac{\partial S_t}{\partial V_t} = -\kappa + \lambda_t q(\theta_t) + E_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial N_{t+1}} q(\theta_t)\right] = 0. \qquad \text{(S25)}$$

Equivalently,

$$\frac{\kappa}{q(\theta_t)} - \lambda_t = E_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial N_{t+1}}\right]. \tag{S26}$$

In addition, differentiating S_t with respect to N_t yields:

$$\frac{\partial S_t}{\partial N_t} = \frac{\partial Y_t}{\partial N_t} - W_t + (1-s)E_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial N_{t+1}}\right]. \tag{S27}$$

Combining the last two equations yields the job creation condition.

B.2.3 The Wage Rate

From equations (S20), (S23), and (S27), the total surplus of the worker-firm relationship is:

$$\begin{aligned}
H_t &= W_t + E_t\left[M_{t+1}\left[(1-s)\frac{\partial J_{t+1}}{\partial N_{t+1}} + s\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]/\phi_{t+1}\right] - b \\
&\quad -E_t\left[M_{t+1}\left[f(\theta_t)\frac{\partial J_{t+1}}{\partial N_{t+1}} + (1-f(\theta_t))\frac{\partial J_{t+1}}{\partial U_{t+1}}\right]/\phi_{t+1}\right] + \frac{\partial Y_t}{\partial N_t} - W_t + (1-s)E_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial N_{t+1}}\right] \\
&= \frac{\partial Y_t}{\partial N_t} - b + (1-s)E_t\left[M_{t+1}\left[\left(\frac{\partial J_{t+1}}{\partial N_{t+1}} - \frac{\partial J_{t+1}}{\partial U_{t+1}}\right)/\phi_{t+1} + \frac{\partial S_{t+1}}{\partial N_{t+1}}\right]\right] \\
&\quad -f(\theta_t)E_t\left[M_{t+1}\left(\frac{\partial J_{t+1}}{\partial N_{t+1}} - \frac{\partial J_{t+1}}{\partial U_{t+1}}\right)/\phi_{t+1}\right] \\
&= \frac{\partial Y_t}{\partial N_t} - b + (1-s-\eta f(\theta_t))E_t\left[M_{t+1}H_{t+1}\right].
\end{aligned} \tag{S28}$$

The sharing rule implies $\partial S_t/\partial N_t = (1-\eta)H_t$, which, combined with equation (S27), yields:

$$(1-\eta)H_t = \frac{\partial Y_t}{\partial N_t} - W_t + (1-\eta)(1-s)E_t\left[M_{t+1}H_{t+1}\right]. \tag{S29}$$

Combining equations (S28) and (S29) yields:

$$\begin{aligned}
\frac{\partial Y_t}{\partial N_t} - W_t + (1-\eta)(1-s)E_t\left[M_{t+1}H_{t+1}\right] &= (1-\eta)\left(\frac{\partial Y_t}{\partial N_t} - b\right) + (1-\eta)(1-s)E_t\left[M_{t+1}H_{t+1}\right] \\
&\quad -(1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}H_{t+1}\right] \\
W_t &= \eta\frac{\partial Y_t}{\partial N_t} + (1-\eta)b + (1-\eta)\eta f(\theta_t)E_t\left[M_{t+1}H_{t+1}\right].
\end{aligned}$$

Using equations (S14) and (S26) to simplify further:

$$W_t = \eta\frac{\partial Y_t}{\partial N_t} + (1-\eta)b + \eta f(\theta_t)E_t\left[M_{t+1}\frac{\partial S_{t+1}}{\partial N_{t+1}}\right] \tag{S30}$$

$$W_t = \eta\frac{\partial Y_t}{\partial N_t} + (1-\eta)b + \eta f(\theta_t)\left[\frac{\kappa}{q(\theta_t)} - \lambda_t\right]. \tag{S31}$$

If $V_t > 0$, then $\lambda_t = 0$, and equation (S31) reduces to equation (18) because $f(\theta_t) = \theta_t q(\theta_t)$. If $V_t \geq 0$ is binding, $\lambda_t > 0$, but $V_t = 0$ means $\theta_t = 0$ and $f(\theta_t) = 0$. Equation (S31) reduces to $W_t = \eta \partial Y_t / \partial N_t + (1 - \eta) b$. Because $\theta_t = 0$, equation (18) continues to hold.

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Table S1 : Basic Properties of Asset Prices in the Historical Sample, with the Longest Possible Sample for Each Moment

The historical cross-country panel is from the Jordà-Schularick-Taylor macrohistory database, except for Canada's asset prices, which we obtain from the Dimson-Marsh-Staunton (2002) database purchased from Morningstar. All (annual) series end in 2015. $E[\widetilde{r}_S]$, $\widetilde{\sigma}_S$, and $E[\widetilde{r}_S - r_f]$ are the average stock market return, stock market volatility, and the equity premium, respectively, without adjusting for financial leverage. $E[r_S - r_f]$ and σ_S are the equity premium and stock market volatility, respectively, after adjusting for financial leverage. $E[r_f]$ is the mean real interest rate, and σ_f the interest rate volatility. All asset pricing moments are in annual percent. We use the longest possible samples of stocks, bills, and bonds described in the second, third, and fourth column, respectively, to calculate each moment. For example, in Australia, the sample for stock market returns starts in 1871, the sample for real interest rates start in 1871, with missing observations from 1945 to 1947, and the sample for long-term government bonds starts in 1900. Other than Italy, which has missing asset prices from 1872 to 1884, all other missing years are in the 20th century.

	Sample, $\widetilde{r}_S$	Sample, r_f	Sample, r_B	$E[\widetilde{r}_S]$	$\widetilde{\sigma}_S$	$E[r_f]$	σ_f	$E[\widetilde{r}_S - r_f]$	$E[r_S - r_f]$	σ_S
Australia	1871	1871 (45–47)	1900	8.39	15.77	2.02	4.44	6.33	4.49	11.76
Belgium	1871	1871 (15–18)	1871 (14–19)	5.89	21.97	1.68	9.94	5.25	3.73	16.22
Canada	1900	1900	1900	7.01	17.00	1.60	4.79	5.41	3.84	12.26
Denmark	1873	1875	1871 (15)	7.54	16.36	2.98	5.77	4.59	3.26	11.88
Finland	1896	1871	1871	8.83	30.57	0.15	10.50	9.57	6.80	22.98
France	1871	1871 (15–21)	1871	3.21	22.14	−0.47	7.78	4.45	3.16	16.75
Germany	1871	1871 (23, 45–49)	1871 (44–48)	9.44	32.04	−0.23	13.17	9.00	6.39	20.15
Italy	1871	1871 (1872–1884, 15–21)	1871	5.75	26.18	0.58	10.50	6.05	4.29	20.41
Japan	1886 (46–47)	1876	1881	8.86	27.69	−0.41	12.90	8.87	6.29	21.10
Netherlands	1900	1871	1871	6.96	21.44	1.37	5.04	6.19	4.39	15.32
Norway	1881	1871	1871	5.67	19.82	1.10	5.96	4.77	3.39	14.53
Portugal	1871	1880	1871	4.05	25.20	−0.01	9.43	3.82	2.71	19.29
Spain	1900	1871 (36–38)	1900 (37–40)	5.77	21.07	0.70	6.83	6.28	4.46	15.88
Sweden	1871	1871	1871	8.00	19.54	1.77	5.60	6.23	4.42	14.26
Switzerland	1900	1871	1900 (15)	6.50	19.09	1.64	5.88	5.70	4.05	14.04
UK	1871	1871	1871	6.86	17.77	1.16	4.82	5.70	4.05	12.96
USA	1872	1871	1871	8.40	18.68	2.23	4.71	6.23	4.43	13.66
Mean				6.89	21.90	1.05	7.53	6.14	4.36	16.08
Median				6.96	21.07	1.16	5.96	6.05	4.29	15.32

Table S2 : Basic Properties of the Real Consumption, Output, and Investment Growth and Asset Prices, 1950–2015

The historical cross-country panel is from the Jordà-Schularick-Taylor macrohistory database. The only exception is asset prices data for Canada, which we obtain from the Dimson-Marsh-Staunton (2002) database purchased from Morningstar. All (annual) series end in 2015. In Panel A, $\overline{g}_C$, σ_C, S_C, K_C, and $\rho_C^{(i)}$ denote the mean (in percent), volatility (in percent), skewness, kurtosis, and ith-order autocorrelation, for $i = 1, 2, \ldots, 5$, of real per capita consumption growth. In Panel B, $\overline{g}_Y$, σ_Y, S_Y, K_Y, and $\rho_Y^{(i)}$ denote the mean (in percent), volatility (in percent), skewness, kurtosis, and ith-order autocorrelation for real per capita output growth. In Panel C, $\overline{g}_I$, σ_I, S_I, K_I, and $\rho_I^{(i)}$ denote the mean (in percent), volatility (in percent), skewness, kurtosis, and ith-order autocorrelation for real per capita investment growth. Finally, in Panel D, $E[\widetilde{r}_S]$, $\widetilde{\sigma_S}$, and $E[\widetilde{r}_S - r_f]$ are the average stock market return, stock market volatility, and the equity premium, respectively, without adjusting for financial leverage. $E[r_S - r_f]$ and σ_S are the equity premium and stock market volatility, respectively, after adjusting for financial leverage. $E[r^f]$ is the mean real interest rate, and σ_f the interest rate volatility. All asset pricing moments are in annual percent.

	Panel A: Real consumption growth									Panel B: Real output growth								
	$\overline{g}_C$	σ_C	S_C	K_C	$\rho_C^{(1)}$	$\rho_C^{(2)}$	$\rho_C^{(3)}$	$\rho_C^{(4)}$	$\rho_C^{(5)}$	$\overline{g}_Y$	σ_Y	S_Y	K_Y	$\rho_Y^{(1)}$	$\rho_Y^{(2)}$	$\rho_Y^{(3)}$	$\rho_Y^{(4)}$	$\rho_Y^{(5)}$
Australia	1.78	2.02	−0.14	3.55	0.17	−0.24	−0.11	0.19	0.30	1.95	1.86	−0.56	4.19	0.19	−0.03	−0.07	0.09	0.24
Belgium	1.89	1.92	0.20	3.42	0.34	0.21	0.41	0.18	0.21	2.22	2.01	−0.28	2.95	0.28	0.27	0.20	0.23	0.05
Canada	2.01	1.81	−0.61	4.00	0.31	0.07	0.17	−0.07	−0.26	1.94	2.25	−0.73	3.73	0.25	−0.04	0.05	0.05	−0.01
Denmark	1.24	2.43	−0.03	2.95	0.22	0.01	0.03	−0.17	−0.30	1.85	2.33	−0.05	3.88	0.26	0.09	0.16	0.16	0.09
Finland	2.62	3.17	−0.40	3.04	0.40	−0.08	−0.05	−0.05	−0.03	2.54	3.24	−0.93	5.23	0.42	0.01	0.08	0.02	0.03
France	2.34	1.79	0.19	2.18	0.65	0.48	0.40	0.42	0.41	2.37	1.87	−0.27	3.35	0.56	0.41	0.47	0.44	0.40
Germany	2.81	2.46	0.71	2.98	0.73	0.53	0.50	0.51	0.49	2.80	2.69	0.18	3.93	0.48	0.17	0.31	0.48	0.36
Italy	2.51	2.72	−0.30	2.97	0.67	0.46	0.52	0.48	0.41	2.69	2.71	−0.79	3.56	0.51	0.35	0.42	0.39	0.39
Japan	3.90	3.53	0.72	3.00	0.74	0.62	0.69	0.66	0.61	3.80	3.69	0.26	2.72	0.69	0.59	0.61	0.53	0.48
Netherlands	1.92	2.47	−0.16	2.45	0.67	0.32	0.15	0.08	0.13	2.22	2.20	−0.12	3.78	0.39	0.05	0.05	0.13	0.12
Norway	2.39	2.19	0.21	3.76	0.23	−0.02	−0.18	−0.14	−0.13	2.53	1.87	−0.52	2.73	0.51	0.28	0.16	0.20	0.19
Portugal	3.05	3.56	−0.58	4.03	0.36	0.16	0.08	−0.14	−0.18	2.93	3.48	−0.34	3.87	0.51	0.23	0.26	−0.01	0.02
Spain	2.79	3.54	0.08	3.20	0.51	0.25	0.20	0.23	0.23	3.15	3.21	0.07	2.62	0.50	0.32	0.23	0.22	0.14
Sweden	1.55	1.92	−0.59	3.12	0.38	0.18	0.08	−0.09	−0.16	2.09	2.14	−1.13	5.31	0.32	−0.02	0.03	0.12	0.15
Switzerland	1.44	1.42	0.11	2.59	0.61	0.24	0.14	0.10	0.11	1.62	2.29	−0.65	4.06	0.30	−0.04	−0.03	0.08	0.02
UK	1.97	2.09	−0.13	3.11	0.45	0.05	−0.11	−0.11	0.00	1.88	1.90	−0.85	4.89	0.33	−0.13	−0.12	−0.01	0.02
USA	2.08	1.73	−0.21	2.49	0.32	0.03	−0.06	0.02	−0.04	1.91	2.21	−0.43	2.88	0.12	−0.01	−0.15	0.06	−0.07
Mean	2.25	2.40	−0.05	3.11	0.46	0.19	0.17	0.12	0.11	2.38	2.47	−0.42	3.75	0.39	0.15	0.16	0.19	0.15
Median	2.08	2.19	−0.13	3.04	0.40	0.18	0.14	0.08	0.11	2.22	2.25	−0.43	3.78	0.39	0.09	0.16	0.13	0.12

	Panel C: Real investment growth									Panel D: Asset prices						
	$\overline{g}_I$	σ_I	S_I	K_I	$\rho_I^{(1)}$	$\rho_I^{(2)}$	$\rho_I^{(3)}$	$\rho_I^{(4)}$	$\rho_I^{(5)}$	$E[\widetilde{r}_S]$	$\widetilde{\sigma}_S$	$E[r_f]$	σ_f	$E[\widetilde{r}_S - r_f]$	$E[r_S - r_f]$	σ_S
Australia	2.33	5.70	−0.45	2.82	0.09	−0.29	−0.08	0.16	0.07	7.33	20.46	1.44	3.97	5.89	4.18	14.90
Belgium	2.63	7.08	−0.73	3.95	0.05	−0.13	−0.09	−0.01	−0.16	8.07	21.05	1.60	2.91	6.47	4.59	15.02
Canada	2.10	5.65	−0.46	3.15	0.23	0.02	−0.21	−0.20	−0.28	7.47	16.33	1.80	3.12	5.66	4.02	11.51
Denmark	1.32	9.06	−1.34	6.86	0.24	0.07	0.06	−0.13	−0.19	9.60	21.37	2.24	2.85	7.36	5.22	15.19
Finland	2.41	9.01	−0.66	4.25	0.49	0.04	−0.19	−0.20	−0.09	12.17	33.86	0.76	4.50	11.41	8.10	24.47
France	1.86	6.18	−2.56	14.73	0.14	−0.03	−0.18	−0.13	−0.19	6.45	26.13	1.08	3.29	5.38	3.82	18.71
Germany	2.60	6.41	0.28	3.78	0.39	−0.06	−0.08	−0.02	0.03	12.09	27.71	1.72	1.78	10.37	7.36	19.62
Italy	2.37	5.53	−0.63	3.12	0.41	0.11	0.18	0.22	0.10	6.02	25.99	1.23	3.09	4.79	3.40	18.69
Japan	4.11	7.86	0.56	2.84	0.52	0.19	0.30	0.30	0.28	9.58	22.37	1.21	3.40	8.37	5.94	16.04
Netherlands	2.21	6.11	0.10	3.42	0.24	0.00	−0.07	−0.11	−0.27	9.43	21.81	1.15	2.83	8.28	5.88	15.58
Norway	2.18	8.58	0.29	4.50	0.13	−0.14	−0.03	−0.10	−0.22	7.25	25.99	−0.21	3.26	7.46	5.30	18.69
Portugal	2.64	9.58	−0.22	3.08	0.22	0.21	0.06	−0.13	0.08	4.86	33.53	−0.73	4.85	5.59	3.97	24.38
Spain	3.60	9.32	−0.20	3.40	0.45	0.30	−0.07	−0.12	−0.27	7.93	24.53	−0.22	4.43	8.15	5.79	17.91
Sweden	2.49	5.32	−1.41	5.32	0.28	−0.09	−0.13	−0.08	0.03	11.14	24.01	0.82	2.58	10.32	7.33	17.23
Switzerland	2.25	7.93	0.46	5.96	0.35	0.03	−0.04	−0.21	−0.24	8.33	21.41	0.06	2.13	8.27	5.87	15.33
UK	2.61	5.75	−0.76	4.16	0.38	0.02	−0.03	0.02	0.05	9.13	22.94	1.21	3.63	7.92	5.62	16.27
USA	1.91	4.98	−0.89	4.45	0.27	−0.12	−0.27	−0.21	−0.08	8.56	16.83	1.41	2.25	7.15	5.08	12.03
Mean	2.45	7.06	−0.51	4.69	0.29	0.01	−0.05	−0.06	−0.08	8.55	23.90	0.97	3.23	7.58	5.38	17.15
Median	2.37	6.41	−0.46	3.95	0.27	0.02	−0.07	−0.11	−0.09	8.33	22.94	1.21	3.12	7.46	5.30	16.27

Table S3 : Gollin's (2002) Labor Share Calculations

For the 12 countries that are in both Gollin (2002) and Jordà-Schularick-Taylor macrohistory database, this table reports the labor shares reported in Gollin's Table 2. The three columns correspond to the last three columns labeled "Adjustment 1," "Adjustment 2," and "Adjustment 3," respectively, in Gollin's table.

	Method 1	Method 2	Method 3
Australia	0.719	0.669	0.676
Belgium	0.791	0.743	0.740
Finland	0.765	0.734	0.680
France	0.764	0.717	0.681
Italy	0.804	0.717	0.707
Japan	0.727	0.692	0.725
Netherlands	0.721	0.680	0.643
Norway	0.678	0.643	0.569
Portugal	0.825	0.748	0.602
Sweden	0.800	0.774	0.723
UK	0.815	0.782	0.719
US	0.773	0.743	0.664
Mean	0.765	0.720	0.677
Median	0.769	0.726	0.681

Table S4 : Dividend Dynamics in the Historical and Post-1950 Samples

Real output is from the Jordà-Schularick-Taylor macrohistory database. Appendix A describes our construction of dividends from their database. We use two detrending methods for real dividends and output. “Prop. dev.” means the HP-filtered proportional deviations from the mean, and “Log dev.” means log deviations from the HP-trend. ρ_{DY} is the correlation between the cyclical components of dividends and output, and σ_D/σ_Y the volatility of the cyclical component of dividends divided by that of output. We examine three frequencies, annual, 3-year, and 5-year. For the 3-year frequency, we sum up the three annual observations within a given 3-year interval. The 3-year intervals are nonoverlapping. The 5-year series are constructed analogously. The HP smoothing parameters for the 1-, 3-, and 5-year series are $1600/4^4 = 6.25$, $1600/12^4 = 0.077$, and $1600/20^4 = 0.01$, respectively, all of which correspond to 1,600 in the quarterly frequency. in Panel A, the column “Sample” indicates the starting year of a country. For Japan, the annual observations from 1946 and 1947 are missing. In Panel B, all countries start their samples in 1950, except for Switzerland, which starts in 1960. In calculating log deviations, the zero-dividend observations are removed.

Panel A: The historical sample

		1-year frequency				3-year frequency				5-year frequency			
		Prop. dev.		Log dev.		Prop. dev.		Log dev.		Prop. dev.		Log dev.	
	Sample	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y
Australia	1870	0.109	6.943	0.121	3.430	0.231	5.144	0.295	2.565	0.551	4.596	0.633	2.593
Belgium	1870	0.183	8.501	0.498	4.706	0.419	9.797	0.825	5.718	0.765	5.622	0.923	2.830
Denmark	1872	0.191	14.885	0.182	6.836	0.263	12.065	0.226	6.513	0.027	10.580	0.092	6.031
Finland	1912	0.083	10.143	0.308	6.883	0.670	8.464	0.504	5.754	0.815	4.225	0.437	5.477
France	1870	0.169	5.590	0.119	2.658	0.234	5.311	−0.029	3.564	0.479	4.443	−0.204	3.234
Germany	1870	0.018	6.052	0.211	20.321	0.114	3.821	0.552	2.924	0.241	1.952	0.894	3.652
Italy	1870	0.035	5.097	0.396	6.373	0.262	5.925	0.847	10.566	0.571	5.603	0.764	7.340
Japan	1886 (46–47)	0.027	10.673	0.612	8.949	0.058	6.347	0.806	10.513	0.110	7.107	0.882	7.470
Netherlands	1950	−0.001	16.807	0.203	14.904	0.548	13.768	0.390	13.042	0.369	18.404	0.258	13.533
Norway	1880	0.216	10.520	0.214	8.517	0.348	5.638	0.440	6.574	0.342	5.413	0.656	5.580
Portugal	1870	−0.021	3.062	0.007	7.762	0.043	1.572	0.597	12.343	0.108	1.570	0.454	14.630
Spain	1899	0.035	11.598	0.269	8.865	0.176	6.008	0.562	11.541	0.266	4.142	0.625	5.039
Sweden	1871	−0.019	9.309	0.152	5.664	0.091	5.756	0.440	5.112	0.565	4.616	0.701	5.631
Switzerland	1960	0.026	11.061	0.051	13.165	0.429	8.658	0.342	7.725	0.034	7.987	0.014	9.190
UK	1871	0.272	4.314	0.094	5.072	0.570	3.001	0.399	2.788	0.114	2.735	0.044	2.538
USA	1871	0.472	3.176	0.415	2.626	0.527	3.367	0.419	2.682	0.307	2.099	0.458	2.118
Mean		0.112	8.608	0.241	7.921	0.312	6.540	0.476	6.870	0.354	5.693	0.477	6.055
Median		0.059	8.905	0.207	6.860	0.262	5.840	0.440	6.133	0.325	4.606	0.542	5.529

Panel B: The post-1950 sample

	1-year frequency				3-year frequency				5-year frequency			
	Prop. dev.		Log dev.		Prop. dev.		Log dev.		Prop. dev.		Log dev.	
	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y	ρ_{DY}	σ_D/σ_Y
Australia	0.166	10.847	0.206	9.315	0.220	10.371	0.217	8.048	0.667	6.390	0.732	4.997
Belgium	−0.037	15.171	−0.020	11.548	−0.064	13.081	0.000	9.053	0.263	11.680	0.368	9.798
Denmark	0.200	37.696	0.127	15.578	0.336	29.277	0.351	14.641	0.369	14.957	0.406	8.189
Finland	0.054	12.063	0.182	11.121	0.573	8.375	0.874	9.303	0.795	5.395	0.712	7.865
France	−0.113	9.507	−0.060	10.510	0.082	9.709	0.105	9.099	0.096	5.515	0.277	5.893
Germany	−0.232	10.350	0.059	10.676	0.126	9.745	0.257	12.919	−0.326	12.597	0.338	11.169
Italy	0.019	8.131	−0.058	10.659	−0.009	11.848	0.157	11.149	0.268	10.223	0.362	14.628
Japan	0.287	4.826	0.393	5.584	0.198	5.750	0.192	4.645	0.489	4.724	0.556	4.506
Netherlands	−0.001	16.807	0.203	14.904	0.548	13.768	0.390	13.042	0.369	18.404	0.258	13.533
Norway	0.188	33.166	0.076	23.009	0.085	19.021	0.070	16.454	0.574	4.214	0.287	6.402
Portugal	−0.243	6.067	0.073	16.699	0.156	5.767	0.720	26.052	0.510	2.865	0.812	20.826
Spain	−0.052	16.567	0.027	12.367	0.044	6.939	0.055	5.929	0.196	5.250	0.109	4.727
Sweden	−0.033	11.772	0.182	8.797	0.615	9.721	0.816	9.321	0.436	3.884	0.513	5.503
Switzerland	0.026	11.061	0.051	13.165	0.429	8.658	0.342	7.725	0.034	7.987	0.014	9.190
UK	0.634	3.837	0.639	3.695	0.734	4.251	0.735	4.165	0.411	3.593	0.482	3.336
USA	0.645	3.801	0.497	2.962	0.710	4.377	0.646	3.508	0.371	3.302	0.506	2.671
Mean	0.094	13.229	0.161	11.287	0.299	10.666	0.370	10.316	0.345	7.561	0.421	8.327
Median	0.022	10.954	0.101	10.898	0.209	9.715	0.299	9.201	0.370	5.455	0.387	7.134

Table S5 : Predicting Excess Returns and Consumption Growth with Log Price-to-consumption in the post-1950 Sample

The historical cross-country panel is from the Jordà-Schularick-Taylor macrohistory database, except for Canada. The annuals series start in 1950 and end in 2015. Panel A performs predictive regressions of stock market excess returns on log price-to-consumption, $\sum_{h=1}^{H}[\log(r_{St+h})-\log(r_{ft+h})]=a+b\log(P_t/C_t)+u_{t+h}$, in which H is the forecast horizon, r_{St+1} real stock market return, r_{ft+1} real interest rate, P_t real market index, and C_t real consumption. r_{St+1} and r_{ft+1} are over the course of period t, and P_t and C_t are at the beginning of t. Excess returns are adjusted for a financial leverage ratio of 0.29. Panel B performs long-horizon predictive regressions of log consumption growth on $\log(P_t/C_t)$, $\sum_{h=1}^{H}\log(C_{t+h}/C_t)=c+d\log(P_t/C_t)+v_{t+h}$. In both regressions, $\log(P_t/C_t)$ is standardized to have a mean of zero and a standard deviation of one. H ranges from one year (1y) to five years (5y). The t-values of the slopes are adjusted for heteroscedasticity and autocorrelations of $2(H-1)$ lags. The slopes and R-squares are in percent.

	Slopes					t-values of slopes					R-squares				
	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
	Panel A: Predicting stock market excess returns														
Australia	−4.79	−7.74	−8.35	−9.52	−9.70	−3.03	−4.13	−4.19	−3.06	−2.46	12.17	19.86	21.17	22.51	21.26
Belgium	−2.39	−5.00	−6.91	−9.44	−10.86	−1.45	−1.57	−1.61	−1.89	−2.39	2.46	5.64	8.16	11.80	15.00
Denmark	−0.43	−1.76	−2.32	−3.05	−3.08	−0.17	−0.41	−0.46	−0.67	−0.76	0.08	0.61	0.79	1.13	1.13
Finland	−3.76	−9.48	−14.03	−17.33	−19.08	−1.36	−2.46	−4.08	−5.30	−5.33	3.68	9.66	14.50	18.23	20.25
France	−1.85	−4.05	−5.95	−8.59	−11.47	−0.97	−1.17	−1.09	−1.21	−1.48	1.26	3.10	4.85	7.24	11.90
Germany	−6.24	−11.41	−14.93	−18.14	−19.06	−2.78	−3.21	−3.15	−3.20	−3.40	12.48	20.22	24.99	29.08	29.57
Italy	−0.98	−2.51	−4.20	−5.61	−6.34	−0.52	−0.63	−0.77	−0.80	−0.76	0.32	0.93	1.76	2.40	2.76
Japan	−4.00	−9.60	−13.90	−17.98	−21.83	−2.30	−2.96	−4.35	−5.80	−5.96	8.19	18.14	25.40	31.70	36.39
Netherlands	−3.04	−6.48	−8.91	−11.12	−13.51	−1.68	−1.87	−2.09	−2.46	−3.06	4.13	8.98	12.71	16.31	20.65
Norway	−3.89	−7.14	−8.74	−9.80	−11.69	−1.99	−2.68	−2.70	−2.59	−2.87	4.99	9.69	12.52	14.56	18.57
Portugal	−2.16	−8.22	−14.17	−17.85	−17.39	−0.48	−0.94	−1.30	−1.51	−1.66	0.77	3.93	6.64	7.60	5.75
Spain	−0.32	−2.18	−4.83	−7.32	−9.22	−0.17	−0.54	−0.86	−1.17	−1.32	0.04	0.78	2.22	3.64	4.76
Sweden	−1.57	−3.12	−4.06	−5.13	−6.09	−0.75	−0.84	−0.91	−1.10	−1.24	0.95	1.88	2.46	3.28	4.10
Switzerland	−3.09	−6.51	−8.50	−10.67	−12.95	−1.70	−2.30	−2.85	−3.89	−4.17	4.02	8.50	11.76	15.72	20.05
UK	−6.50	−11.41	−13.92	−14.44	−16.54	−3.01	−4.32	−4.54	−5.95	−6.68	17.37	30.67	38.71	42.28	49.39
USA	−2.89	−5.59	−7.18	−9.65	−12.36	−2.18	−2.27	−2.24	−2.47	−2.79	5.83	10.67	13.61	18.59	23.90
Mean	−2.99	−6.39	−8.81	−10.98	−12.57	−1.53	−2.02	−2.32	−2.69	−2.90	4.92	9.58	12.64	15.38	17.84
Median	−2.96	−6.49	−8.42	−9.73	−12.02	−1.56	−2.07	−2.16	−2.47	−2.63	3.85	8.74	12.14	15.14	19.31
	Panel B: Predicting consumption growth														
Australia	0.40	0.41	0.36	0.81	1.20	1.79	0.93	0.58	1.35	1.85	4.04	1.78	1.08	5.54	10.21
Belgium	0.09	0.09	0.21	0.41	0.54	0.42	0.25	0.43	0.68	0.76	0.24	0.09	0.27	0.62	0.78
Denmark	−0.08	−0.42	−0.69	−1.05	−1.38	−0.27	−0.55	−0.61	−0.79	−0.94	0.10	1.04	1.61	2.57	3.51
Finland	0.31	0.06	−0.40	−0.73	−0.90	0.95	0.08	−0.39	−0.63	−0.72	0.97	0.01	0.37	0.89	1.10
France	0.95	1.81	2.68	3.51	4.37	4.45	3.67	3.84	4.18	4.69	28.51	32.88	37.06	40.26	43.62
Germany	−0.10	−0.47	−1.05	−1.43	−1.84	−0.29	−0.51	−0.69	−0.72	−0.73	0.15	1.07	2.65	3.20	3.74
Italy	1.58	3.04	4.43	5.68	6.84	5.69	4.47	4.07	3.74	3.51	33.87	37.49	40.15	40.67	40.62
Japan	0.51	0.86	1.44	1.86	2.17	1.48	0.89	0.90	0.80	0.71	2.12	1.79	2.65	2.63	2.30
Netherlands	0.67	1.12	1.46	1.87	2.35	2.43	1.49	1.24	1.17	1.14	7.43	6.49	5.94	6.42	7.60
Norway	0.23	0.38	0.56	0.73	1.00	0.78	0.65	0.77	0.95	1.29	1.16	1.26	1.78	2.36	3.78
Portugal	0.19	0.05	0.13	0.62	1.54	0.36	0.04	0.08	0.38	0.98	0.26	0.01	0.03	0.53	2.66
Spain	1.75	3.04	4.02	4.90	5.62	4.78	3.94	3.36	2.99	2.72	24.75	26.39	25.77	24.87	23.66
Sweden	0.00	−0.22	−0.39	−0.56	−0.74	−0.01	−0.44	−0.46	−0.48	−0.53	0.00	0.45	0.74	1.01	1.30
Switzerland	0.22	0.31	0.36	0.35	0.34	1.32	0.84	0.61	0.43	0.33	2.52	1.40	1.00	0.64	0.44
UK	0.45	0.46	0.45	0.23	−0.12	2.14	1.14	0.80	0.30	−0.13	4.78	1.73	0.99	0.19	0.04
USA	0.28	0.16	0.11	0.19	0.22	1.34	0.31	0.15	0.20	0.20	2.69	0.30	0.09	0.19	0.20
Mean	0.47	0.67	0.86	1.09	1.33	1.71	1.07	0.92	0.91	0.95	7.10	7.14	7.64	8.29	9.10
Median	0.30	0.34	0.36	0.51	0.77	1.33	0.74	0.59	0.56	0.73	2.32	1.33	1.35	2.46	3.08

Table S6 : Predicting Volatilities of Stock Market Excess Returns and Consumption Growth with Log Price-to-consumption in the Post-1950 Sample

The historical cross-country panel is from the Jordà-Schularick-Taylor macrohistory database, except for Canada. The annuals series start in 1950 and end in 2015. For a given horizon, H, we measure excess return volatility as $\sigma_{St,t+H-1} = \sum_{h=0}^{H-1} |\epsilon_{St+h}|$, in which ϵ_{St+h} is the h-period-ahead residual from the first-order autoregression of excess returns, $\log(r_{St+1}) - \log(r_{ft+1})$ (adjusted for a financial leverage ratio of 0.29). Panel A performs long-horizon predictive regressions of excess return volatilities, $\log \sigma_{St+1,t+H} = a + b \log(P_t/C_t) + u^{\sigma}_{t+H}$. For a given H, consumption growth volatility is $\sigma_{Ct,t+H-1} = \sum_{h=0}^{H-1} |\epsilon_{Ct+h}|$, in which ϵ_{Ct+h} is the h-period-ahead residual from the first-order autoregression of log consumption growth, $\log(C_{t+1}/C_t)$. Panel B performs long-horizon predictive regressions of consumption growth volatilities, $\log \sigma_{Ct+1,t+H} = c + d \log(P_t/C_t) + v^{\sigma}_{t+H}$. $\log(P_t/C_t)$ is standardized to have a mean of zero and a standard deviation of one. H ranges from one year (1y) to five years (5y). The t-values are adjusted for heteroscedasticity and autocorrelations of $2(H-1)$ lags. The slopes and R-squares are in percent.

	Slopes					t-values of slopes					R-squares				
	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y	1y	2y	3y	4y	5y
	Panel A: Predicting stock market volatility														
Australia	1.71	6.76	3.98	4.74	2.83	0.11	0.55	0.31	0.36	0.21	0.01	0.59	0.36	0.62	0.27
Belgium	1.96	2.38	1.97	1.08	−0.63	0.20	0.32	0.28	0.15	−0.10	0.04	0.17	0.19	0.08	0.03
Denmark	13.67	11.12	11.79	11.24	10.74	1.14	0.91	0.82	0.73	0.66	1.95	2.30	3.25	3.28	3.18
Finland	19.65	14.49	14.23	11.41	8.21	1.48	1.48	1.85	2.04	1.72	2.78	5.01	6.96	5.39	3.28
France	−9.75	−10.93	−9.49	−10.00	−10.89	−0.74	−1.43	−1.64	−2.47	−3.97	0.97	3.74	6.17	10.88	16.47
Germany	17.77	15.12	15.12	15.11	13.28	1.07	1.76	1.92	1.93	1.74	1.56	5.04	10.80	16.58	16.29
Italy	−33.16	−28.63	−23.29	−19.12	−18.45	−1.75	−2.07	−2.52	−2.80	−3.46	4.55	10.16	15.51	20.36	27.24
Japan	6.13	12.73	12.65	11.56	10.48	0.41	1.24	1.25	1.22	1.16	0.33	4.57	6.30	6.41	6.37
Netherlands	6.06	8.47	11.33	11.06	8.98	0.42	0.67	1.04	1.20	1.16	0.32	1.44	4.74	5.73	5.26
Norway	−34.27	−29.37	−25.24	−25.28	−26.05	−2.43	−4.10	−3.70	−3.45	−3.36	3.56	12.38	22.44	28.85	35.65
Portugal	−42.17	−42.76	−43.85	−46.49	−46.10	−2.14	−2.42	−2.93	−3.25	−2.89	11.85	22.46	28.69	34.92	31.02
Spain	−18.04	−22.59	−19.14	−18.62	−17.79	−1.41	−2.40	−2.09	−2.05	−1.94	2.42	9.73	11.80	15.50	17.10
Sweden	15.21	17.32	19.29	18.48	19.27	1.48	1.88	2.37	2.46	2.73	3.54	9.61	19.61	22.61	28.04
Switzerland	7.05	11.57	9.51	11.11	11.03	0.39	0.87	0.90	1.18	1.30	0.27	2.01	3.01	5.79	7.64
UK	1.05	6.22	11.23	14.44	16.17	0.07	0.56	1.26	2.11	2.62	0.01	0.88	3.89	7.43	11.03
USA	12.24	10.13	11.34	12.42	13.72	0.83	1.17	1.93	2.60	3.07	1.42	2.42	5.21	9.98	17.62
Mean	−2.18	−1.12	0.09	0.20	−0.32	−0.06	−0.06	0.07	0.12	0.04	2.23	5.78	9.31	12.15	14.16
Median	4.01	7.62	10.37	11.08	8.59	0.30	0.62	0.86	0.95	0.91	1.49	4.15	6.23	8.71	13.66
	Panel B: Predicting consumption growth volatility														
Australia	−4.80	12.99	14.07	13.44	12.20	−0.20	1.60	1.88	2.03	2.09	0.17	4.54	8.03	9.83	9.27
Belgium	−4.34	0.39	5.59	10.58	12.26	−0.33	0.04	0.65	1.26	1.63	0.24	0.00	1.16	5.51	8.86
Denmark	−23.77	−22.00	−16.49	−14.52	−15.41	−1.83	−2.23	−1.65	−1.55	−1.85	3.69	7.15	5.85	6.89	12.03
Finland	−25.16	−14.09	−8.95	−5.96	−5.41	−1.84	−1.03	−0.63	−0.42	−0.40	4.60	3.10	1.74	0.91	0.89
France	16.54	17.63	16.51	16.50	16.37	1.28	1.95	2.11	2.74	3.44	2.07	6.56	9.91	13.57	18.08
Germany	−6.27	−1.99	−0.64	1.33	4.44	−0.47	−0.18	−0.07	0.17	0.57	0.21	0.07	0.01	0.07	1.11
Italy	8.31	5.18	5.95	7.19	9.02	0.73	0.56	0.65	0.93	1.56	0.73	0.79	1.33	2.60	5.04
Japan	1.35	−8.36	−6.69	−7.02	−8.53	0.06	−0.67	−0.55	−0.59	−0.73	0.01	1.09	1.05	1.36	2.26
Netherlands	6.40	9.28	11.58	11.02	10.08	0.54	0.80	0.96	1.02	1.05	0.42	2.10	4.59	5.09	5.23
Norway	−22.89	−23.25	−24.00	−21.67	−19.13	−1.86	−1.76	−2.01	−2.28	−2.66	3.70	8.06	11.96	12.72	12.60
Portugal	−18.51	−12.71	−10.28	−9.25	−9.76	−2.00	−1.38	−1.02	−0.82	−0.77	5.10	3.87	3.57	3.38	3.57
Spain	41.05	34.10	31.91	30.02	29.87	2.88	2.55	2.50	2.32	2.22	11.21	14.65	18.82	23.51	28.32
Sweden	−12.16	−20.44	−17.17	−13.44	−12.42	−0.78	−1.43	−1.41	−1.44	−1.60	0.91	6.08	6.43	5.21	6.27
Switzerland	−13.49	−13.67	−13.37	−9.64	−6.97	−1.01	−1.06	−1.11	−0.84	−0.69	1.40	2.71	3.78	2.63	1.61
UK	−24.73	−16.02	−16.09	−16.80	−16.53	−1.91	−1.85	−2.24	−2.79	−2.77	3.67	4.52	7.87	11.74	14.83
USA	−4.93	−13.20	−10.05	−10.31	−10.96	−0.31	−1.14	−0.98	−1.02	−1.07	0.12	2.32	2.44	3.14	4.42
Mean	−5.46	−4.14	−2.38	−1.16	−0.68	−0.44	−0.33	−0.18	−0.08	0.00	2.39	4.23	5.53	6.76	8.40
Median	−5.60	−10.54	−7.82	−6.49	−6.19	−0.40	−0.85	−0.59	−0.50	−0.55	1.16	3.48	4.19	5.15	5.75

Do Investors Recognize Biases in Securities Analysts' Forecasts?

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Abstract

This study presents direct evidence on the question whether investors recognize the widely documented biases in securities analysts' earnings forecasts. The internal rate of return implied by current stock price and consensus earnings forecasts is found to be correlated with indicators of bias in a manner consistent with investors discounting optimistic earnings forecasts at higher rates of return and less optimistic forecasts at lower rates of return. In a departure from studies of excess returns, the evidence in implied returns indicates that investors recognize the biases in analysts' earnings forecasts.

JEL Codes: G11, G12, G14, G41
Keywords: analyst forecast bias, behavioral bias, market efficiency, earnings

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Abstract

This study presents direct evidence on the question whether investors recognize the widely documented biases in securities analysts' earnings forecasts. The internal rate of return implied by current stock price and consensus earnings forecasts is found to be correlated with indicators of bias in a manner consistent with investors discounting optimistic earnings forecasts at higher rates of return and less optimistic forecasts at lower rates of return. In a departure from studies of excess returns, the evidence in implied returns indicates that investors recognize the biases in analysts' earnings forecasts.

1. Introduction

A substantial literature investigating analysts' earnings forecasts supports the conclusion that they are biased. A more recent and growing body of research asserts that because investors fail to optimally process available information, they overweight analyst forecasts resulting in substantial mispricing of common stock. This assertion is based on evidence purporting to show the existence of profitable trading strategies formed on indicators of bias. However, on the question whether investors fail to recognize analyst bias, the evidence from realized returns is circumstantial and open to varying interpretation. By now, analyst biases have been extensively documented. Thus, without a compelling explanation of investors' inability to account for them in valuing common stock, the attribution of seemingly profitable trading strategies to deficiencies in investor judgment must be considered tenuous and needing additional corroborating evidence. The present study takes a new approach to the question whether investors fail to recognize analyst forecast bias and investigates the determinants of expected return in a recent cross section of U.S. public companies.

Clearly, from the perspective of financial market efficiency, the inability of investors to recognize analyst bias is troubling. But, is it true? If investors are able to recognize biases in analyst earnings forecasts, then in valuing stocks they will apply higher discount rates to forecasts they believe are biased upward (i.e., optimistic) and lower rates to those they believe are biased downward (pessimistic). It should be the case, then, that stock price relative to the consensus earnings forecast is correlated with indicators of bias. That is, for a given consensus forecast, stock price will be lower (higher) to the extent investors perceive the forecast to be optimistic (pessimistic). If investors are unable to recognize analyst bias (or, equivalently, if they believe analyst forecasts are unbiased), then stock price relative to the consensus forecast will be uncorrelated with indicators of bias. In this study, the relation of stock price to consensus forecast is measured by reverse engineering an equity valuation model to obtain the internal rate of return implied by current stock price and the consensus forecast. The implied return is found to be strongly correlated with indicators of bias in a fashion consistent with investors discounting optimistic (pessimistic) consensus forecasts at higher (lower) rates of return. Hence, in contrast to assertions made in previous studies, the results presented here support the view that equity investors are indeed capable of recognizing and adjusting for analyst bias. As a preliminary indication of this, the sample median implied return of stocks rated by analysts as Buy, Hold and Sell are 10.7%, 8.6% and 7.6%, respectively. Differences among them are highly statistically significant.

The rest of the paper is organized as follows. Section 2 reviews the literature on analyst earnings forecasts as well as attempts to model earnings forecast error and to profit therefrom. Against this backdrop, the contribution of the present study is articulated. The empirical methodology and data are described in section 3. Section 4 presents and discusses the findings, and section 5 summarizes and concludes.

2. Review of Literature

The literature on analysts' earnings and stock price forecasts indicates that long-range forecasts are optimistic, short-range forecasts are pessimistic, and forecasts generally do not fully reflect available information. Companies report earnings that on average fall short of consensus long-range forecasts (e.g., Abarbanell & Lehavy, 2003; Agrawal & Chen, 2006; Bradshaw et al., 2006; Brous, 1992; Brous & Kini, 1993; Butler & Lange, 1991; Dreman & Berry, 1995; Easterwood & Nutt, 1999; Francis & Philbrick, 1993; Fried & Givoly, 1982; Kang et al., 1994; Richardson et al., 2004), and stock prices tend not to reach analysts' long-range price targets (e.g., Cowen et al., 2006; Szakmary et al., 2008). Researchers attempting to understand the factors driving these biases have considered analysts' relationships with their employers, with the firms they cover, and with their investor clients. Forecast optimism has been attributed to the investment banking and trading activities of analysts' sell-side employers, to the tendency of analysts to cover firms about which they are optimistic, and to analysts' desire to appease company executives in order to maintain access to valuable information. Management guidance and analysts' desire to establish and maintain credibility with investor clients act to dampen analyst optimism (Cowen et al., 2006; Dugar & Nathan, 1995; Francis & Philbrick, 1993; Lin & McNichols, 1998; Ljungqvist et al., 2007; Michaely & Womack, 1999; Raedy et al., 2006; Richardson et al., 2004).

Apart from being biased, consensus earnings forecasts do not fully incorporate available information and are therefore inefficient. Forecast errors are correlated with prior forecast errors, past stock returns, and past earnings changes (Ali et al., 1992; Abarbanell and Bernard, 1992; Shane and Brous, 2001), and Cohen and Lys (2003) report that analysts underreact to prior information. Attempts to explain these inefficiencies have relied on the existence of defects in analysts' judgment. Conservatism bias, for example, is alleged to cloud analyst judgment. However, Raedy et al. (2006) provide a rational explanation for underreaction in terms of analyst credibility. For a forecast error of given magnitude, credibility is damaged when later information causes a forecast revision in the direction opposite the analyst's previous revision. Hence, analyst forecast inefficiency could arise from rational incentives as opposed to defective judgment.

A related stream of research seeks to model earnings forecast error in order to improve earnings forecasts. Laroque (2013), for example, models earnings forecast error in terms of lagged forecast error, lagged abnormal stock return, and lagged equity market value. Mohanram and Gode (2013) model forecast error in terms of lagged accruals, lagged sales growth, lagged analyst forecasts of long-term growth, lagged change in property, plant and equipment, lagged change in other total assets, lagged stock return, and the revision in analyst forecasts from the prior year. Easton and Monahan (2016) conclude that while these methods are effective in removing errors in earnings forecast levels, they are less effective in removing errors in forecasts of earnings changes.

Another related research stream seeks to identify profitable trading strategies by exploiting predictable earnings forecast error. Kothari et al. (2016) survey the literature on analysts' forecasts and asset pricing and conclude that investors only partially unravel the biases in analysts' forecasts resulting in predictable stock prices. Their conclusion is based on