

Control Number: 34800



Item Number: 239

Addendum StartPage: 0

### SOAH DOCKET NO. 473-08-0334 PUC DOCKET NO. 34800

APPLICATION OF ENTERGY	§		
GULF STATES, INC. FOR	§	BEFORE THE	
AUTHORITY TO CHANGE	§	STATE OFFICE OF ADMINISTI	RATIVE
RATES AND RECONCILE	§	HEARINGS C	211
FUEL COSTS	§	in the second se	5

## RESPONSE OF ENTERGY GULF STATES, INC. TO TIEC'S SECOND REQUEST FOR INFORMATION

Now comes, Entergy Gulf States, Inc. ("Entergy Gulf States" or "the Company") and files its Response to Texas Industrial Energy Consumers' ("TIEC") Second Request for Information. The responses to such requests are attached hereto and are numbered as in the request. An additional copy is available for inspection at the Company's office in Austin, Texas.

Entergy Gulf States believes the foregoing responses are correct and complete as of the time of the responses, but the Company will supplement, correct or complete the responses if it becomes aware that the responses are no longer true and complete, and the circumstances are such that failure to amend the answer is in substance misleading. The parties may treat these responses as if they were filed under oath.

Respectfully submitted,

## L. Richard Westerburg, Jr.

L. Richard Westerburg, Jr. Steve Neinast Entergy Services, Inc. 919 Congress Avenue, Suite 701 Austin, Texas 78701 (512) 487-3957 telephone (512) 487-3958 facsimile

Attachments: TIEC 2: 6, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26

## **CERTIFICATE OF SERVICE**

I certify that a copy of the foregoing Response of Entergy Gulf States, Inc. to TIEC's First Request for Information has been sent by either hand delivery, facsimile, overnight delivery, or U.S. Mail to all parties on the attached service list on this the <u>27th</u> day of <u>November</u>, 2007.

L. Richard Westerburg, Jr. L. Richard Westerburg, Jr.

### EGSI 07 TX Rate Case Docket No 34800

Dick Westerburg

**EGSI** 

919 Congress Ave. Suite 701

Austin, Texas 78701

Barry Howell EGSI

919 Congress Ave. Suite 840

Austin, Texas 78701

John Williams EGSI (Clark, Thomas)

Clark, Thomas

300 West 6<sup>th</sup> Street, 15<sup>th</sup> Floor

Austin, Texas 78701

James Galloway PUC Filing Clerk

1701 N. Congress Ave. Austin, Texas 78711

Patrick Sullivan STAFF

1701 N. Congress Ave Austin, Texas 78711

Dan Lawton CITIES

816 Congress Avenue, Suite 1120

Austin, Texas 78701

Rex VanMiddlesworth TIEC

111 Congress Avenue, Suite 1700

Austin, Texas 78701

Mark Gladney OPC

Assistant Public Counsel

1701 N. Congress Avenue, Suite 9-180

P.O. Box 12397

Austin, Texas 78711-2397

Stephen J. Davis ARM

Law Offices of Stephen J. Davis, PC

701 Brazos, Suite 970 Austin, Texas 78701

Michael L. Kurtz

Kurt J. Boehm

**KROGER** 

Boehm, Kurtz & Lowry

36 East Seventh Street, Suite 1510

Cincinnati, Ohio 45202

Susan M. Kelley

**STATE** 

Bryan L. Baker

Consumer Protection & Public Health Division

Office of the Attorney General

P.O. Box 12548

Austin, Texas 78711-2548

Physical Address: W.P. Clements State Office Bldg., 9<sup>th</sup> Floor

300 West 15<sup>th</sup> Street, Austin, Texas 78701

Carrie Tournillon

TLSC

Randall Chapman

Texas Legal Services Center

815 Brazos St. Suite 1100 Austin, Texas 78701

Carol Biedrzycki

Texas ROSE

Texas Ratepayers' Organization To Save Energy

815 Brazos St. Suite 1100

Austin, Texas 78701

Frederick H. Ritts

**ETEC** 

**ETEC** 

John Conway

Brickfield Burchette Ritts & Stone, P.C.

1025 Thomas Jefferson, N.W.

800 West Tower

Washington, D.C. 20007

Mark C. Davis

Nelson Nease

John T. Wright

Brickfield Burchette Ritts & Stone, P.C.

1005 Congress Avenue, Suite 400

Austin, Texas 78701

Response of: Entergy Gulf States, Inc. to the Second Set of Data Requests

of Requesting Party: TIEC

Prepared By: Samuel C. Hadaway

Sponsoring Witness: Samuel C. Hadaway

Beginning Sequence No. Ending Sequence No.

Question No.: TIEC 2-6

Part No.:

Addendum:

Question:

The following questions relate to the direct testimony of Dr. Samuel C. Hadaway:

On an electronic spreadsheet with all formula intact, please provide a copy of all analyses and workpapers used in support of Dr. Hadaway's direct testimony and schedules in this proceeding.

Response:

Please see the attached CD containing spreadsheets.



## ENTERGY GULF STATES, INC. PUBLIC UTILITY COMMISSION OF TEXAS

Docket No. 34800 - 2007 Texas Rate Case

Response of: Entergy Gulf States, Inc.

to the Second Set of Data Requests

of Requesting Party: TIEC

Prepared By: Samuel C. Hadaway

Sponsoring Witness: Samuel C. Hadaway

Beginning Sequence No.

Ending Sequence No.

Ouestion No.: TIEC 2-10

Part No.:

Addendum:

Question:

The following questions relate to the direct testimony of Dr. Samuel C. Hadaway:

Referring to page 20 of Dr. Hadaway's direct testimony, he states that "long-term growth rate estimates have been highly uncertain" since the mid-1800s. Does Dr. Hadaway agree that the current long-term growth rate estimates are relatively high?

#### Response:

No. Dr. Hadaway's testimony, page 20, references growth rate since the mid 1980's. As shown in Dr. Hadaway's Direct Testimony, Exhibit SCH-3, Value Line's earnings growth rate estimates are currently over 75 basis points lower than they were five years ago in 2002, and the sustainable growth "b times r" growth rate based on Value Line's projections is over 150 basis points lower.

Response of: Entergy Gulf States, Inc.

to the Second Set of Data Requests

of Requesting Party: TIEC

Prepared By: Samuel C. Hadaway

Sponsoring Witness: Samuel C. Hadaway

Beginning Sequence No.

Ending Sequence No.

Question No.: TIEC 2-11

Part No.:

Addendum:

Question:

The following questions relate to the direct testimony of Dr. Samuel C. Hadaway:

At page 28 of his direct testimony Dr. Hadaway states that the current three-tofive year analysts' projections are extremely low. Please provide copies of all reports, analyses and reference material in support of this testimony.

#### Response:

Dr. Hadaway cannot find the statement referenced in the data request on page 28 of his direct testimony. On page 32, Dr. Hadaway states that "Value Line's current three-to-five projections are lower than they have been in previous years." Please see further testimony, at page 41, lines 1-10, and Exhibit SCH-3 for Dr. Hadaway's support of this statement.

TH671

34800

## ENTERGY GULF STATES, INC. PUBLIC UTILITY COMMISSION OF TEXAS

Docket No. 34800 - 2007 Texas Rate Case

Response of: Entergy Gulf States, Inc.

to the Second Set of Data Requests

of Requesting Party: TIEC

Prepared By: Samuel C. Hadaway

Sponsoring Witness: Samuel C. Hadaway

Beginning Sequence No.

Ending Sequence No.

Ouestion No.: TIEC 2-12

Part No.:

Addendum:

**Ouestion:** 

The following questions relate to the direct testimony of Dr. Samuel C. Hadaway:

Concerning page 32 of Dr. Hadaway's testimony, he references utility growth expectations. Please explain if Dr. Hadaway believes that the utilities' sustainable growth rates can exceed the growth rate of the U. S. economy as measured by the GDP over an indefinite period of time.

#### Response:

Dr. Hadaway does not believe, nor has he testified, that utilities' sustainable growth rates can exceed the long-term growth rate in the economy as measured by nominal gross domestic product (GDP). None of his average growth rates exceed his 6.6 percent estimated long-term growth rate for GDP.

## ENTERGY GULF STATES, INC. PUBLIC UTILITY COMMISSION OF TEXAS

Docket No. 34800 - 2007 Texas Rate Case

Response of: Entergy Gulf States, Inc. to the Second Set of Data Requests

of Requesting Party: TIEC

Prepared By: Samuel C. Hadaway

Sponsoring Witness: Samuel C. Hadaway

Beginning Sequence No.

Ending Sequence No.

Question No.: TIEC 2-13

Part No.:

Addendum:

Question:

The following questions relate to the direct testimony of Dr. Samuel C. Hadaway:

Referring to Exhibit SCH-5 of Dr. Hadaway's direct testimony, please provide this exhibit using the methodology Dr. Hadaway has applied in previous regulatory proceedings. For example, he replaced the average of his sustainable growth rate (b x r), historical GDP growth rate of 6.60%, and the Zack's growth rate projections with the analysts growth rate projections obtained from Zack's, Thomson Financial/First Call, Reuters and SNL.

Response:

Dr. Hadaway has not performed the requested analysis nor prepared such an exhibit.

Response of: Entergy Gulf States, Inc.

to the Second Set of Data Requests

of Requesting Party: TIEC

Prepared By: Samuel C. Hadaway

Sponsoring Witness: Samuel C. Hadaway

Beginning Sequence No.

Ending Sequence No.

Ouestion No.: TIEC 2-14

Part No.:

Addendum:

Question:

The following questions relate to the direct testimony of Dr. Samuel C. Hadaway:

Referring to page 43 of his direct testimony, please state weather Dr. Hadaway has previously opposed the use of the Capital Asset Pricing Model (CAPM). Please explain the reasons for including this model in the calculation of the return on equity for EGSI.

#### Response:

Dr. Hadaway has opposed the use of the CAPM in some cases in which a witness utilizing the CAPM may have used unreasonable input assumptions and produced unreasonable estimates of the required rate of return. As Dr. Hadaway explains on pages 4 and 5 of his present Direct Testimony, however, current market and utility industry conditions require additional model estimates beyond the traditional DCF approach, and the CAPM is an additional approach that is currently appropriate to consider, as the Texas PUC Staff routinely does.

Response of: Entergy Gulf States, Inc.

to the Second Set of Data Requests

of Requesting Party: TIEC

Prepared By: Samuel C. Hadaway

Sponsoring Witness: Samuel C. Hadaway

Beginning Sequence No.

Ending Sequence No.

Question No.: TIEC 2-15

Part No.:

Addendum:

Question:

The following questions relate to the direct testimony of Dr. Samuel C. Hadaway:

Please explain Dr. Hadaway's position on the use of geometric mean returns as opposed to arithmetic mean returns.

#### Response:

The geometric mean is sometimes called compound or compounded rate of return. It is the rate of return that equates a beginning and ending wealth amount assuming that each year's return on investment is retained and reinvested. The arithmetic rate of return is the simple average or expected value calculated by adding up each of the years' returns and dividing the sum by the number of years. For estimating the required rate of return, the issue is what investors expect or require for each year. Unless the returns are the same each year, the geometric mean is always smaller than the arithmetic mean. For this reason, economists who use the geometric mean exclusively always find a lower ROE estimate. Economists who use the arithmetic mean exclusively always find a higher ROE. The best academic research (see attached article by Professor Marshall Blume) indicates that the correct answer is somewhere in between. For this reason, in his CAPM analysis Dr. Hadaway uses the average of the geometric and arithmetic mean market risk premiums. This approach is consistent with the academic research and produces a middle ground estimate of ROE.

Response of: Entergy Gulf States, Inc. to the Second Set of Data Requests

of Requesting Party: TIEC

Prepared By: Counsel Sponsoring Witness: N/A Beginning Sequence No. Ending Sequence No.

Question No.: TIEC 2-16

Part No.:

Addendum:

Question:

The following RFIs relate to the Jurisdictional Separation Plan ("JSP") referred to in EGSI's testimony and in Application of Entergy Gulf States, Inc., Entergy Gulf States Louisiana, LLC, and Entergy Texas, Inc., For Authorization to Implement Jurisdictional Separation Plan, Federal Energy Regulatory Commission ("FERC") Docket No. EC07-66-000. Reference to "EGS-TX" refers to Entergy Texas, Inc. as described in FERC Docket No. EC07-66-000 or the Entergy Corp. affiliate or subsidiary that will serve EGSI's customers in Texas upon jurisdictional separation.

Does EGSI contend that it is not required to seek Commission approval of the JSP? Please state EGSI's contention and the legal and factual bases for that contention.

#### Response:

Yes, EGSI contends that it is not required to seek Commission approval for the JSP. EGSI reserves the right to fully brief this issue as appropriate at the later stages of this proceeding. Subject to this reservation, the legal and factual basis for EGSI's contention are set forth in: 1) PURA §§ 14.101(d)(3) and 39.452(e); 2) EGSI's Comments filed May 7, 2007 in Project No. 34038: Rulemaking Proceeding to Amend PUC Substantive Rules Relating Notification of Transactions Affecting the Ownership of Electric Utilities; 3) PUC SUBST. R. 25.74(e); and 4) the Preamble to PUC SUBST. R. 25.74.

Response of: Entergy Gulf States, Inc. to the Second Set of Data Requests

of Requesting Party: TIEC

Prepared By: Counsel Sponsoring Witness: N/A Beginning Sequence No. Ending Sequence No.

Question No.: TIEC 2-17

Part No.:

Addendum:

#### **Ouestion:**

The following RFIs relate to the Jurisdictional Separation Plan ("JSP") referred to in EGSI's testimony and in Application of Entergy Gulf States, Inc., Entergy Gulf States Louisiana, LLC, and Entergy Texas, Inc., For Authorization to Implement Jurisdictional Separation Plan, Federal Energy Regulatory Commission ("FERC") Docket No. EC07-66-000. Reference to "EGS-TX" refers to Entergy Texas, Inc. as described in FERC Docket No. EC07-66-000 or the Entergy Corp. affiliate or subsidiary that will serve EGSI's customers in Texas upon jurisdictional separation.

Does EGSI contend that the Commission lacks authority to review and approve the JSP prior to its closing? Please state EGSI's contention and the legal and factual bases for that contention.

#### Response:

Yes, EGSI contends that the Commission lacks authority to review and approve the JSP prior to its closing. EGSI reserves the right to fully brief this issue as appropriate at the later stages of this proceeding. Subject to this reservation, the legal and factual basis for EGSI's contention are set forth in: 1) PURA §§ 14.101(d)(3) and 39.452(e); 2) EGSI's Comments filed May 7, 2007 in Project No. 34038: Rulemaking Proceeding to Amend PUC Substantive Rules Relating Notification of Transactions Affecting the Ownership of Electric Utilities; 3) PUC SUBST. R. 25.74(e); and 4) the Preamble to PUC SUBST. R. 25.74.

Response of: Entergy Gulf States, Inc. to the Second Set of Data Requests

of Requesting Party: TIEC

Prepared By: Counsel Sponsoring Witness: N/A Beginning Sequence No. Ending Sequence No.

Question No.: TIEC 2-18

Part No.:

Addendum:

#### Question:

The following RFIs relate to the Jurisdictional Separation Plan ("JSP") referred to in EGSI's testimony and in Application of Entergy Gulf States, Inc., Entergy Gulf States Louisiana, LLC, and Entergy Texas, Inc., For Authorization to Implement Jurisdictional Separation Plan, Federal Energy Regulatory Commission ("FERC") Docket No. EC07-66-000. Reference to "EGS-TX" refers to Entergy Texas, Inc. as described in FERC Docket No. EC07-66-000 or the Entergy Corp. affiliate or subsidiary that will serve EGSI's customers in Texas upon jurisdictional separation.

Does EGSI contend that the Commission lacks authority to review the JSP after its closing? Please state EGSI's contention and the legal and factual bases for that contention.

#### Response:

Yes, EGSI contends that the Commission lacks authority to review the JSP after its closing. EGSI reserves the right to fully brief this issue as appropriate at the later stages of this proceeding. Subject to this reservation, the legal and factual basis for EGSI's contention are set forth in: 1) PURA §§ 14.101(d)(3) and 39.452(e); 2) EGSI's Comments filed May 7, 2007 in Project No. 34038: Rulemaking Proceeding to Amend PUC Substantive Rules Relating Notification of Transactions Affecting the Ownership of Electric Utilities; 3) PUC SUBST. R. 25.74(e); and 4) the Preamble to PUC SUBST. R. 25.74.

Response of: Entergy Gulf States, Inc. to the Second Set of Data Requests

of Requesting Party: TIEC

Prepared By: Counsel Sponsoring Witness: N/A Beginning Sequence No. Ending Sequence No.

Question No.: TIEC 2-19

Part No.:

Addendum:

Question:

The following RFIs relate to the Jurisdictional Separation Plan ("JSP") referred to in EGSI's testimony and in Application of Entergy Gulf States, Inc., Entergy Gulf States Louisiana, LLC, and Entergy Texas, Inc., For Authorization to Implement Jurisdictional Separation Plan, Federal Energy Regulatory Commission ("FERC") Docket No. EC07-66-000. Reference to "EGS-TX" refers to Entergy Texas, Inc. as described in FERC Docket No. EC07-66-000 or the Entergy Corp. affiliate or subsidiary that will serve EGSI's customers in Texas upon jurisdictional separation.

Does EGSI contend that, if the Commission finds that the JSP is not in the public interest, the Commission has authority to take the effect of the transaction into consideration in ratemaking proceedings and disallow the effects of the transaction if it will unreasonably affect rates or service? Please state EGSI's contention and the legal and factual bases for that contention.

#### Response:

This RFI is phrased in terms of the provisions of PURA § 14.101, which EGSI contends is not applicable to the JSP. EGSI reserves the right to fully brief this issue as appropriate at the later stages of this proceeding. Subject to this reservation, the legal and factual basis for EGSI's contention are set forth in: 1) PURA §§ 14.101(d)(3) and 39.452(e); 2) EGSI's Comments filed May 7, 2007 in Project No. 34038: Rulemaking Proceeding to Amend PUC Substantive Rules Relating Notification of Transactions Affecting the Ownership of Electric Utilities; 3) PUC SUBST. R. 25.74(e); and 4) the Preamble to PUC SUBST. R. 25.74.

Response of: Entergy Gulf States, Inc. to the Second Set of Data Requests

of Requesting Party: TIEC

Prepared By: Counsel Sponsoring Witness: N/A Beginning Sequence No.

Ending Sequence No.

Question No.: TIEC 2-20

Part No.:

Addendum:

Question:

The following RFIs relate to the Jurisdictional Separation Plan ("JSP") referred to in EGSI's testimony and in Application of Entergy Gulf States, Inc., Entergy Gulf States Louisiana, LLC, and Entergy Texas, Inc., For Authorization to Implement Jurisdictional Separation Plan, Federal Energy Regulatory Commission ("FERC") Docket No. EC07-66-000. Reference to "EGS-TX" refers to Entergy Texas, Inc. as described in FERC Docket No. EC07-66-000 or the Entergy Corp. affiliate or subsidiary that will serve EGSI's customers in Texas upon jurisdictional separation.

After jurisdictional separation, what corporate entity will own EGSI's certificate of convenience and necessity ("CCN")?

Response:

After the jurisdictional separation, Entergy Texas, Inc. will own EGSI's CCN.

Response of: Entergy Gulf States, Inc. to the Second Set of Data Requests

of Requesting Party: TIEC

Prepared By: Counsel Sponsoring Witness: N/A Beginning Sequence No. Ending Sequence No.

Question No.: TIEC 2-21

Part No.:

Addendum:

**Ouestion:** 

The following RFIs relate to the Jurisdictional Separation Plan ("JSP") referred to in EGSI's testimony and in Application of Entergy Gulf States, Inc., Entergy Gulf States Louisiana, LLC, and Entergy Texas, Inc., For Authorization to Implement Jurisdictional Separation Plan, Federal Energy Regulatory Commission ("FERC") Docket No. EC07-66-000. Reference to "EGS-TX" refers to Entergy Texas, Inc. as described in FERC Docket No. EC07-66-000 or the Entergy Corp. affiliate or subsidiary that will serve EGSI's customers in Texas upon jurisdictional separation.

Does EGSI contend that it is not required to seek Commission approval of the transfer of its CCN? Please state EGSI's contention and the legal and factual bases for that contention.

#### Response:

Yes, EGSI contends that it is not required to seek Commission approval of the transfer of its CCN. EGSI reserves the right to fully brief this issue as appropriate at the later stages of this proceeding. Subject to this reservation, the legal and factual basis for EGSI's contention are set forth in: 1) Texas Business Corporation Act, Article 5.06(A)(2); 2) PURA §39.452(e); 3) EGSI's Comments filed May 7, 2007 in Project No. 34038: Rulemaking Proceeding to Amend PUC Substantive Rules Relating Notification of Transactions Affecting the Ownership of Electric Utilities; 4) PUC SUBST. R. 25.74(e); and 5) the Preamble to PUC SUBST. R. 25.74.

Response of: Entergy Gulf States, Inc. to the Second Set of Data Requests

of Requesting Party: TIEC

Prepared By: Counsel Sponsoring Witness: N/A Beginning Sequence No. Ending Sequence No.

Ouestion No.: TIEC 2-22

Part No.:

Addendum:

Question:

The following RFIs relate to the Jurisdictional Separation Plan ("JSP") referred to in EGSI's testimony and in Application of Entergy Gulf States, Inc., Entergy Gulf States Louisiana, LLC, and Entergy Texas, Inc., For Authorization to Implement Jurisdictional Separation Plan, Federal Energy Regulatory Commission ("FERC") Docket No. EC07-66-000. Reference to "EGS-TX" refers to Entergy Texas, Inc. as described in FERC Docket No. EC07-66-000 or the Entergy Corp. affiliate or subsidiary that will serve EGSI's customers in Texas upon jurisdictional separation.

How will EGS-TX be organized and capitalized? Identify the amount of equity and capital that EGS-TX will have on its balance sheet upon jurisdictional separation.

#### Response:

EGS-TX will be organized as a corporation pursuant to the Texas Business Corporation Act. With respect to EGS-TX's capital structure, the JSP has been structured such that EGS-LA LLC and EGS-TX each will emerge from the JSP with approximately the same capital structure as EGSI's pre-JSP capital structure (with only minor differences related to the conversion of EGSI preference stock into EGS-LA LLC preferred membership interest). EGS-TX's final capital structure will be reflected in its Annual Report on Form 10-K to be filed with the SEC no later than February 29, 2008.

Response of: Entergy Gulf States, Inc. to the Second Set of Data Requests of Requesting Party: TIEC

Prepared By: Counsel Sponsoring Witness: N/A Beginning Sequence No. Ending Sequence No.

**Question No.: TIEC 2-24** 

Part No.:

Addendum:

Question:

The following RFIs relate to the Jurisdictional Separation Plan ("JSP") referred to in EGSI's testimony and in Application of Entergy Gulf States, Inc., Entergy Gulf States Louisiana, LLC, and Entergy Texas, Inc., For Authorization to Implement Jurisdictional Separation Plan, Federal Energy Regulatory Commission ("FERC") Docket No. EC07-66-000. Reference to "EGS-TX" refers to Entergy Texas, Inc. as described in FERC Docket No. EC07-66-000 or the Entergy Corp. affiliate or subsidiary that will serve EGSI's customers in Texas upon jurisdictional separation.

If EGS-TX is capitalized with debt, identify the amount of debt, the cost of the debt, and whether such debt will be capitalized with EGS-TX's assets.

#### Response:

EGS-TX will assume its share of EGSI current long-term debt (the Assumed Debt") consisting of First Mortgage Bonds and Pollution Control Bonds (the "Outstanding Debt"), such amounts to be paid on the date and manner provided by the Outstanding Debt. The Assumed Debt will constitute the initial component of EGS-TX's capital structure and EGS-TX will have 3 years to complete the refinancing of the Assumed Debt. The cost of this refinancing will depend upon market conditions at the time the new debt is issued. The specific amount of EGS-TX's debt capitalization will be determined no later than February 29, 2008 at which time the Company must file its Annual Report of Form 10-K with the SEC.

Response of: Entergy Gulf States, Inc. to the Second Set of Data Requests

of Requesting Party: TIEC

Prepared By: Counsel Sponsoring Witness: N/A Beginning Sequence No. Ending Sequence No.

Question No.: TIEC 2-25

Part No.:

Addendum:

Question:

The following RFIs relate to the Jurisdictional Separation Plan ("JSP") referred to in EGSI's testimony and in Application of Entergy Gulf States, Inc., Entergy Gulf States Louisiana, LLC, and Entergy Texas, Inc., For Authorization to Implement Jurisdictional Separation Plan, Federal Energy Regulatory Commission ("FERC") Docket No. EC07-66-000. Reference to "EGS-TX" refers to Entergy Texas, Inc. as described in FERC Docket No. EC07-66-000 or the Entergy Corp. affiliate or subsidiary that will serve EGSI's customers in Texas upon jurisdictional separation.

At what step of the JSP does EGS-TX's stock transfer from EGSI to Entergy Corp.?

#### Response:

The JSP consists of five steps or their functional equivalent. These five steps are generally described in EGSI's September 24, 2007 letter filed in FERC Docket No. ES07-26-000 (p. 4). In the first step, EGSI will be divided into EGS-TX and Entergy Gulf States Louisiana, Inc. ("EGS-LA Inc."), and EGS-TX will be allocated all of EGSI's Texas-related assets and liabilities. In consideration thereof, EGS-TX will issue approximately 46,525,000 shares of common stock to Entergy Corporation, the result of which EGS-TX will be a wholly-owned subsidiary of Entergy Corporation.

# Unbiased Estimators of Long-Run Expected Rates of Return

MARSHALL E. BLUME\*

This article documents the biases in using sample arithmetic or geometric means of one-period returns to assess long-run expected rates of return. The formulas developed are applicable to other compound growth processes. For types of distributions of one-period returns likely to be encountered for bonds and stocks, numerical values for these biases are given. Four unbiased estimators of long-run expected rates of return are developed and their relative efficiency examined.

#### 1. INTRODUCTION

In a variety of financial decisions, an individual or firm must assess the long-run expected rates of return of some investment vehicle. As one example, a professor whose institution invests in TIAA/CREF on his behalf would certainly try to assess the magnitude of his retirement fund in determining his current schedule of savings. As another example, an actuary in calculating premiums for a life insurance policy would need to make some assumption about long-run expected rates of return. Such persons as these would typically base their assessments of future expected rates of return upon past experience.

Assume, for instance, that this past experience consists of T monthly relatives, defined as the ratio of the value at the end of the month to the value at the end of the previous month. Now, assume that one wishes to determine the expected increase in value of this asset if it were to be held N months, where this increase is measured by the ratio of the terminal value to the initial value—a so-called N-period relative. If it can be assumed that the relatives in each single period approximate identically distributed independent normal variates, the expected N-period relative is given by the population expected one-period relative to the Nth power.

In practice, one does not know the population statistic and therefore must make an estimate. Some might be tempted to estimate the expected N-period relative by raising the arithmetic average of the T one-period relatives to the Nth power. As long as N exceeds one, this procedure will yield an upward biased estimate. Others would take the geometric mean of the T observations and raise this number to the Nth power to derive an estimate of the expected N-period relative. This estimate is downward biased if N is less than T.

The article develops formulas for the magnitude of these biases which, when evaluated at reasonable values for the stock market, show that the biases are sometimes substantial. More generally, these formulas can be used to calculate their magnitude for any compound process.

An unbiased estimate of the expected N-period relative for N < T will therefore be between the arithmetic mean raised to the Nth power and the geometric mean raised to the Nth power. Finally, this article will propose and evaluate various unbiased estimators of the expected N-period relative for data like those found in the bond and stock markets.

#### 2. THE BIAS IN THE ARITHMETIC MEAN

Let  $R_t$  represent a one-period relative or one plus the interest rate. Further, assume that  $R_t$  is an independent, normally distributed random variate with positive  $\mu$  and nonzero  $\sigma(R)$ —stationary over time. It is convenient to define a new random variable  $\epsilon_t$  as

$$R_t = \mu + \epsilon_t. \tag{2.1}$$

The random variable  $\epsilon_t$  is thus independently and normally distributed with mean zero and a standard deviation the same as  $R_t$ .

The expected N-period relative, denoted by  $E(W_N)$ , is given by

$$E(W_N) = E(\prod_{t=1}^N R_t),$$
  
=  $E[\prod_{t=1}^N (\mu + \epsilon_t)].$  (2.2)

Because of independence, (2.2) becomes

$$E(W_N) = \mu^N. \tag{2.3}$$

Lawrence Fisher [8] observed in empirical data the biases associated with the use of one-period geometric and arithmetic means to estimate long-run rates of return. He however did not give a full explanation of this phenomenon.

Cheng and Deets [6] have previously shown that the geometric mean raised to the Nth power is downward biased for N less than T and upward biased for N greater than T. They however did not calculate the magnitude of the bias. In the process of deriving a formula for such a calculation and in extending the theoretical results to a comparison of the relative efficiencies of arithmetic and geometric estimates of one-period expected returns, this article will again demonstrate these biases but more concisely.

Journal of the American Statistical Association
 September 1974, Volume 69, Number 347
 Applications Section

21

<sup>\*</sup> Marshall E. Blume is professor, Finance Department, The Wharton School, University of Pennsylvania, Philadelphia, Penn. 19174. The author wishes to thank the Rodney L. White Center for Financial Research of the Wharton School for financial support, as well as Professors Stephen Ross and Randolph Westerfield whose helpful comments were greatly appreciated. The contents and the opinions expressed in this article are the sole responsibility of the author.

<sup>&</sup>lt;sup>1</sup> The author has been familiar for some time with the biases in using the geometric mean to estimated expected one-period or long-run rates of return. Research on a recent paper, coauthored with Irwin Friend [5], suggested that the arithmetic mean may yield substantially biased estimates of expected long run rates of return. This article is in part an outgrowth of this research.

Equation (2.3) shows that the population expected N-period relative is the population expected one-period relative raised to the Nth power.

From a sample of T observations,  $R_t$ ,  $t = 1, \ldots, T$ , an unbiased estimate of the expected one-period return is

$$A = \mu + (\sum_{t=1}^{T} \epsilon_t)/T, \qquad (2.4)$$

where A denotes the arithmetic mean of one-period relatives. Raising (2.4) to the Nth power in the spirit of (2.3) and letting

$$h = \left(\sum_{t=1}^{T} \epsilon_t\right) / T, \tag{2.5}$$

one obtains the following estimate of  $E(W_N)$ :

$$A^{N} = (\mu + h)^{N}. (2.6)$$

It follows directly from (2.6) that the estimator  $A^N$  is asymptotically unbiased and consistent. Since h is an average of normally distributed and independent random variables, h is itself a normal variate. As T approaches infinity for fixed values of N, the variance of h will approach zero and therefore the probability limit of  $A^N$  is  $\mu^N$ .

Although  $A^N$  is asymptotically unbiased and consistent, it is upward biased for finite T and N greater than one. Applying expected value operators to (2.6) yields

$$E(A^N) = E[(\mu + h)^N].$$
 (2.7)

Jensen's inequality shows that the term on the right is equal to or greater than  $\mu^N$ , so that the arithmetic estimate is upward biased.

To measure the magnitude of the bias,  $E(A^N)$  was evaluated for values of  $\mu$  from 1.00 to 1.01 and  $\sigma(R)$  from 0.03 to 0.15. The values assigned to N and T ranged up to 100. These ranges are roughly the ranges one might encounter in empirical work with monthly relatives for bonds and common stocks. A comparison of the estimated expected N-period relatives with the corresponding population statistic discloses that the biases are in many cases substantial. For instance, for E(R) = 1.01 and  $\sigma(R) = 0.15$ , the expected 40-period relative estimated from 80 observations is 1.8416 compared to the population statistic of 1.4888.

### 3. THE BIAS IN THE GEOMETRIC MEAN

From a sample of T observations, the sample geometric mean is calculated as

$$G = \left[\prod_{t=1}^{T} (R_t)\right]^{1/T} \tag{3.1}$$

where G denotes geometric mean and where it is now assumed that every  $R_t$  exceeds zero. An estimate of the

$$E(A^N) \, \approx \, \mu^N \, + \, \, \sum_{i=1}^n \binom{N}{2i} \mu^{N-2i} (2i!/2^ii!) \left[ \sigma^2(\epsilon)/T \right]$$

where n is the largest integer equal to or less than (N/2).

expected N-period relative is given by

$$G^{N} = \left[\prod_{i=1}^{T} (R_{i})\right]^{N/T} \tag{3.2}$$

If every  $R_t$  must exceed zero,  $R_t$  cannot be normally distributed as was assumed in Section 2. Nonetheless, for monthly relatives of stocks or bonds in which  $\mu$  will be somewhat greater than 1.0 and  $\sigma(R_t)$  around 0.15 or less, the distribution of  $R_t$  may closely approximate a normal distribution.<sup>3</sup>

The estimate of the expected N-period relative given by the geometric mean in (3.2) is downward biased when N is less than T. The demonstration of this bias follows from first substituting (2.1) into (3.2), which gives (3.3):

$$G^{N} = \left[\prod_{i=1}^{T} (\mu + \epsilon_{i})\right]^{N/T}. \tag{3.3}$$

Defining Y as  $\prod_{i=1}^{T} (\mu + \epsilon_i) - \mu^T$  and taking expected values, (3.3) becomes

$$E(G^N) = E\lceil (\mu^T + Y)^{N/T} \rceil. \tag{3.4}$$

Since  $\mu + \epsilon_i$  must be assumed positive for the use of the geometric mean, since a positive variable raised to the N/T power is a concave function for N less than T, and since E(Y) equals zero, the following inequality holds

$$E(G^N) < (\mu^T)^{N/T} = \mu^N,$$
 (3.5)

providing at least one & is nonzero.

If N equals T, the geometric mean raised to the Nth power provides an unbiased estimator, but this is not surprising in that this estimator is merely one drawing from the distribution of N-period relatives. A single drawing from a distribution is of course an unbiased estimator of the mean. If N is greater than T, the estimator of  $E(W_N)$  provided by the geometric mean is biased upwards. Further, the estimate provided by the geometric mean is not consistent.

To measure the magnitude of the bias in the geometric mean,  $E[G^N]$  was evaluated numerically for the same

This statement is based on the following: taking the probability limit of (3.2), one obtains

 $plim G^N = \exp \left\{ plim \left( N/T \right) \sum_{i=1}^{T} \log R_i \right\}.$ 

The term in the braces is N  $E[\log R]$ , which is less than N  $\log \mu$  for nondegenerate distributions since the logarithmic function is concave. Thus,

$$plim G^N < \exp(N \log \mu) = \mu^N.$$

Of course, for fixed N, taking the probability limit implies that T > N.

A formula to calculate these values was obtained by taking expected values of (3.3) and rearranging terms to yield

$$E[G^N] = \mu^N E\{\prod_{i=1}^T [1 + (\epsilon_i/\mu)]^{NiT}\}.$$

Using the exponential function, this becomes

$$E[G^N] = \mu^N E\{\prod_{i=1}^T \exp\left((N/T)\log\left(1 + (\epsilon_i/\mu)\right)\right)\}.$$

On the assumption that  $\epsilon_i/\mu$  is less than one in absolute value, the logarithmic function can be expanded in an infinite series as

$$E[G^N] = \mu^N E\{\prod_{i=1}^T \exp((N/T) \sum_{j=1}^m [(-1)^{j+1}/j] (\epsilon_i/\mu)^j)\}.$$

The expansion of the exponential function yields

$$E[G^N] = \mu^N E\{\prod_{i=1}^T \left[ \sum_{i=0}^{\infty} \{(N/T) \sum_{j=1}^{\infty} \{(-1)^{j+j}/j\} (\epsilon_i/\mu)^j\}^j/i! \} \right].$$

<sup>&</sup>lt;sup>2</sup> The computational formula for  $E(A^N)$  was derived by first expanding (2.6) with the Binomial expansion and taking expected values. Noting that h is normally distributed, all odd moments in the expansion can be set to zero and  $E(h^i)$  for even i replaced by  $(i!)/[2^{i+2}(i/2)!][\sigma^2(h)]^{i/2}$ . After changing the index of summation and setting  $\sigma^2(h)$  to  $\sigma^2(s)/T$ , the resulting formula is

<sup>\*</sup>Some investigators (e.g., [7], [11]) prefer lognormal distributions on the assumption that the distribution of  $R_i$  would be skewed to the right. However, [3] finds no evidence of asymmetry in the distributions of  $R_i$  for monthly data. In fact for monthly data, there may well be no distinguishable empirical differences whether  $\ln R_i$  or  $R_i$  is used. For longer periods, saymmetry will become more pronounced (cf. [1]).

values of N, T, E(R), and  $\sigma(R)$  as for the arithmetic mean. These biases are sometimes substantial. For instance, for E(R) = 1.01 and  $\sigma(R) = 0.15$ , the expected 40-period relative estimated from 80 observations is 1.1880 compared to the population statistic of 1.4888. It may be recalled that the corresponding estimate provided by the arithmetic mean was 1.8416.

The analytical and the numerical results of this section and the previous one show that estimators of the expected N-period relative derived either from arithmetic means or geometric means of T observations may be substantially biased for distributions of relatives for common stocks and bonds. More specifically, for N less than T and N greater than one—a case of importance for empirical work, the arithmetic estimate of  $E(W_N)$  will be upward biased while the geometric estimate will be downward biased. Thus, an unbiased estimate of  $E(W_N)$  will be between the arithmetic and geometric estimates. The remainder of this article explores methods of obtaining unbiased estimates of  $E(W_N)$ .

Before proceeding, it may be worthwhile to record an explicit comparison for the case in which N equals one since a large number of empirical studies of stock market returns are based upon this case. The arithmetic mean provides an unbiased and consistent estimate of the expected one-period relative, while the geometric mean provides a biased and inconsistent estimate. Further, a formula to be developed in Section 4 can be used to show that the geometric mean has a larger sample variance than the arithmetic mean. It therefore appears that if one can assume that the relatives,  $R_t$ , are distributed by independent, stationary, normal distributions, the arithmetic mean provides a superior estimate of the expected one-period relative compared to that provided by the geometric mean.

#### 4. UNBIASED ESTIMATES

This section proposes four different methods of obtaining unbiased estimates of the expected N-period relative for N less than T.<sup>8</sup> Section 5 uses Monte Carlo techniques to obtain an insight into the distributional properties of these unbiased estimators as well as the

Noting that  $\alpha$  and  $\alpha$ ,  $k \neq l$ , are independent and the formula for the *i*th moment about the mean for a normal distribution, one can calculate the desired numbers to any degree of accuracy by tedious but straightforward numerical calculations for any specific values of the parameters.

$$G \approx \mu \{1 + (1/T) \sum_{i=1}^{T} ((\epsilon_i/\mu) - [(T-1)\epsilon_i^2/2T\mu^2])\}.$$

Subtracting E(G) given by the expression in Footnote 10 from the above, squaring, and taking expected values, one obtains

$$E[G - E(G)]^2 \approx [\sigma^2(a)/T] + [(T - 1)/2T\mu^2]E[(\Sigma_{i=1}^T a_i^2) - \sigma^2(a)]^2].$$

Since the first term on the right is the variance for the arithmetic mean, the geometric mean should have a somewhat larger variance that the arithmetic mean.

generally biased estimators provided by the arithmetic geometric means discussed previously.

The first type of estimator will be dubbed the "simple unbiased" estimator. This estimator is appropriate where the number of observations in the sample, T, is an integral multiple of the number of periods, N, for which the expected relative is calculated. To calculate this estimate, multiply the first N relatives together, the second N relatives, and so on until the T one-period relatives are exhausted. Then, average these products or N-period relatives, T/N in number, to obtain an unbiased estimate of the expected N-period relative. The reader should note that this procedure makes no assumptions about the independence of the distributions of the one-period relatives.

The second type of estimator, discussed in [6], will be called the "overlapped unbiased" estimator. This estimator proceeds by calculating N-period relatives. T-N+1 in number, by multiplying the first through the Nth one-period relatives together, the second through the (N+1)st one-period relatives together, and so on. These overlapped relatives are then averaged to obtain an unbiased estimate. Intuitively, some investigators might anticipate that this estimator would be more efficient than the previous one in that it incorporates somehow more information. Nonetheless, it is easy to construct a counter example which shows that it may be less efficient.9 Indeed, the Monte Carlo simulations in Section 5 will show for data likely to be observed in the stock market that the "overlapped unbiased" estimator is probably less efficient than the "simple unbiased" estimator.

The third type of estimator will be termed a "weighted unbiased" estimator because it is calculated as a weighted average of the biased estimators provided by the arithmetic and geometric means. Formulas developed earlier in the paper imply<sup>10</sup> that an approximately unbiased

10 To derive this weighted average, the expected value of the estimate of the N-period relative can be approximated from the formula in Footnote 1 by dropping all terms involving momenta of greater than the second order. The resulting approximation is

$$\begin{split} E[A^N] &\approx \mu^N + [N(N-1)\sigma^2(\epsilon)\mu^{N-2}/2T] \\ &= \mu^N \{1 + (N-1)[N\sigma^2(\epsilon)/2T\mu^2]\}. \end{split}$$

For the geometric mean, a similar type of approximation can be derived from the last equation in Footnote 5 by expanding it and then by dropping terms involving moments of e greater than the second order. The resulting approximation is

$$\begin{split} E[Q^N] \approx \mu^N \{1 - [N\sigma^2(\epsilon)/2\mu^2] + [N^2\sigma^2(\epsilon)/2T\mu^2] \} \\ &= \mu^N \{1 - (T-N)[N\sigma^2(\epsilon)/2T\mu^2] \}. \end{split}$$

23

<sup>&</sup>lt;sup>8</sup> A representative sample of such articles includes [2, 4, 7, and 10]. Each of these articles contain bibliographies which point to a large number of other articles.

<sup>&</sup>lt;sup>7</sup> This statement about efficiency is hased on a lengthy algebraic calculation for which an outline follows: remove the expected value operators from the last equation in Footnote 5 and set N equal to one. Dropping from the expansion of this equation all terms of degree greater than two and all terms involving cross products of  $\epsilon_i$  and  $\epsilon_i$ ,  $k \neq l$ , one obtains

 $<sup>^{3}</sup>$  The techniques are designed for N greater than one, although the first three reduce to the arithmetic mean when N equals one.

<sup>\*</sup> A counter example using four one-period relatives  $R_t$ ,  $t=1,\cdots,4$ , to estimate the expected two-period relative is as follows: since the Ri's are independent and stationary, the variance of the simple unbiased estimator will be  $o^{2}(R_{1}R_{2})/2$ , while the variance of the overlapped unbiased estimator is  $\sigma^2(R_1R_2)/3 + [4 \cos (R_1R_2)]$  $R_1R_1/9$ ]. Assuming  $E(R_1)$  equals 1.0,  $\sigma^2(R_1R_2)$  can be rewritten as  $[E(R_1^2)]^2-1$ and cov  $(R_1R_2, R_2R_3)$  as  $E(R_1^2) = 1$ . It is easily verified that for  $E(R_1^2)$  greater than 1.0 and less than 5/3, the variance of the simple unbiased estimator is less than the variance of the overlapped estimator. For example, if  $E(R_i^2) = 4/3$ , the variance of the simple average will be 21/54 compared to 22/54 for the overlapped average. It might be noted that Cheng and Deets [6] showed that the expected sample variance of an average calculated from a single drawing of two overlapped twoperiod relatives is smaller than that calculated from two nonoverlapped two-period relatives. Though correct, this observation about expected sample variances has no obvious implications for the efficiency of these two estimators which should be judged by a comparison of the population variances of the estimators and not by the variances calculated from single samples. The situation they analyzed is similar to a regression with autocorrelated residuals, for which it is well known that the expected standard error calculated from a single sample is downward biased.

N	Estimators	$\mu = 1.00, \ \sigma(R) = 0.03$				μ = 1.01, σ(R) = 0.15					
		Average	Standard Error	Fractiles				Fractiles			
				0.05	0.50	0.95	Average	Standard Error	0.05	0.50	0.95
5		1.0000					1.0510	فر			
•	ARITHMETIC	1.0005	0.0193	0.9734	1.3008		1.0564	0.0871	0 0101		
	GEOMETRIC	0.0003	0.0191	0.9712	0.9986	1.0275			0.9181	1.0553	1.2005
	SIMPLE UNBLASED	1.0005	0.0193	0.9733		1.0250	9,9991	0.0847	0.8618	0.9983	1,1424
	CVERLAPPED UNBLASED	1.0005	0.0192	0.9722	1.0008	1.0277	1.0545	0.0893	0.9149	1.0523	1.2038
	WEIGHTED UNGLASED	1.0004	0.0192	0.9722	1.0007	1.0292	1-0544	0.0898	0.9094	1.0524	1.2078
	ADJ. UNB. TEST S(R))		0.0192	0.9733	1.0007	1.0274	1-0535	0.0969	0.9160	1.0528	1.1970
	ADJ. UMB. (POP S(R))		0.0192	0.9732		1.0274	1.0532	0.0868	0.9159	1.0526	1.1964
	P034 0104 1119F 31M11	1.0004	0.0142	0.9132	1.0007	1.0274	1.0532	J.0368	0.9153	1.0521	1-1968
10		1.0000					1 1044				
	APITHMETIC	1.0014	0.0345	0.9474	1.0016	1.0553	1.1046	5.1844	0.0133		
	GEOMETRIC	0.9970	0.0343	0.9432	0.9973	1.0507	1.1237	0.1699	0.8433 0.7427	1.1137	1.4411
	SIMPLE UNBLASED	1.0011	0.0346	7.9465	1.0009	1.0560	1.1134	0.1079	9.8257	0.9966 1.1043	1.3050
	OVERLAPPED UNBLASED	1.0009	3.0357	3.9438	1.0012	1.0584	1.1115	9.1962	0.8132	1.1020	1.4591
	HEIGHTED UNBIASED	1.0009	0.0345	0.9470	1.0013	1.0552	1.1102	7.1827	0.8331	1.1023	1.4401
	ADJ. UNB. (EST S(P))	1.0009	0.0345	0.9470	1-0013	1.0552	1.1796	0.1821	0.8335	1.1016	1.4227
	ADJ. UNB. (PDP S(P))	1.0009	7.0344	0.9469	1.0011	1.0553	1.1096	0.1821	0.8324	1.0997	1.4210
			33.3.7	347407	110011	1.0333	171040	2.1071	0.0324	1.0997	1.4231
20		1.0000					1.2202				
GEDMET SIMPLS SIMPLS OVERLA MEIGHT LOA	ARITHMETIC	1-0041	0.0672	0.8976	1.0033	1.1147	1. 2966	0.4303	0.7106	1.2402	2.0769
	GEOMFTR I C	0.9957	0.0667	0.8896	0.9945	1.1040	1.0397	0.3553	0.5516	0.9933	1.7030
	SIMPLE UNBIASED	1.0022	0.0673	0.8956	1.0009	1.1150	1.2411	0.4508	0.6344	1.1813	2.0488
	OVERLAPPED UNBIASED	1.0016	0.0736	J.8863	1.0010	1.1239	1.2332	0.4802	0.6136	1.1519	2.0897
	MEIGHTED UMBIASED	1.0019	0.0671	3.8960	1.0015	1.1121	1.2348	0.4119	0.0743	1.1805	1.9732
	ADJ. UNB. TEST SIRTI	1.0020	0.0670	9-8961	1.0015	1.1121	1.2313	0.4087	0.6747	1.1786	1.9664
	ADJ. UNB. (POP S(R))	1.0070	0.0671	0.8957	1.0012	1.1124	1.2317	0.4085	0.6747	1.1775	1.9718
40		1.0000					1.4889				
••	ARITHMETIC	1.0127	0.1347	9.8057	1.0066	1.2425	1.8661	1.3247	0.5050	1 5700	
	GEOMETRIC	0.9949	0.1324	0.7915	0.9892	1.2188	1.2070	0.9818	0.3042	1.5382	4.3135
	SIMPLE UNRIASED	1-0040	3.1342	0.7964	0.9999	1.2305	1.5147	1.2007	0.3528	0.9866	2.9001
	DVERLAPPED UNBLASED	1.0040	0.1567	0.7691	0.9929	1.2720	1.5282	1.4104		1.2255	3.6189
	WEIGHTED UNBIASED	1.0039	3.1336	0.7994	0.9992	1.2306	1.5407	1.1045	0.2863	1.1472	3.8228
	ADJ. UNB. (EST SIRI)	1.0040	0.1335	3.7997	0.9994	1.2337	1.5093	1.0699	0.4115	1.2691	3.5893
	ADJ. HNB. (POP S(R))	1.0940	0.1335	0.7988	0.9980	1.2319	1.5070	1.0698	0.4978		3.5213
	· · · · · · · · · · · · · · · · · ·	,0		401700	V4 970U	4.6217	1+20+0	1.0070	0.4510	1.2422	3.4836

estimator of  $E(W_N)$  is given by the weighted average:

$$\hat{E}(W_N) \approx \frac{T-N}{T-1}A^N + \frac{N-1}{T-1}G^N.$$
 (4.1)

The coefficients of  $A^N$  and of  $G^N$  in (4.1) sum to one and can be used to form a weighted average of the estimates of  $\mu$  provided by the arithmetic and geometric means. These weights which are functions of T and N make intuitive sense. When N equals one, all the weight is given to the arithmetic mean. When N equals T, all the weight is given to the geometric mean. As N drops from T, more and more weight is given to the arithmetic mean and less to the geometric mean. Since the arithmetic mean is consistent while the geometric mean is not, the weighting is sensible.

The fourth type of estimator adjusts  $A^N$  with an appropriate adjustment factor. This estimator will be termed the "adjusted unbiased" estimator. For monthly data from the bond or stock markets,  $\mu$  is likely to be in the interval from 1.00 to 1.01 while  $\sigma(R)$  may range as high as 0.15. For these ranges of  $\mu$  and  $\sigma(R)$  and for

Solving one of these equations for  $[N\sigma^2(\epsilon)]/[2T\mu^2]$  and substituting the resulting expression into the other, one obtains by solving for  $\mu^N$  the basis for the expression in the text. Although this development involves substituting approximations into approximations—a treacherous procedure, the absolute bias in the weighted estimator for 1 < N < T will always be less than the absolute maximum of the biases in  $A^N$  or  $G^N$ .

 $N \le 80$ ,  $T \le 100$ , and  $N \le T$ , the following regression which does not include  $\mu$  as an independent variable fits the bias calculations extremely well:<sup>12</sup>

$$\ln \left\{ \frac{[E(A^N)]^{1/N} - \mu}{\mu} \right\} = -0.9174 + 1.9958 \ln \sigma(R) + 1.0441 \ln N - 0.9989 \ln T, \bar{R}^2 = 0.9990. \quad (4.2)$$

The values of  $[E(A^N)]^{1/N}$ , implicitly defined by (4.2), differ in absolute values from their true values by 1.1 percent on average and by 4.2 percent at most.<sup>13</sup> Using the sample estimate to measure  $\sigma(R)$ , (4.2) implies for any particular N and T a value of the ratio of  $E(A^N)$  to  $\mu^N$ . Dividing this ratio into  $A^N$  should yield an approximately unbiased estimator of  $E(W_N)$ . Section 5 will examine the degree of approximation introduced by using a sample value of  $\sigma(R)$  instead of the population value.

#### 5. A MONTE CARLO ANALYSIS

To examine the empirical properties of these various estimators of the expected N-period relative, 80,000 randomly distributed unit normal variates were calculated using the procedure found in [14]. These variates,

<sup>&</sup>quot;This technique is not the same as employed in [8].

<sup>12</sup> This regression was fitted for E(R) equal to 1.000, 1.005, and 1.010, for  $\sigma(R)$  equal to 0.030, 0.060, 0.100, and 0.150 and for N running from 10 to 80 and from 10 to 100 in increments of 10.

<sup>12</sup> Including  $\mu$  as an explanatory variable increased the value of  $\tilde{R}^2$  to 0.99994.

reexpressed so as to have appropriate values of E(R) and  $\sigma(R)$ , were partitioned sequentially into 1,000 separate samples of 80 observations to correspond to a T of 80.

For each sample,  $A^N$ ,  $G^N$ , and the four unbiased estimators just discussed were obtained for N equals 5, 10, 20, and 40. The adjusted unbiased estimator was calculated using both the estimated and population values of  $\sigma(R)$ . Although in any application only the estimated value could be used, a comparison of the two estimates will indicate the magnitude of the error introduced by using an estimate rather than the population value.<sup>14</sup>

The table gives descriptive statistics of the distributions of the various estimates for two cases: (a)  $\mu = 1.00$ and  $\sigma(R) = 0.03$  and (b)  $\mu = 1.01$  and  $\sigma(R) = 0.15$ . A comparison of the arithmetic and geometric means of T one-period relatives raised to the Nth power to the population statistic,  $\mu^N$ , show that the estimates are biased in the anticipated directions. The simple unbiased estimate, as well as the overlapped estimate, are very close to the population statistic as would be expected. The averages for the three remaining estimators show that any errors introduced into the estimates because of the approximations used in deriving the formulas do not create any substantial biases. Further, a comparison of the two estimates provided by the adjusted unbiased estimator show that little error is introduced in using an estimate of  $\sigma(R)$  instead of the population value in calculating the adjustment factor.

The figures in the table additionally suggest that the overlapped unbiased estimator is markedly less efficient (say as measured by the standard deviation or the 0.05 and 0.95 fractiles) than the other unbiased estimators. In addition, the simple unbiased estimator appears somewhat less efficient than both the weighted unbiased estimator and the adjusted unbiased estimator, where the adjustment factor is estimated with the sample value of  $\sigma(R)$ . Finally, the reader may note that the sample distributions are skewed to the right with the skewness more pronounced for the case in which E(R) = 1.010 and  $\sigma(R) = 0.150$ .

#### 6. CONCLUSION

The theoretical and empirical results of this article suggest that one should proceed very cautiously in using arithmetic or geometric means of one-period relatives to assess the expected N-period relatives. More explicitly, an estimate of the expected N-period relative derived by raising either of these statistics to the Nth power would usually be biased.

If one can assume that the one-period relatives are distributed by an independent normal process, the article shows, for data like those which might be encountered in the stock or bond markets, that an average of overlapped data may be much less efficient than merely a simple average of nonoverlapped data. The article then goes on to suggest two nonlinear methods of assessing unbiased estimates which appear somewhat more efficient than a simple average of N-period relatives: (1) a weighted unbiased estimator and (2) an adjusted unbiased estimator. Although there is little difference in efficiency between the weighted unbiased estimator and the adjusted estimator, the weighted unbiased estimator is probably safer to use than the other. One could easily visualize types of departures from stationary independent normal distributions which might lead to absurd estimates from the adjusted unbiased estimator but not from the weighted unbiased estimator. Yet, if one cannot assume independence of successive one-period relatives or if there is even a slight chance that these relatives are dependent, the simple average of N-period relatives would appear preferable to the nonlinear estimators which, even under ideal conditions, yield only a modest increase in efficiency.

[Received October 1972. Revised September 1973.]

#### REFERENCES

- Arditti, Fred D., "Risk and the Required Return on Equity," Journal of Finance, 22 (March 1967), 19-36.
- [2] Black, Fisher, Jensen, Michael and Scholes, Myron, "The Capital Asset Pricing Model: Some Empirical Tests," in Michael C. Jensen, ed., Studies in the Theory of Capital Markets, New York: Praeger Publishing, 1973.
- [3] Blume, Marshall, "Portfolio Theory: A Step Towards Its Practical Application," Journal of Business, 43 (April 1970), 152-72
- [4] —— and Friend, Irwin, "A New Look at the Capital Asset Pricing Model," Journal of Finance, 28 (March 1973), 19-33.
- [5] —— and Friend, Irwin, "Risk, Investment Strategy and the Long-Run Rates of Return," The Review of Economics and Statistics, 56 (August 1974).
- [6] Cheng, Pao L. and Deets, M. King, "Statistical Biases and Security Rates of Return," Journal of Financial and Quantilative Analysis, 6 (June 1971), 977-94.
- [7] Fama, Eugene, et al., "Adjustment of Stock Prices to New Information," International Economic Review, 10 (February 1969), 1-21.
- [8] Fisher, Lawrence, "Some New Stock Market Indexes," Journal of Business, 39 (January 1966), 191-225.
- [9] Hoel, Paul G., Introduction to Mathematical Statistics, New York: John Wiley and Sons, Inc., 1958.
- [10] Jensen, Michael, "Risk, the Pricing of Capital Assets, and the Evaluation of Investment Portfolios," Journal of Business, 42 (April 1969), 167-247.
- [11] Osborne, M.F.M., "Brownian Motion in the Stock Market," Operations Research, 7 (March-April 1959), 145-73.
- [12] Rie, Daniel, "Single Parameter Risk Measures and Multiple Sources of Risk: A Reexamination of the Data Based on Changes in Determinants of Price Over Time," Rodney L. White Center for Financial Research, Working Paper No. 14-72.
- [13] Samuelson, Paul, "Lifetime Portfolio Selection by Dynamic Stochastic Programming," The Review of Economics and Statistics, 51 (August 1969), 239-46.
- [14] "System/360 Scientific Subroutine Package: Version III: Programmer's Manual," International Business Machines Corporation, August 1970.

<sup>&</sup>quot;Similarly, estimates of E(Wn) were obtained for N equals 5, 10, and 20 and T equals 40 from the first half of each of the 1,000 samples. For reasons of space, these are not presented. They, however, give substantially the same conclusions.

Response of: Entergy Gulf States, Inc. to the Second Set of Data Requests

of Requesting Party: TIEC

Prepared By: Counsel Sponsoring Witness: N/A Beginning Sequence No.

Ending Sequence No.

Ouestion No.: TIEC 2-26

Part No.:

Addendum:

**Question:** 

The following RFIs relate to the Jurisdictional Separation Plan ("JSP") referred to in EGSI's testimony and in Application of Entergy Gulf States, Inc., Entergy Gulf States Louisiana, LLC, and Entergy Texas, Inc., For Authorization to Implement Jurisdictional Separation Plan, Federal Energy Regulatory Commission ("FERC") Docket No. EC07-66-000. Reference to "EGS-TX" refers to Entergy Texas, Inc. as described in FERC Docket No. EC07-66-000 or the Entergy Corp. affiliate or subsidiary that will serve EGSI's customers in Texas upon jurisdictional separation.

At what step of the JSP will EGS-TX become an electric utility under Texas law?

Response:

EGS-TX will become an electric utility under Texas law upon its creation.

